Reminders

• Where is the class webpage?
  – Announcements
  – Syllabus
  – Lecture Slides
  – Office hours schedule
  – Lecture Examples

• All students must attend discussion for which they are officially registered
• You are expected to attend every class session
• No electronic devices during class
• Convention for order of variables/interpretations in truth tables
Logical Equivalence

Recall: Two statements are **logically equivalent** if they have the same truth values for every possible interpretation.

Notation:
\[ p \equiv \sim \sim p \]

How can we check whether or not two statements are logically equivalent?

Example:
\[ \sim (p \lor \sim q) \lor (\sim q \land \sim p) \equiv? \sim p \]
Logical Equivalence

Show that \((p \land q \land r \land s) \lor u\) is \textbf{not} logically equivalent to \((p \land q \land r) \land (s \lor u)\)
Tautology and Contradiction

• A statement is a tautology if it is true under every possible interpretation.

• A statement is a contradiction if it is false under every possible interpretation.

• Examples
# Equivalencies in Propositional Logic

Given any statement variables $p$, $q$, and $r$, a tautology $t$ and a contradiction $c$, the following logical equivalences hold:

<table>
<thead>
<tr>
<th></th>
<th>Commutative laws:</th>
<th>$p \land q \equiv q \land p$</th>
<th>$p \lor q \equiv q \lor p$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.</td>
<td>Associative laws:</td>
<td>$(p \land q) \land r \equiv p \land (q \land r)$</td>
<td>$(p \lor q) \lor r \equiv p \lor (q \lor r)$</td>
</tr>
<tr>
<td>2.</td>
<td>Distributive laws:</td>
<td>$p \land (q \lor r) \equiv (p \land q) \lor (p \land r)$</td>
<td>$p \lor (q \land r) \equiv (p \lor q) \land (p \lor r)$</td>
</tr>
<tr>
<td>3.</td>
<td>Identity laws:</td>
<td>$p \land t \equiv p$</td>
<td>$p \lor c \equiv p$</td>
</tr>
<tr>
<td>4.</td>
<td>Negation laws:</td>
<td>$p \lor \sim p \equiv t$</td>
<td>$p \land \sim p \equiv c$</td>
</tr>
<tr>
<td>5.</td>
<td>Double negative law:</td>
<td>$\sim(\sim p) \equiv p$</td>
<td></td>
</tr>
<tr>
<td>6.</td>
<td>Idempotent laws:</td>
<td>$p \land p \equiv p$</td>
<td>$p \lor p \equiv p$</td>
</tr>
<tr>
<td>7.</td>
<td>DeMorgan’s laws:</td>
<td>$\sim(p \land q) \equiv \sim p \lor \sim q$</td>
<td>$\sim(p \lor q) \equiv \sim p \land \sim q$</td>
</tr>
<tr>
<td>8.</td>
<td>Universal bounds laws:</td>
<td>$p \lor t \equiv t$</td>
<td>$p \land c \equiv c$</td>
</tr>
<tr>
<td>9.</td>
<td>Absorption laws:</td>
<td>$p \lor (p \land q) \equiv p$</td>
<td>$p \land (p \lor q) \equiv p$</td>
</tr>
<tr>
<td>10.</td>
<td>Negations of $t$ and $c$:</td>
<td>$\sim t \equiv c$</td>
<td>$\sim c \equiv t$</td>
</tr>
</tbody>
</table>

- You don’t need to memorize this
- Posted on class webpage (under “resources”)
- We can substitute long expressions for the variables above
- Let’s derive a few of these with truth tables
Simplifying Using the “Laws”

Caution:

- Don’t confuse variables in the chart with variables you have defined
- Don’t skip any steps!
  - Double negation
  - Commutativity

Let’s use the “Laws of Equivalence” to simplify this sentence:

\(~(\sim p \land q) \land (p \lor \sim q)\)
Deriving Equivalencies Using “Laws”

Previously, we showed (using a truth table):

\[ \neg(p \lor \neg q) \lor (\neg q \land \neg p) \equiv \neg p \]

Let’s now demonstrate this equivalency a different way by using the established “laws” of equivalence.

There are many different ways to prove this using the “laws”!
Conditional Connective

\( p \rightarrow q \) represents “If p then q” or “p implies q”

- If \( p \) is true then \( q \) must also be true
- If \( p \) is false then \( q \) could be either true/false

<table>
<thead>
<tr>
<th>( p )</th>
<th>( q )</th>
<th>( p \rightarrow q )</th>
</tr>
</thead>
<tbody>
<tr>
<td>T</td>
<td>T</td>
<td>T</td>
</tr>
<tr>
<td>T</td>
<td>F</td>
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<td>F</td>
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<td>T</td>
</tr>
</tbody>
</table>

- Note: precedence is lower than conjunction/disjunction
- Examples translating from English
- Differences between logical connective and everyday English
More Equivalencies

Add this rule to your table of “laws”:

12. [Defn of $\rightarrow$] $p \rightarrow q \equiv \sim p \lor q$

Let’s prove this:

$\sim(p \rightarrow q) \equiv p \land \sim q$
Definitions for Conditional Statements

The **converse** of \( p \rightarrow q \) is \( q \rightarrow p \)
The **inverse** of \( p \rightarrow q \) is \( \sim p \rightarrow \sim q \)
The **contrapositive** of \( p \rightarrow q \) is \( \sim q \rightarrow \sim p \)

- Examples from English

- Is an implication logically equivalent to its inverse, its converse, or its contrapositive? Let’s check!

- Given an implication, what is the relationship between its converse and its inverse?
“If and Only If”

Biconditional Connective: \( p \leftrightarrow q \)

“\( p \) if and only if \( q \)”

<table>
<thead>
<tr>
<th>p</th>
<th>q</th>
<th>( p \leftrightarrow q )</th>
</tr>
</thead>
<tbody>
<tr>
<td>T</td>
<td>T</td>
<td>T</td>
</tr>
<tr>
<td>T</td>
<td>F</td>
<td>F</td>
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<tr>
<td>F</td>
<td>F</td>
<td>T</td>
</tr>
</tbody>
</table>

Add this to your table of “laws”:

13. [Defn of \( \leftrightarrow \)] \( p \leftrightarrow q \equiv (p \rightarrow q) \land (q \rightarrow p) \)
Arguments

An argument is a conjecture that says:
If you make certain assumptions, then a particular statement must follow.

- The assumptions are called premises
- The statement that (supposedly) follows is the conclusion

Example:

\[ p \lor q \]
\[ q \rightarrow r \]
\[ \sim p \]
\[ \therefore r \]

Premises
Conclusion