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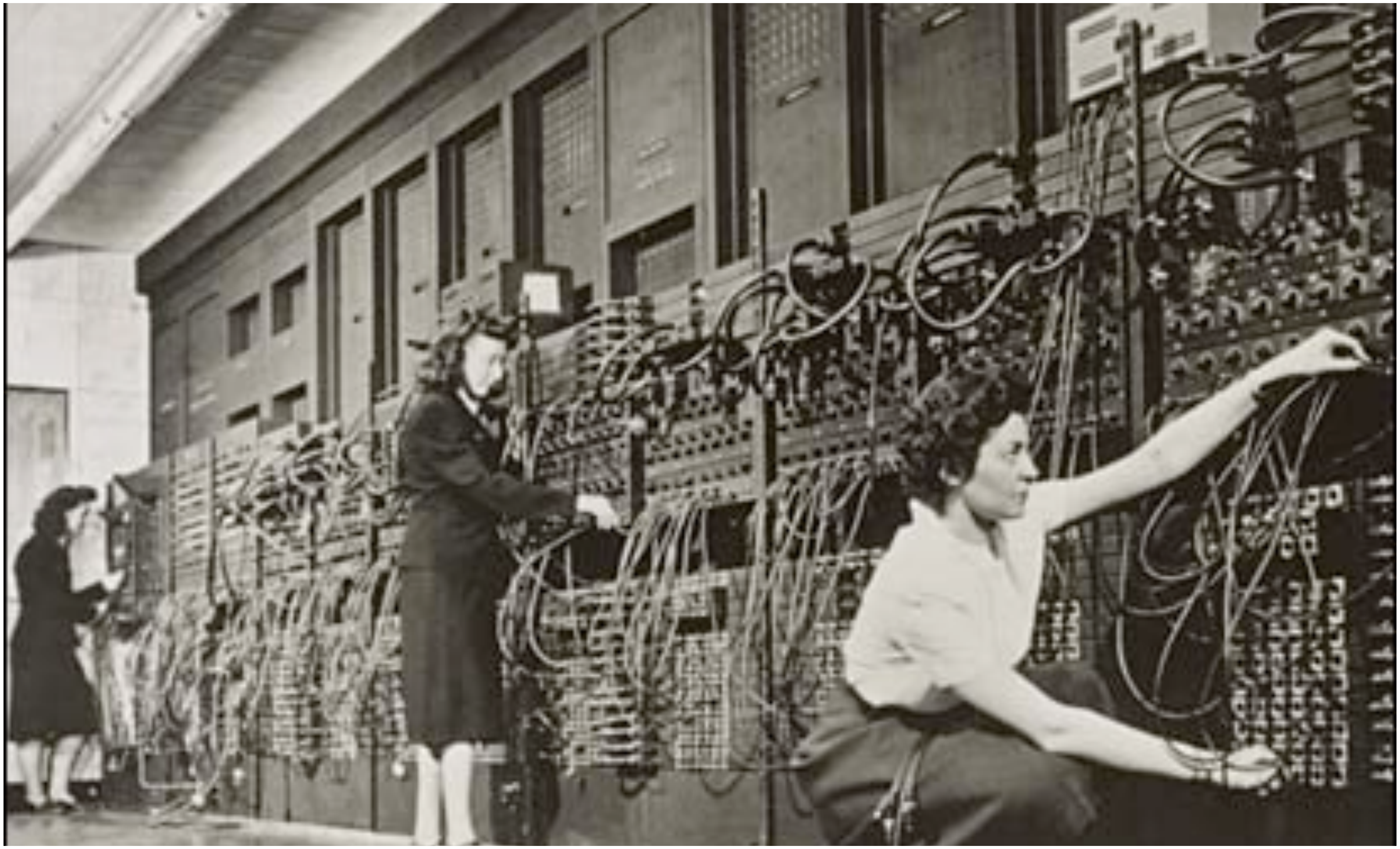
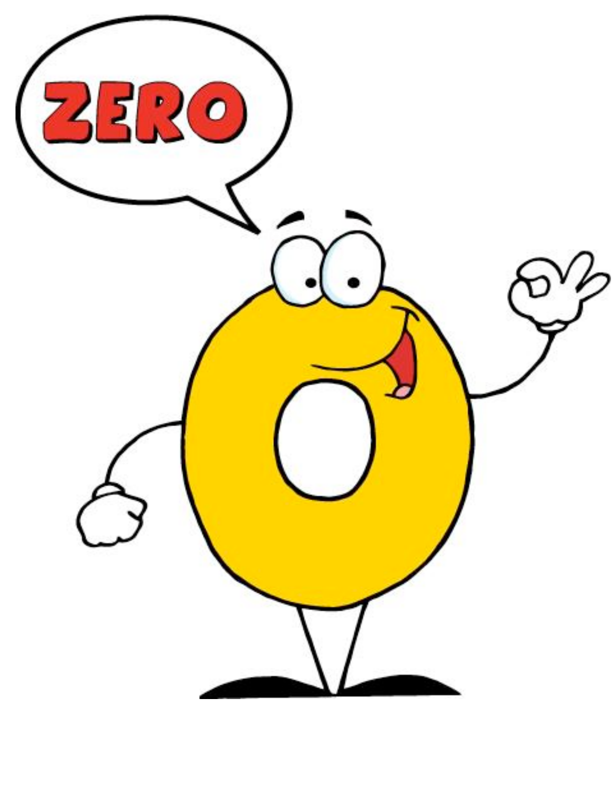


Secure computation

With material from Matthew Green, Elaine Shi, CS Unplugged, others

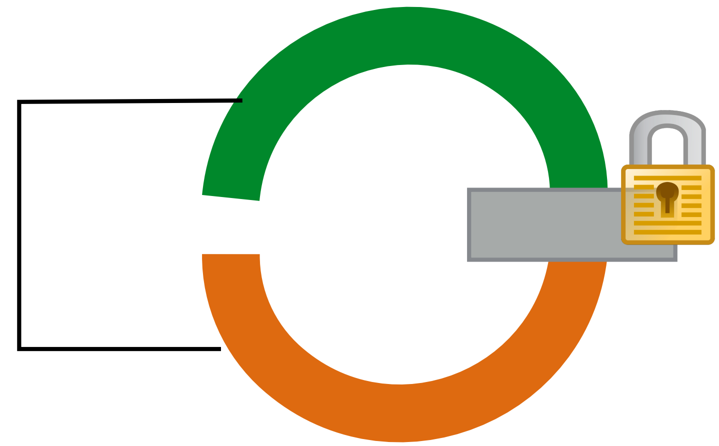
- Secure computation
    - Zero-knowledge proofs
    - Commitment schemes
    - Multiparty computation

Zero-knowledge proofs



- Goal: P proves to V that some statement is true
  - ***Without*** conveying additional information
- In general, probabilistic
  - Repeat a bunch of times as proof

Example 1: Hallway password



- Does Peggy have the key?
- Both stand in the entrance.
  - When Victor isn’t looking, Peggy picks one hall
  - Victor then yells “GREEN” or “ORANGE”
  - Peggy must come back via the chosen color
- Repeating many times “proves” Peggy has password
  - With high probability

Example 2: Two baseballs

- Peggy has two baseballs: One red, one green
  - Otherwise identical
- Victor is color-blind, thinks they are the same
  - Peggy's goal: To prove she can distinguish
- Peggy places them in Victor's hands
  - Victor puts them behind his back, may switch
  - Peggy tells whether he switched
  - As before, repeat many times

Security properties

- Complete: Honest V will be convinced by honest P
- Sound: Honest V can’t* be convinced by cheating P
- Proves nothing to outside observers either way
  - Peggy and Victor can **collude** by ***precomputing***
- Peggy could cheat with a time machine
  - Victor gets the same info either way
  - Implies that real protocol does not leak

Burning questions

- Why is this crypto?
- Does everyone have to be in the same place?
- Why do we care in real life?

Commitment schemes

Commitment schemes

- Commit to a value but do not show it
  - ***Open*** it later and prove it hasn't changed

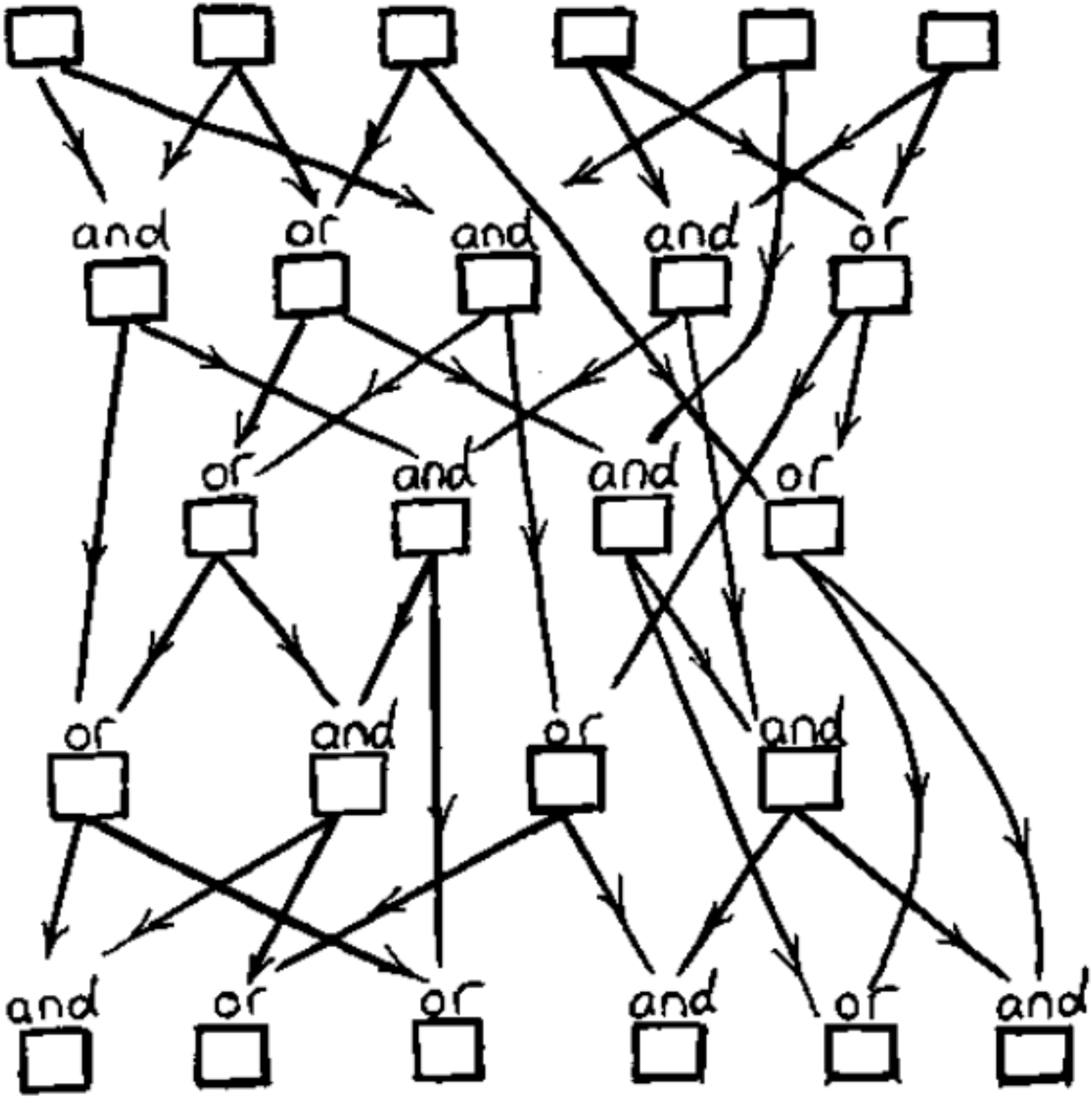
- Analogy:
  - I pick a number between 1 and 100
    - Write it down and seal it in an envelope
  - You pick odd or even
  - If you're right, I pay you; else you pay me
    - ***Why did I have to write it down?***

Required properties

- Hiding: Commitment reveals nothing about value
- Binding: Can’t open to a different value

Remote coin-flip

- Goal: Flip coin over the telephone
  - Alice flips, Bob chooses heads or tails
- Requires Alice to ***commit*** her output
  - In essence, need a one-way function
- Example/activity: Using and/or circuits



Heads = Even input parity
Tails = Odd input parity

http://csunplugged.org/wp-content/uploads/2014/12/unplugged-17-cryptographic_protocols_0.pdf

Try it! (Small groups)

- “Bob” draws a circuit
- “Alice” commits to an outcome
- “Bob” chooses odd or even parity
- Declare a winner
- Can either of you cheat? How?

Cheating

- Alice can cheat IFF she has two opposite-parity inputs that produce the same output
- Bob can cheat IFF he can predict the input from the output

Commitment via hash

- Alice, Bob pick a random numbers X, Y
  - Alice publishes H(X); Bob publishes H(Y)
- Bob chooses odd or even
  - Reveal X, Y and add them; check sum parity
- Collision resistance: Can’t fake X or Y
- Pre-image resistance: Can’t calculate X or Y

Multiparty computation

- Everyone has a private input
- Together, we compute some related result
- No one’s private input is given away

Example 1: How old are we?

- Goal: Find our average age
  - Without anyone giving away their own age
- Activity: Need five volunteers
  - And five sheets of paper

http://csunplugged.org/wp-content/uploads/2014/12/unplugged-16-information_hiding_0.pdf

Setup

- Alice, Bob are **honest but curious**
  - Don't lie, follow protocol correctly
  - But try to learn from available info
- Security equivalent to **fully trusted** third party

Defining leakage

- Learning f(a,b) gives some information
- What if f(a,b) = (a + b)?
- Final security property:
  - Alice learns only info computable from f(a,b), a
  - Bob learns only info computable from f(a,b), b

Example 2: Truth in dating

- After meeting and chatting, Alice and Bonnie want to find out whether they want to date each other
- If Bonnie says no, Alice doesn't reveal her answer
  - And vice versa
- Essentially secure AND

Alice	Bonnie	Result
NO DATE	NO DATE	NO DATE
NO DATE	DATE	NO DATE
DATE	NO DATE	NO DATE
DATE	DATE	DATE

Solution using 5 cards

- Alice and Bob each get two emoji cards: ❤️, 💣
  - Plus one public ❤️
- Place cards face down on table as follows:

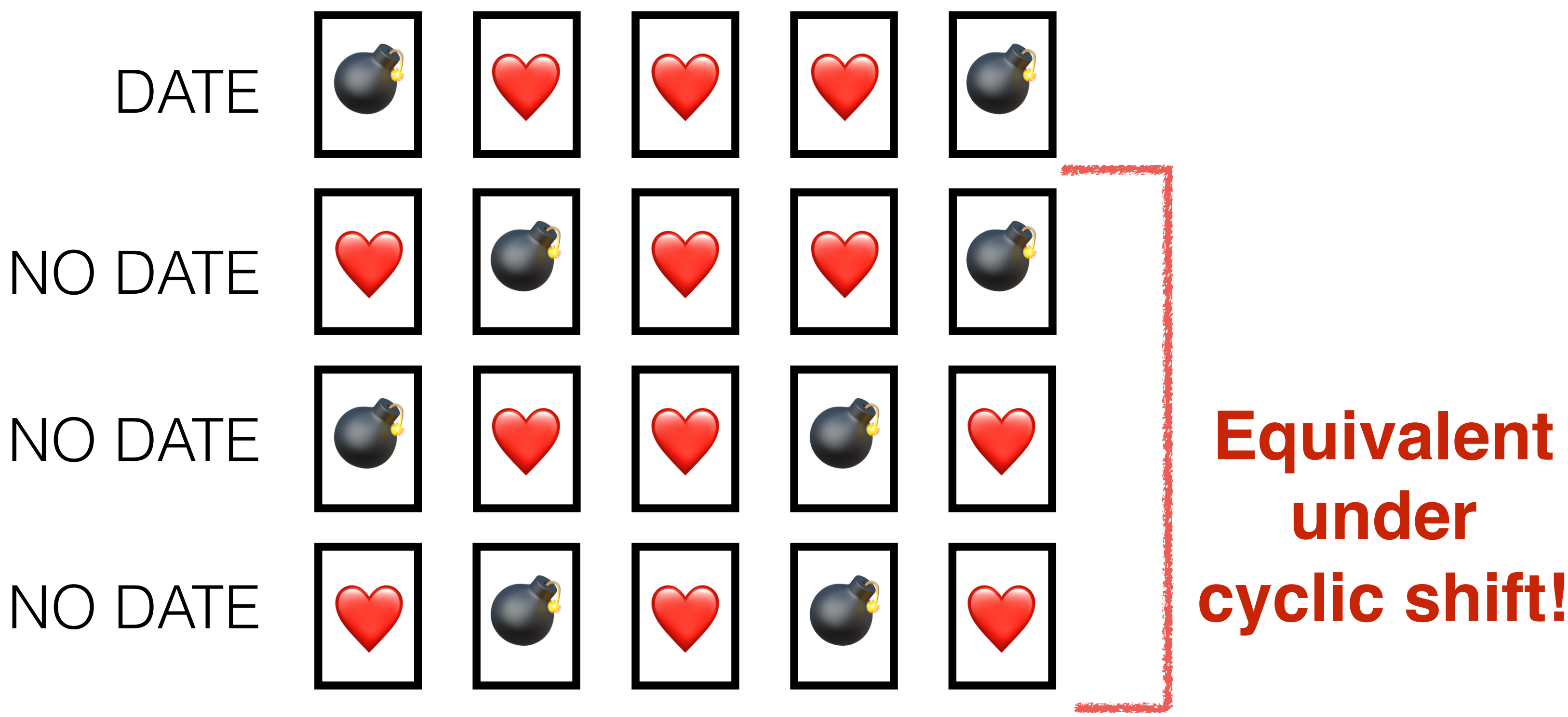
A A ❤️ B B

- Using this chart:

	ALICE		BONNIE	
DATE	💣	❤️	❤️	💣
NO DATE	❤️	💣	💣	❤️

Solution, ctd.

- Each gets to privately cyclic-shift the cards X times
- Final results: 3 hearts in a row = match



Other sample problems

- Two reporters compare confidential sources
  - To see if they are the same person
- Check for secret society password
- Find out who bid more without revealing your bid
- etc.

Desired properties

- **Resolution**: Find out desired outcome
- **Privacy**:
  - No involved party learns anything else
  - No third party learns anything
- **Security**: No one profits by cheating
  - Can’t know outcome unless other party does
- **Simplicity**: Easy to implement, understand
- **Remoteness**: Don’t need to be co-located

Example: Who is richer?

Yao’s millionaire’s problem (1982)

- Alice (i) and Bob (j) have $1 <= i,j <= $6
  - Assumption for simplicity
  - Generalizable to more people, more numbers
  - Later improvements in efficiency
- Also has security limitations
  - For conceptual purposes only

1. Bob's turn

- Bob chooses a large random number x
- Bob computes $m = E(PK_A, x)$
- Bob sends to Alice: $B = m - j + 1$

- *Example: $j = 5$, $B = m - 4$*

2. Alice's turn

- Alice generates $y_u = D(SK_A, B + u - 1)$ for $u = 1:6$
  - $y_u = D(SK_A, m - j + u)$
- Alice picks a prime p and generates $z_u = y_u \bmod p$
  - Ensure all z's at least 2 apart or try again
- *Example:*
  - *$z_3 = D(SK_A, m - 2) \bmod p$*
  - *$z_5 = D(SK_A, m) \bmod p = x \bmod p$*

2.5 Still Alice's turn

- Alice sends p to Bob
- Alice sends 6 numbers to Bob as follows:
  - z_1 .. z_i
  - $z_{i+1} + 1$ .. $z_6 + 1$

- *Example: $i = 2$*
  - *$z_1, z_2, z_3 + 1, z_4 + 1, z_5 + 1, z_6 + 1$*

3. Bob's turn

- Bob looks at the jth number in Alice's list
  - If it equals x mod p then i >= j
  - If not, then i < j
- Bob tells Alice the answer

- *Example: 5th number = $z_5 + 1$*
  - *$z_5 + 1 = (x \bmod p) + 1 \neq x \bmod p$*

Security caveats

- Brute force: Bob looks for q s.t. E(q) = m - j + 2
  - Can figure out whether i <= 2
- What if Bob lies to Alice?
- Lots of extensions, generalizations, etc.

Sec. Comp in real life

- Compute over private data
  - Health records
  - Military cooperation
  - Auctions
  - Boston wage equity
- ZCash