Announcements

- Homework #4 has been posted.
- Midterm #1 is on Tuesday, 3/12

Quotient-Remainder Theorem

$(\forall a \in Z)(\forall n \in Z^+)(\exists q,r \in Z)[(a = qn + r) \wedge (0 \leq r < n)]$

[Typically: a is larger than n, and a is being "divided" by n]

Examples.

Quotient-Remainder Theorem and Proof by Cases

Common proof technique:

Claim: : **($\forall x \in \mathbb{Z}$) ["Something about mod 7"].**
Proof:

Let $a \in \mathbb{Z}$, selected arbitrarily.
By the QRT, there exists $k \in \mathbb{Z}$ satisfying one of these cases:
case $a = 7k$:

case $a = 7k + 1$:

case $a = 7k + 2$:

case $a = 7k + 3$:

case $a = 7k + 4$:

case $a = 7k + 5$:

case $a = 7k + 6$:

These cases are exhaustive…

Using Quotient Remainder Theorem

- Claim: For all n, $2n^2 + 3n + 2$ is not divisible by 5
- Claim: $(\forall n \in Z)\ [3 \nmid n \rightarrow n^2 \equiv_3 1]$

Now we can prove…

Claim: For all a, b$\in\mathbb{N}$, the following are equivalent:

1. $a \equiv_n b$
2. $n|(a - b)$
3. $(\exists k \in \mathbf{Z})\ [a = b + kn]$

[How do we prove several things are equivalent?]

Floor and ceiling

- Definitions:
  - For all $x \in \mathbf{R}$, $n \in \mathbf{Z}$

    $$\lfloor x \rfloor = n \leftrightarrow n \leq x < n+1$$

  - For all $x \in \mathbf{R}$, $n \in \mathbf{Z}$

    $$\lceil x \rceil = n \leftrightarrow n-1 < x \leq n$$

Proofs with Floor/Ceiling

Claim: $(\forall x \in \mathbf{R})(\forall y \in Z)[\lfloor x+y \rfloor = \lfloor x \rfloor + y]$

Claim: The floor of (n/2) is either

a) n/2 when n is even

b) (n–1)/2 when n is odd

Unit 6

Review of Sequences, Summations and Products

Practice Finding an explicit formula

- Figure out formula for this sequence:

$$1, -\frac{1}{4}, \frac{1}{9}, -\frac{1}{16}, \frac{1}{25}, \ldots$$

$a_i = ?$

Summation & product notation

- Sum of items specified

$$\sum_{k=1}^{6} 2^k = 2^1 + 2^2 + 2^3 + 2^4 + 2^5 + 2^6$$

- Product of items specified

$$\prod_{k=1}^{5} 2k = 2(1) * 2(2) * 2(3) * 2(4) * 2(5)$$

Variable ending point

- n as the index of the final term

$$\sum_{k=0}^{n} \frac{k+1}{n+k}$$

- Evaluate for for n = 2

Nesting of sum/product notation

- Variations (same or different??):

$$\sum_{j=1}^{n} \sum_{i=1}^{m_j} Y_{ij}^{\ 2} \qquad \sum_{j=1}^{n} (\sum_{i=1}^{m_j} Y_{ij})^2 \qquad (\sum_{j=1}^{n} \sum_{i=1}^{m_j} Y_{ij})^2$$

Telescoping series

$$\sum_{k=1}^{n}\left(\frac{k}{k+1}-\frac{k+1}{k+2}\right)$$

$$\prod_{i=1}^{n}\left(\frac{i}{i+1}\right)$$

Properties

- Merging and splitting

$$\sum_{k=m}^{n} a_k + \sum_{k=m}^{n} b_k = \sum_{k=m}^{n} (a_k + b_k)$$

$$\sum_{k=m}^{n} a_k = \sum_{k=m}^{i} a_k + \sum_{k=i+1}^{n} a_k$$

$$\prod_{k=m}^{n} a_k * \prod_{k=m}^{n} b_k = \prod_{k=m}^{n} (a_k * b_k)$$

$$\prod_{k=m}^{n} a_k = \prod_{k=m}^{i} a_k * \prod_{k=i+1}^{n} a_k$$

Properties, con't.

- Distribution

$$c * \sum_{k=m}^{n} a_k = \sum_{k=m}^{n} (c * a_k)$$

Factorial

- $n! = n * (n - 1) * (n - 2) * \ldots * 2 * 1$

- Definition:

  $0! = 1$

  $n! = n * (n - 1)!$