Announcements

- Homework due tomorrow (and HW #9 will be assigned).

Permutations

Different ways of ordering objects in a list are called **permutations.**

Examples:

- On your vacation you will visit Germany, Italy, France, and Greece. How many different ways are there to organize this trip?
- How many ways are there to shuffle a deck of cards?

Can we derive a formula for this?

"Number of permutations of n objects" = ???

n!

Ways to select *some* members from a set

Suppose we have a set of 7 colors and we want to select three of them. How many ways can it be done?

It depends… 4 different scenarios:

1. No repeats; order matters (lists with no repeats)
   RGY, BGR, GYB, GBY, BGY, etc.
2. No repeats; order doesn't matter (sets)
   RGY, BGR, GYB, etc. (Can't **also** include GBY, BGY)
3. Repeats allowed; order matters (lists)
   RRY, BBB, RGG, GRG, GGR, etc.
4. Repeats allowed; order doesn't matter (bags)
   RRY, BBB, RGG, etc. (Can't **also** include GRG, GGR)

1. No repeats; order matters

Examples:

- How many ways are there to go on a trip where you visit 5 of the 50 states?
- The Kentucky Derby has 20 horses. How many outcomes (win, place, and show) are possible?

These are called **R-Permutations**

Can we derive a formula for this?

"Number of permutations of n objects taken r at a time" = ???

$$P(n, r) = {}_nP_r = \frac{n!}{(n-r)!}$$

2. No repeats; order doesn't matter

Examples:

- Three senators will be chosen to form a sub-committee. How many ways can this be done?
- How many different 5-card poker hands are possible?

These are called **Combinations**. (Note that this is the same as **subsets** of a fixed size.)

Can we derive a formula for this?

"Number of combinations of n objects taken r at a time" = ???

$$C(n,r) = \binom{n}{r} = \frac{P(n,r)}{r!} = \frac{n!}{(n-r)!r!}$$

3. Repeats allowed; order matters

Examples:

- Every day the forecast is either rainy, sunny, or cloudy. How many forecasts are possible for a given week?
- How many different ways can you answer a quiz with 10 multiple choice questions, labelled A, B, C, and D?

These are called **Tuples**

Can we derive a formula for this?

"Number of ways to select an r-tuple from a set of size n" = ???

$\mathbf{n^r}$

4. Repeats allowed; order doesn't matter

Examples:

- I have a bag full of: 7 snickers bars, 12 milkyway bars, and 15 KitKats. How many ways are there to reach in and grab 4 pieces of candy?
- M&M's come in 6 colors. How many different handfuls of 25 M&M's are possible?

These are called **multi-sets**.

Can we derive a formula for this?

"Number of multisets of size r taken from set of size n" = $\binom{n+r-1}{r}$

More multiset questions

Examples:

- I have 330 students. How many grade distributions are possible? (For example, a grade distribution might be: 100 A's, 50 B's, 100 C's, 50 D's, and 30 F's.)

- How many ways are there to distribute 50 tennis balls amoung 4 containers. (Any number of balls could be put into any container, including 0.)

Summary: Choosing r elements out of n elements

	order matters	order doesn't matter
repetition allowed	$\underbrace{n \times \cdots \times n}_{r \text{ times}} = n^r$	$\binom{n+r-1}{r}$
repetition not allowed	$P(n,r) = \frac{n!}{(n-r)!}$	$\binom{n}{r} = \frac{n!}{(n-r)!\,r!}$

Another kind of Question

- Examples:
  - Arrangements of the word “mississippi”
  - Assume you have a set of 15 beads:
    - 6 green
    - 4 orange
    - 3 red
    - 2 black

  How many ways are there to arrange them in a row?