Homework 7 – Due Friday, April 5th

1. Prove each of the following claims using strong induction. Be sure to:

    - Assert that you are invoking strong induction on a particular variable.
    - State the element(s) for which the base case(s) apply, and prove them.
    - Carefully state the inductive hypothesis. (Be sure to follow the examples from class!)
    - Label the inductive step and state what you must show
    - Prove the inductive step, being careful to label the point at which the inductive hypothesis is being applied.

    a. Let a_n be the recurrence defined by: $a_0 = 12, a_1 = 29$, and for all $n \geq 2, a_n = 5a_{n-1} - 6a_{n-2}$. Prove that for all $n \geq 0$: $a_n = 5 \cdot 3^n + 7 \cdot 2^n$

    b. Let a_n be the recurrence defined by: $a_0 = 1, a_1 = 2, a_2 = 3$, and for all $n \geq 3, a_n = a_{n-1} + a_{n-2} + a_{n-3}$. Prove that for all $n \geq 0$: $a_n \leq 3^n$

    c. Let a_n be the recurrence defined by: $a_1 = 1, a_2 = 3$, and for all $n \geq 3, a_n = a_{n-1} + a_{n-2}$. Prove that for all $n \geq 1$: $a_n \leq \left(\frac{7}{4}\right)^n$

    d. Prove that if you only have 3 cent and 5 cent coins available, you can generate any amount of money that is greater than or equal to 13 cents.

2. Let a_n be the recurrence defined by: $a_0 = 4, a_1 = 7$, and for all $n \geq 2, a_n = 2a_{n-1} + 5a_{n-2}$. Using constructive induction, find integer constants A and B such that for all $n \geq 0, a_n \leq AB^n$. Try to make B as small as possible.