MATH299M/CMSC389W – *Visualization Through Mathematica*

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Model H9.1: Visualizing Trig Functions on the Complex Plane (Part II)

Assigned: Tuesday March 26th, 2019

Due: Tuesday April 2nd, 2019 11:59PM

Note: Between models H7.1, H7.2, H8.1, H8.2, H9.1, and H9.2 (Group 2) you need only complete 3 assignments.

We've already done an assignment investigating the trig functions over the complex domain – but at the time we were restricted to the plane! Now we have tools to visualize things in 3D B)

This assignment has the same mission statement and possible paths as Model H8.1, however now we can get a different kind of insight.

Mathematica doesn't have a super nice way to do complex numbers (although it can do algebra with them just fine and has some functions for them like Conjugate[] and Norm[]) so you'll have to do conversions between $\mathbb{C}$ to $\mathbb{R}^2$ yourself (you'll see what I mean by this when you try to implement any visual with complex numbers). Remember that you can create the imaginary unit i with Esc-ii-Esc, which gives you a double-struck i, which in Mathematica is the root of -1.

One thing to do is to make one surface over the complex plane that is the real part of the function, and another one that is the imaginary part of the function, using Re[] and Im[], respectively. These surfaces are interesting for a few reasons:

- How much is happening that's invisible to our real eyes?
- Does the function look like you expected on the real line?
- What's the imaginary part doing on the real line?
- Does the whole structure look symmetrical in certain ways? Periodic?

And more deeply, the real and imaginary parts of the function have to satisfy a super nice relationship. All *analytic* functions (continuous and differentiable for complex) satisfy the *Cauchy-Riemann Equations*:

$$f(x+iy) = u(x,y) + v(x,y)i$$
$$\frac{\partial u}{\partial x} = \frac{\partial v}{\partial y} \; and \; \frac{\partial u}{\partial y} = -\frac{\partial v}{\partial x}$$

Can you see this with these surfaces?

Also interesting to do would be to plot just the Norm[] over the complex plane, to see how the magnitude of the function varies over the complex plane. Along the lines, how about seeing just the Arg[], which is just the angle if the vector representing the point on the plane from the x-axis? How does that vary over the complex plane?

Does it make sense where the singularities are? Can you how these functions are related to the exponential function e^{ix}? How are the derivatives are related? And like with the last project, we can also investigate the hyperbolic trig functions:

$$sinh(\theta) = -isin(i\theta)$$

$$cosh(\theta) = cos(i\theta)$$
$$tanh(\theta) = -itan(i\theta)$$
$$coth(\theta) = icot(i\theta)$$
$$sech(\theta) = sec(i\theta)$$
$$csch(\theta) = icsc(i\theta)$$

There's a LOT of intuition to uncover and developed for complex analysis in general, specifically in this assignment you can enrich your understanding of conic sections that you've been gradually learning more about since grade school (first as Algebra II functions, then as level sets of surfaces (a Calc III concept), then as things with suspiciously nice derivative properties, now as things with extremely suggestive complex definitions). Have fun!