# CMSC 250

Exam II Review

# Topics Covered

Modular Arithmetic Theorem through Counting and Probability

# Modular Arithmetic Theorem

Evaluate:

- $(6^{15}+77^{11}) \; mod \; 7$

# Quotient-Remainder Theorem

Prove:

Claim: For all natural numbers, n: $5n^2$+11n-2 is not divisible by 3

(Must prove using Q-R Theorem.)

# Floor / Ceiling

Claim: If x is not an integer, then floor(x)+ floor(-x) = -1.

# Induction

Claim: For all natural numbers, n: $n^2 + n$ is even.

(Must prove using induction!)

# Induction

For all natural numbers > 0:

A $2^n$ by $2^n$ checkerboard with one square removed can always be tiled with right-triominoes. (OK, what's a right-triomino?)

# Strong Induction

$a_0 = 5,$
$a_1 = 16,$

For i > 1: $a_i = 7a_{i-1} - 10a_{i-2}.$

Prove: For all natural numbers, n: $a_n = 3(2^n) + 2(5^n).$

# Sets

- Claim: If A, B, and C are sets, then $A - (B \cup C) = (A - B) \cap (A - C)$.

(Prove this from the definitions, only).

- Know about powersets

# Recall: Choosing $r$ elements from $n$ elements

| | order matters | order doesn't matter (like a "set") |
|---|---|---|
| repetition allowed | $\underbrace{n \times \cdots \times n}_{r \text{ times}} = n^r$ | $\binom{n+r-1}{r}$ |
| repetition not allowed | $P(n,r) = \frac{n!}{(n-r)!}$ | $\binom{n}{r} = \frac{n!}{(n-r)!\,r!}$ |

# Probability and Counting

You are playing a card game in which the deck has 10 yellow cards, 10 red cards, 7 blue cards, and 8 green cards.

1. You are dealt five cards. How many hands are possible? (A “hand” could be, for example: “1 red, 2 blue, 2 yellow”).
2. What is the probability that you get 3 red cards and 2 blue cards?

# Probability and Counting

1. How many ways can the 9 supreme court justices be seated in a row?

2. If they are seated randomly, what is the probability that Breyer will be seated next to Alito?

# Probability and Counting

1. You have 64 crayons.  How many ways can you color in the keys of an 88-key piano.  (Each key will be a single color)?

2. What is the probability that no two adjacent keys will share the same color?