Announcements

- Homework #7 is due Friday 04/05.

Power set

$\mathcal{P}$(A) = the set of **all** subsets of A

Examples- what are $\mathcal{P}(\{a\})$?

$\mathcal{P}(\{a,b,c\})$?

$\mathcal{P}(\varnothing)$?

$\mathcal{P}(\{\varnothing\})$?

$\mathcal{P}(\{\varnothing,\{\varnothing\}\})$?

Recall Basic Definitions

Union: $x \in A \bigcup B \leftrightarrow x \in A \vee x \in B$

Intersection: $x \in A \bigcap B \leftrightarrow x \in A \wedge x \in B$

Complement: $x \in A^c \leftrightarrow x \notin A$

Difference: $x \in A - B \leftrightarrow x \in A \wedge x \notin B$

Proving Subset Relationship

Claim: **$A \subseteq B$.**

Proof:

Let $x \in A$.

…

$\therefore x \in B$.

Some Properties of Sets

- Inclusion $A \cap B \subseteq A$ $A \cap B \subseteq B$

  $A \subseteq A \cup B$ $B \subseteq A \cup B$

- Transitivity $A \subseteq B \wedge B \subseteq C \rightarrow A \subseteq C$

- Let's prove a few of these

Proving two sets are equal

Two (basic) techniques:

Claim: A = B.

Proof:

$x \in A \leftrightarrow$

$S1 \leftrightarrow$

$S2 \leftrightarrow$

$S3 \leftrightarrow$

…

$x \in B$

Claim: A = B.

Proof:

Part I. [Show $A \subseteq B$]

…

Part II. [Show $B \subseteq A$]

…

More Properties of Sets

- DeMorgan's for complement

$$(A \cup B)' = A' \cap B'$$

$$(A \cap B)' = A' \cup B'$$

- Distribution of union and intersection

$$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$$

$$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$$

- Let's prove a couple.
- There are a number of others as well; see the handout that is on the class webpage

Proving two sets are equal (Using the Rule Sheet)

A *third* way to prove two sets are equal:

<u>Claim:</u> A = B.

<u>Proof:</u>

A = X   [By rule called…]

= Y   [By rule called…]

= Z   [By rule called…]

= B   [By rule called…]

Deriving new properties using rules (or from definitions)

$$B-(A\cap C)=(B-A)\cup(B-C)$$

$$A-B=A-(A\cap B)$$

$$A\subseteq B\wedge A\subseteq C\rightarrow A\subseteq(B\cap C)$$

Using Venn diagrams to help find counterexamples

$$A \cup (B \cap C) =?= (A \cap B) \cup (A \cap C)$$

$$A \cup (B - C) =?= (A \cup B) - C$$

Trick: Draw the Venn diagrams and find a cell where they disagree. Make sure your counterexample has an element in that cell.