MATH299M/CMSC389W – *Visualization Through Mathematica* Spring 2019 – Ajeet Gary, Devan Tamot, Vlad Dobrin

Model H9.2: Drawing Conic Sections as Level Sets of Paraboloids

Assigned: Tuesday March 26th, 2019

Due: Tuesday April 2nd, 2019 11:59PM

Note: Between models H7.1, H7.2, H8.1, H8.2, H9.1, and H9.2 (Group 2) you need only complete 3 assignments.

This week we expanded our toolkit to the 3D versions of the Plot functions! Now you can draw surfaces explicitly and as level sets, as well as arbitrary curves in 3-space. You learned in Algebra II in middle or high school that all *conic sections* are cross sections of the *double cone* (a cone with an upside-down cone on top of it) at different angles, including the degenerate cases. The 2D (yes, "2D", because they're surfaces embedded in 3-space) are the *paraboloids*. A paraboloid is a "quadric surface with exactly one axis of symmetry" – complicated. Here's a table of the non-degenerate conics:

Ellipsoid	$1 + y^2 + z^2 = 1$	
Elliptic Paraboloid	$1+y^2-z=0$	
Hyperbolic Paraboloid	1 - y ² - z == 0	
Hyperbolic Paraboloid of One Sheet	$1 + y^2 - z^2 = 1$	
Hyperbolic Paraboloid of Two Sheets	$1 + y^2 - z^2 = -1$	

(Made in Mathematica, of course).

Show me side-by-sides of these quadric surfaces with their level sets! Their level sets are all conic sections. Can you find how these are related? Does it shed some light on how the conics are related? Is there a general form for all quadric surfaces – a general form for all conic sections?

Hint: Statement 1: All conic sections are level sets of quadric Statement 2: All level sets of quadric surfaces are conic sections Conclusion: There's a generalized quadric surface formula ⇒ there's a generalized conic sections formula

The world of conics sections is enormous, and there's tons to investigate. If you can learn and showcase one feature you haven't understood or noticed before, that's fantastic, pooling everyone's findings should be a good step towards a comprehensive "intro. to conic sections" notebook!

Here are some ideas of what you could show:

Eccentricity

You learned the concept of eccentricity in Algebra II, but probably haven't applied it to anything at all so far. The general idea is that eccentricity is a measure of how distorted the circle is. A circle has eccentricity 0, and ellipse between 0 and 1, a parabola 1, and a hyperbola >1. This is intriguing! We can determine which conic we're talking about just by eccentricity – can you make a model that makes it clear exactly what eccentricity is, and why the type of conic depends on it?

Conics as Double Cone Level Sets

Like you learned in grade school, the conics are all cross sections of the double cone. With the tools we've learned so far it's not clear how to implement this – can you figure it out? Of course, a 3D graphic side-by-side would be awesome.

Rotating Quadrics – Rotating Conics

What does rotating a quadric do to the conic level set?