Announcements

• Homework #7 is due Friday 04/05.
Power set

\[ \mathcal{P}(A) = \text{the set of all subsets of } A \]

Examples- what are \( \mathcal{P}({a})? \)
\[ \mathcal{P}({a,b,c})? \]
\[ \mathcal{P}(\emptyset)? \]
\[ \mathcal{P}({\emptyset})? \]
\[ \mathcal{P}({\emptyset,\emptyset})? \]
\[ \mathcal{P}({\emptyset,\{\emptyset}\})? \]
Recall Basic Definitions

Union: \[ x \in A \cup B \iff x \in A \lor x \in B \]

Intersection: \[ x \in A \cap B \iff x \in A \land x \in B \]

Complement: \[ x \in A^c \iff x \notin A \]

Difference: \[ x \in A - B \iff x \in A \land x \notin B \]
Proving Subset Relationship

Claim: \( A \subseteq B \).

Proof:

Let \( x \in A \).

\[
\vdots \\
\therefore x \in B.
\]
Some Properties of Sets

• Inclusion
  \[ A \cap B \subseteq A \quad A \cap B \subseteq B \]
  \[ A \subseteq A \cup B \quad B \subseteq A \cup B \]

• Transitivity
  \[ A \subseteq B \land B \subseteq C \rightarrow A \subseteq C \]

• Let’s prove a few of these
Proving two sets are equal

Two (basic) techniques:

Claim:  \( A = B \).
Proof:
\[
\begin{align*}
& x \in A \iff \\
& S1 \iff \\
& S2 \iff \\
& S3 \iff \\
& \cdots \\
& x \in B
\end{align*}
\]

Claim:  \( A = B \).
Proof:
\[
\begin{align*}
\text{Part I. } & \text{ [Show } A \subseteq B ] \\
& \cdots \\
\text{Part II. } & \text{ [Show } B \subseteq A \text{ ]} \\
& \cdots
\end{align*}
\]
More Properties of Sets

- DeMorgan’s for complement
  
  \[(A \cup B)' = A' \cap B'\]
  
  \[(A \cap B)' = A' \cup B'\]

- Distribution of union and intersection
  
  \[A \cup (B \cap C) = (A \cup B) \cap (A \cup C)\]
  
  \[A \cap (B \cup C) = (A \cap B) \cup (A \cap C)\]

- Let’s prove a couple.

- There are a number of others as well; see the handout that is on the class webpage.
A third way to prove two sets are equal:

**Claim:** $A = B$.  
**Proof:** 

$A = X$ [By rule called…]  
$= Y$ [By rule called…]  
$= Z$ [By rule called…]  
$= B$ [By rule called…]
Deriving new properties using rules (or from definitions)

\[ B - (A \cap C) = (B - A) \cup (B - C) \]

\[ A - B = A - (A \cap B) \]

\[ A \subseteq B \land A \subseteq C \rightarrow A \subseteq (B \cap C) \]
Using Venn diagrams to help find counterexamples

\[ A \cup (B \cap C) = ? = (A \cap B) \cup (A \cap C) \]

\[ A \cup (B - C) = ? = (A \cup B) - C \]

Trick: Draw the Venn diagrams and find a cell where they disagree. Make sure your counterexample has an element in that cell.