MATH299M/CMSC389W – *Visualization Through Mathematica*
Spring 2019 – Ajeet Gary, Devan Tamot, Vlad Dobrin
Project 2c: Fourier Analysis Visualization
Assigned: Friday March 15th
Due: Monday April 15th, 11:59PM

Now that you have plenty of visualization tool under your belt for drawing things, you're equipped to start doing some awesome math stuff. For the second project you need only complete one of Project 2a, Project 2b, or Project 2c (this one). If you've ever dealt, this is one is a good choice.

For this project I want you to make model(s) to explain the ideas behind Fourier Analysis. An obvious starting point is to make a model for the Fourier transform itself.

Unfortunately I don't know much about the Fourier transform, I'm only used in Physics because it's how you switch from expressing the basis of a wave function from position-space to momentum-space and is the theory behind the Heisenberg Uncertainty Principle (and uncertainty principles in general) – but that's no use to you. I haven't done any harmonic analysis or signal processing – you can certainly research these things yourself though. Below is Plancherel's Theorem, which I've used in QM and looks important, and a link to an *amazing* 3Blue1Brown video on the Fourier Transform.

Plancherel's Theorem

$$f(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{+\infty} F(k) e^{ikx} dk \Leftrightarrow F(k) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{+\infty} f(x) e^{-ikx} dx$$

Where F(k) is the *Fourier Transform* of f(x) and f(x) is the *inverse Fourier Transform* of F(k).

https://www.youtube.com/watch?v=spUNpyF58BY

Experiment with the Fourier Transform, try to do some signal analysis! Try taking the transform of sinusoids and of the delta function, and the inverse transforms of those, and see what happens.

Good luck! Make something awesome that they could (nay, should) use in a Transform Methods or ODEs class. The final product should be cleaned up, that is, there should be a title, your name and the date on it, text in Text cells, and no extraneous code or outputs floating around. After making your model, you should play around with it, and maybe in a Text cell describe some interesting behavior you found.

In case you're curious: *QM* (Quantum Mechanics) says that everything exists in a *state* which is a *wave function* that is a solution to the complex PDE called *Schrodinger's Equation* and the momentum of the particle in that state is related to how wave-like (sinusoidal) the wave function is, while the position is related to how localized (delta-function-esque) it is, and since these are conflicting qualities, momentum and position are *incompatible observables* (the

Heisenberg Uncertainty Principle).