MATH299M/CMSC389W – *Visualization Through Mathematica*

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Model H8.2: Visualizing Integrals on Washers and Wedges

Assigned: Friday March 15th, 2019

Due: Monday April 1st, 2019 11:59PM

Note: Between models H7.1, H7.2, H8.1, H8.2, H9.1, and H9.2 (Group 2) you need only complete 3 assignments.

In Multivariate Calculus you take integrals over two-dimensional regions. When you do this in polar coordinates you can easily parameterize certain regions such as *washers* and *wedges*, a washer (also called an *annulus*) being a disc with a central disc removed, and a wedge being a certain angle range of a washer. Now that you have PolarPlot and ParametricPlot at your disposal, drawing these regions is simple.

For this assignment create a model that allows the user to take the integral of an arbitrary function over a washer or wedge. It would be cool if the model gave the user the ability to specify what washer or wedge they're integrating over, and of course to see it on a plot. Here are some things to think about:

- Do you see how a line segment is a particular case of a wedge?
- Do you see how a washer is a particular case of a wedge?
- Do you see how a disc is a particular case of a washer?
- Do you see how a ring (a circumference) is a particular case of a washer?
- Do you see how any wedge can be described with five variables?
  Hint: their names could plausibly be x, y, r, R, and θ.
- Does the integral change if the wedge has boundary?

Remember that the function specified will be of two variables. It's up to you whether you want the user to input the function in terms of x and y or in terms of r and θ. You can integrate wrt x and y if you want, but the elegant thing to do here is definitely to use polar coordinates.

Note that if you integrate the constant function over a region (with the correct Jacobian, for polar coordinates this is r $drd\theta$, you get the area of the surface. What is the geometric interpretation (in 3-space) of the integral of a function f(x,y) over a region R in the plane? Understanding this will illuminate what we're *really* doing when we take integral over regions.

Region integral example:
Area of the unit circle

$$\int_R 1\,dR = \int_0^{2\pi}\int_0^{r} 1\,rdrd\theta = \int_0^{2\pi}\frac{1}{2}r^2\,d\theta = \frac{1}{2}r^2\theta|_{\theta=0}^{\theta=2\pi} = \pi r^2$$

Go learn something about integrals over washers and wedges!