Announcements

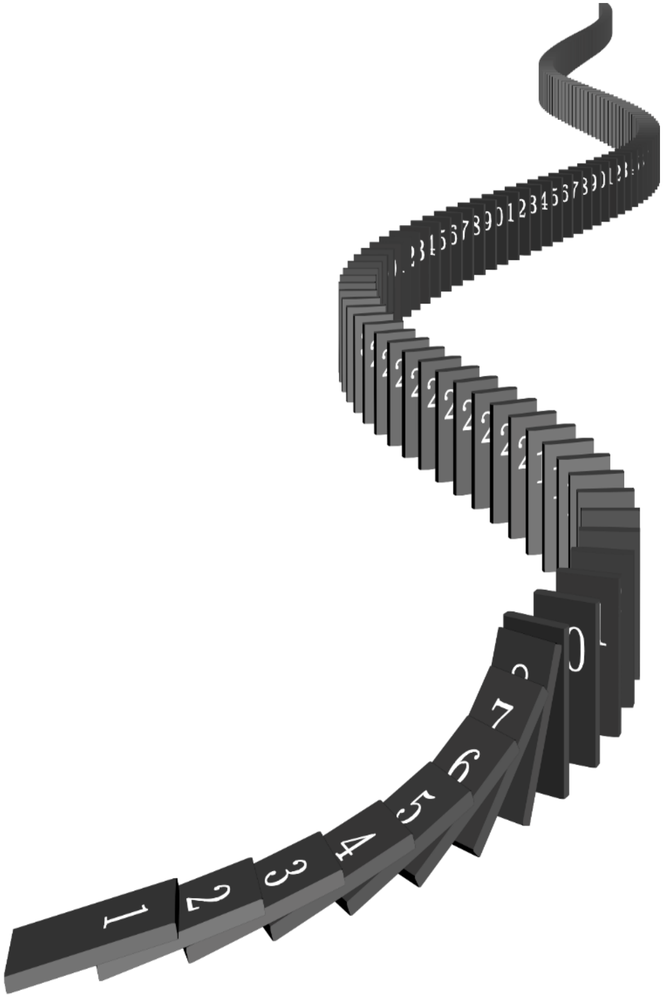
Homework #4 is due tomorrow

Homework #5 posted

- We made not grade every problem
- In Gradescope each problem says “0 points”

Unit 7
Induction

Basic Idea



1. I know that the FIRST domino falls (because I am knocking it over).
2. I can prove that if any particular domino falls, then the very next one must also fall.

What can I conclude?

Simple Induction

Claim: ($\forall n \in \mathbb{N}$) [P(n)].
Proof:

I will induct on n.

Base case: Show P(0) directly**.** (Usually obvious.)

Inductive Hypothesis: Assume P(k) is true, for some $k \in \mathbb{N}$

Inductive Step: Prove P(k+1) must also be true, based on your assumption that P(k) is true.

Domino "Proof"

Claim: **($\forall n \in \mathbb{N}^{>0}$) [Domino #n will fall].**

Proof:

I will induct on n.

Base case: Domino #1 will fall because I push it over.

Inductive Hypothesis: Assume Domino k falls, for some $k \in \mathbb{N}^{>0}$

Inductive Step: Since Domino k is falling, it will strike Domino k+1, knocking it over ('cause that's how physics works.)

Simple Example

Recall the Modular Arithmetic Theorem:

Let a, b, c, d, n $\in$ Z, and n > 1. Suppose $a \equiv_n c$ and $b \equiv_n d$. Then:

1. $(a + b) \equiv_n (c + d)$
2. $(a – b) \equiv_n (c – d)$
3. $ab \equiv_n cd$
4. $a^m \equiv_n c^m$ for all natural numbers m

- Let's prove #4!

Another example with modular congruence

$$(\forall n \in N)[n^3 \equiv_3 n]$$

Examples with Summations

- Claim: $(\forall n \geq 1)\left[\sum_{i=1}^{n} 4i - 2 = 2n^2\right]$
- Claim: $(\forall n \geq 1)\left[\sum_{i=1}^{n} i = \frac{n(n+1)}{2}\right]$
- Claim: $(\forall n \geq 0)\left[\sum_{i=0}^{n} 2^i = 2^{n+1} - 1\right]$

Another example- geometric series

$$(\forall r \in R^{>1})(\forall n \in Z^{\geq 0})\left[\sum_{k=0}^{n} r^k = \frac{r^{n+1}-1}{r-1}\right]$$