Homework 6 – Due Friday, 03/29

1. Evaluate the following expressions:

$$\sum_{i=-2}^{2}(i^2+1) \qquad \sum_{i=1}^{3}\sum_{j=1}^{3}(3i+j) \qquad \prod_{i=1}^{3}\sum_{j=1}^{3}(ij)$$

2. For each of the following claims:

   - Re-state the claim using summation notation, if applicable.
   - Prove the claim by induction. Be sure to carefully show all of the following steps:
     - Assert that you are inducting on a particular variable.
     - State the element for which the base case applies, and prove it.
     - State the inductive hypothesis
     - Label the inductive step and state what you must show
     - Prove the inductive step, being careful to label the point at which the inductive hypothesis is being applied.

   a. Claim: For all $n > 0$: $2 + 4 + 6 + \cdots + 2n = n^2 + n$

   b. Claim: For all $n \geq 3$: $4^3 + 4^4 + 4^5 + \cdots + 4^n = 4(4^n - 16)/3$

   c. Claim: For all $n \geq 1$: $1^2 + 2^2 + 3^2 + \cdots + n^2 = \frac{n(n+1)(2n+1)}{6}$

   d. Claim: For all $n \geq 0$: $6 \mid 7^n - 1$

   e. Claim: For all $n \geq 4$: $2^n < n!$

   f. Claim: For all $n \geq 1$:

$$\sum_{i=1}^{n}\frac{1}{i^2} \leq 2$$

   Hint: This proof is much easier if you first prove that (for all $n \geq 1$):

$$\sum_{i=1}^{n}\frac{1}{i^2} \leq 2 - \frac{1}{n}$$