Reminders

- Where is the class webpage?
  - Announcements
  - Syllabus
  - Lecture Slides
  - Office hours schedule
  - Lecture Examples

- All students must attend discussion for which they are officially registered
- You are expected to attend every class session
- No electronic devices during class
- Convention for order of variables/interpretations in truth tables

Logical Equivalence

Recall: Two statements are **logically equivalent** if they have the same truth values for every possible interpretation.

Notation:

$p \equiv {\sim}{\sim}p$

How can we check whether or not two statements are logically equivalent?

Example:

${\sim}(p \vee {\sim}q) \vee ({\sim}q \wedge {\sim}p) \overset{?}{\equiv} {\sim}p$

Logical Equivalence

Show that $(p \wedge q \wedge r \wedge s) \vee u$ is **not** logically equivalent to $(p \wedge q \wedge r) \wedge (s \vee u)$

Tautology and Contradiction

- A statement is a **tautology** if it is *true* under every possible interpretation.
- A statement is a **contradiction** if it is *false* under every possible interpretation.
- Examples

Equivalencies in Propositional Logic

Given any statement variables p, q, and r, a tautology t and a contradiction c, the following logical equivalences hold:		
1. Commutative laws:	$p \wedge q \equiv q \wedge p$	$p \vee q \equiv q \vee p$
2. Associative laws:	$(p \wedge q) \wedge r \equiv p \wedge (q \wedge r)$	$(p \vee q) \vee r \equiv p \vee (q \vee r)$
3. Distributive laws:	$p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$	$p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r)$
4. Identity laws:	$p \wedge t \equiv p$	$p \vee c \equiv p$
5. Negation laws:	$p \vee {\sim}p \equiv t$	$p \wedge {\sim}p \equiv c$
6. Double negative law:	${\sim}({\sim}p) \equiv p$	
7. Idempotent laws:	$p \wedge p \equiv p$	$p \vee p \equiv p$
8. DeMorgan's laws:	${\sim}(p \wedge q) \equiv {\sim}p \vee {\sim}q$	${\sim}(p \vee q) \equiv {\sim}p \wedge {\sim}q$
9. Universal bounds laws:	$p \vee t \equiv t$	$p \wedge c \equiv c$
10. Absorption laws:	$p \vee (p \wedge q) \equiv p$	$p \wedge (p \vee q) \equiv p$
11. Negations of t and c:	${\sim}t \equiv c$	${\sim}c \equiv t$

- You don’t need to memorize this
- Posted on class webpage (under “resources”)
- We can substitute long expressions for the variables above
- Let’s derive a few of these with truth tables

Simplifying Using the “Laws”

Caution:

- Don’t confuse variables in the chart with variables you have defined
- **Don’t skip any steps!**
  - Double negation
  - Commutativity

Let’s use the “Laws of Equivalence” to simplify this sentence:

$\sim(\sim p \wedge q) \wedge (p \vee \sim q)$

Deriving Equivalencies Using "Laws"

Previously, we showed (using a truth table):

$$\sim(p \vee \sim q) \vee (\sim q \wedge \sim p) \equiv \sim p$$

Let's now demonstrate this equivalency a different way by using the established "laws" of equivalence.

There are many different ways to prove this using the "laws"!

Conditional Connective

$p \rightarrow q$ represents "If p then q" or "p implies q"

- If p is true then q must also be true
- If p is false then q could be either true/false

p	**q**	**$p \rightarrow q$**
T	T	T
T	F	F
F	T	T
F	F	T

- Note: precedence is **lower** than conjunction/disjunction
- Examples translating from English
- Differences between logical connective and everyday English

More Equivalencies

Add this rule to your table of “laws”:

12. [Defn of $\rightarrow$] $\quad \mathbf{p \rightarrow q \equiv {\sim}p \vee q}$

Let’s prove this:

$\sim(p \rightarrow q) \equiv p \wedge \sim q$

Definitions for Conditional Statements

The **converse** of $p \rightarrow q$ is $q \rightarrow p$

The **inverse** of $p \rightarrow q$ is $\sim p \rightarrow \sim q$

The **contrapositive** of $p \rightarrow q$ is $\sim q \rightarrow \sim p$

- Examples from English

- Is an implication logically equivalent to its inverse, its converse, or its contrapositive? Let's check!

- Given an implication, what is the relationship between its converse and its inverse?

"If and Only If"

Biconditional Connective: **$p \leftrightarrow q$**

"p if and only if q"

p	q	$p \leftrightarrow q$
T	T	T
T	F	F
F	T	F
F	F	T

Add this to your table of "laws":

13. [Defn of $\leftrightarrow$] $\quad$ $p \leftrightarrow q \equiv (p \rightarrow q) \wedge (q \rightarrow p)$

Arguments

An **argument** is a conjecture that says:

If you make certain assumptions, then a particular statement must follow.

- The assumptions are called **premises**
- The statement that (supposedly) follows is the **conclusion**

Example:

$$\left.\begin{array}{l} p \vee q \\ q \to r \\ \sim p \end{array}\right\} \text{Premises}$$

$$\therefore\ r \qquad \text{Conclusion}$$