Announcements

- Homework #5 due Friday
- Homework #6 will be assigned tomorrow, but not due until Friday 3/29

An example with an inequality

Claim:

$$(\forall n \in Z^{\geq 3})\left[2n + 1 < 2^n\right]$$

Another example with an inequality

$$(\forall n \in Z^{\geq 2})(\forall x \in R^{+})\left[1 + nx \leq (1+x)^{n}\right]$$

A less-mathematical example

- Claim: If all we had was 2-cent coins and 5-cent coins, we could form any value greater than 3 cents.

An example with a recurrence relation

- Assume the following definition of a sequence:

$$a_1 = 1$$

$$(\forall k \geq 2)\left[a_k = a_{k-1} + (2k-1)\right]$$

- Let's write out a few terms…

- Prove : $$(\forall n \geq 1)\left[a_n = n^2\right]$$

Recurrence relation and summation together

- Assume the following definition of a sequence:

$$a_0 = 1$$

$$\text{For } n \geq 1:\ a_n = \left[\sum_{i=0}^{n-1} a_i\right] + 1$$

- Let's write out a few terms…
- Prove :

$$(\forall n \geq 0)\left[a_n = 2^n\right]$$

Another recurrence to consider

- Assume the following definition of a function:

$$a_0 = 1 \qquad a_1 = 1 \qquad a_2 = 3$$

$$(\forall n \in Z^{\geq 3})\left[a_n = a_{n-1} + a_{n-2} - a_{n-3}\right]$$

:

- Let's write out a few terms

Claim: $$(\forall n \in Z^{\geq 0})\left[a_n \in Z^{odd}\right]$$

Can we prove this with induction?

Dominos (Revisited)

Suppose you connect your line of dominos together with a string and line them all up along the edge of a table.

[Allow me to draw this…]

What if the weight of a single domino falling off the table is not enough to pull the next one off. Maybe it takes the weight of three dominos to pull the next one down.

What happens if we knock the first three dominos off the table manually?

Strong induction

For (ordinary) induction:

- We assume one particular domino falls and then show that the next one must also fall.

For strong induction:

- We assume *all* of the dominos before domino k+1 fall and then show domino k+1 must also fall.

This is frequently EASIER because our Inductive Hypothesis is *stronger* (We are assuming more stuff.)

Strong Induction

<u>Claim:</u> $(\forall n \in \mathbb{N})$ [P(n)].
<u>Proof:</u>

I will apply *strong* induction on n.

Base case: Show P(0), P(1), P(2)…P(7) directly**.**
[It doesn't have to be 7.]

Inductive Hypothesis: For some $k \geq 7$: Assume P(i) holds for all $i \leq k$.

Inductive Step: Prove P(k+1) must also be true, based on your assumption that P holds for all previous values.

Let's draw a picture…

Now prove the recurrence relation property, using strong induction

- Here's the function definition again:

$$a_0 = 1 \qquad a_1 = 1 \qquad a_2 = 3$$

$$(\forall k \in Z^{\geq 3})\left[a_k = a_{k-1} + a_{k-2} - a_{k-3}\right]$$

- This is the property to be proven:

$$(\forall n \in Z^{\geq 0})\left[a_n \in Z^{odd}\right]$$