

Procedural shapes

CMSC425.01 Spring 2019

Administrivia

- Exam being graded ...
- Project 2b due Tuesday, April 16th

Today's question

How to create interesting shape assets

Autogenerating terrain, objects, animations

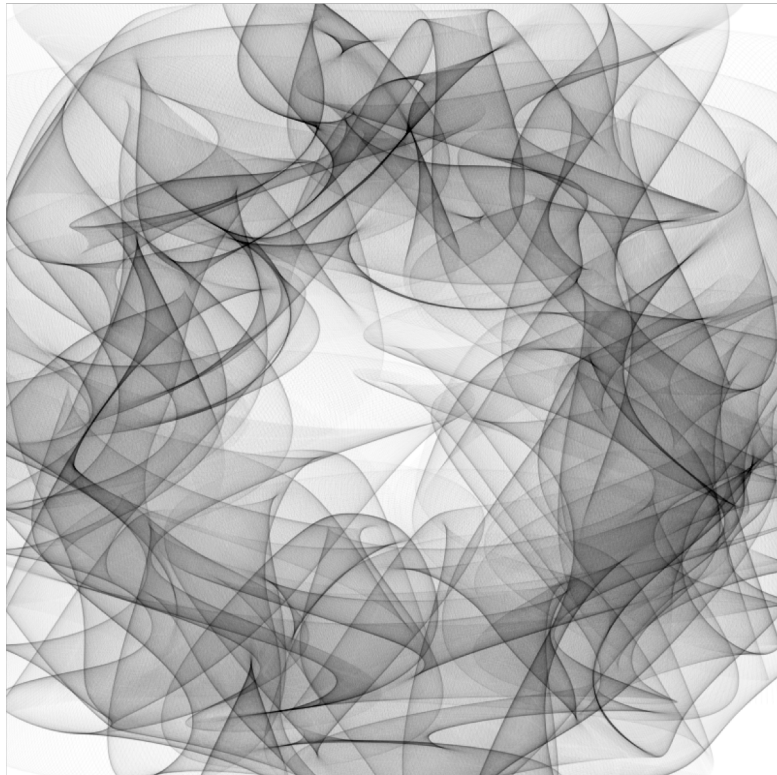
- Why
 - Efficiency – faster than by hand
 - Variety – generate variations on a template
 - LOD – Generate level of detail as needed (*billboarding* in Unity)
- How:
 - Offline procedural generation: create, store mesh, add
 - Online – during game: shape represented by subroutine/object, not a fixed
- What:
 - 3D shapes, 2D textures
 - Fake real (eg, trees), non-real things



Smooth vs. articulated shapes

Bezier curves (Processing)

Continuous, smooth, parametric



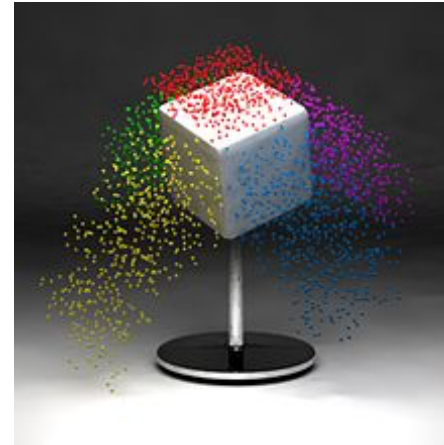
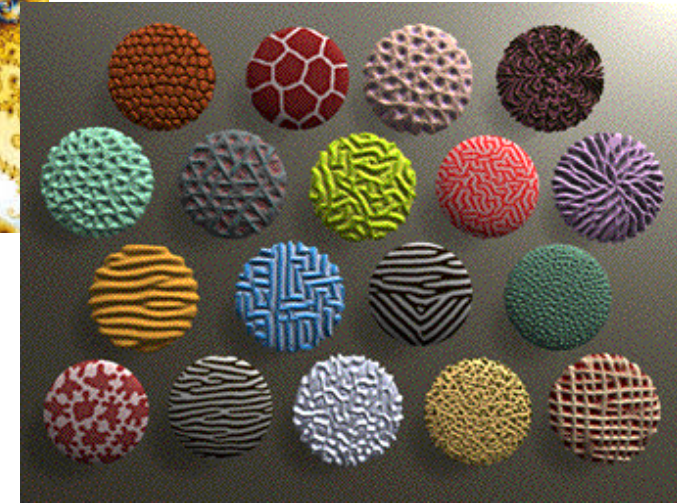
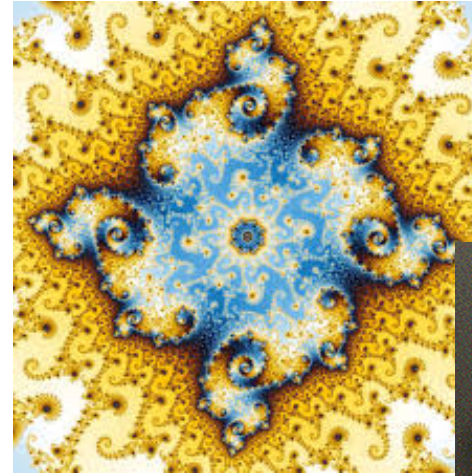
“Fractal” shapes

Articulated, subparts, discontinuous



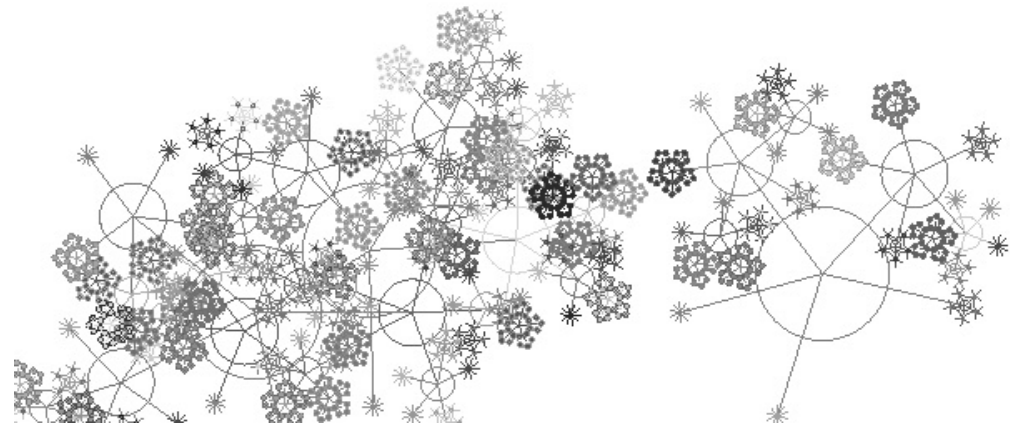
“Fractal” examples

- Midpoint displacement terrain
- Particle systems
- Lindermeyer Systems (L-systems)
- Perlin noise
- Attractor sets
- Iterated function systems (IFS)
- Reaction-diffusion textures
- Recursive fractals



Lindenmayer Systems

- *The Algorithmic Beauty of Plants*
 - by P. Prunskiewicz and A. Lindenmayer, 2004. Chapters 1 and 2.
 - It can be accessed online from <http://algorithmicbotany.org/papers/>
- Grammar based approach to branching structures, including trees



shiftcontrol studio Processing

L-system

- Generative grammar + interpreter
- Use grammar to generate a string
- Use interpreter to interpret the string as a shape
- 2D and 3D versions
- <http://www.kevs3d.co.uk/dev/lsystems/>

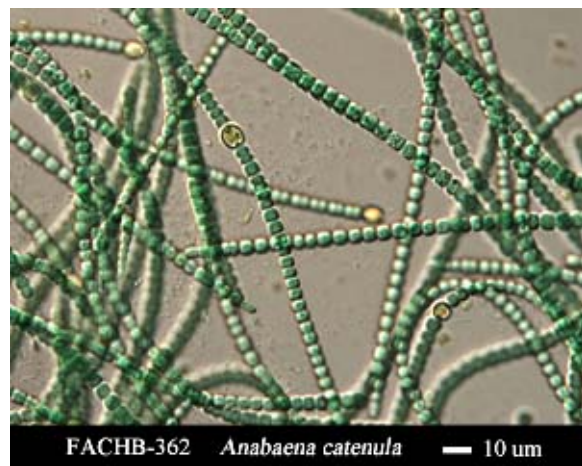
L-System definition

- Grammar $\mathbf{G} = (V, \omega, P)$
- V Alphabet of terminals (constants) and non-terminals (variables)
- ω Start symbol or axiom
- P Production rules

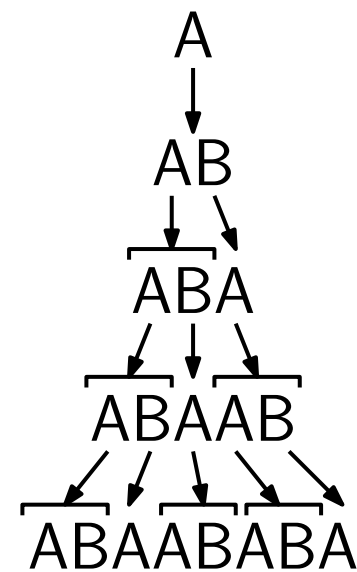
L-System definition

- Grammar $\mathbf{G} = (V, \omega, P)$
- V Alphabet of terminals (constants) and non-terminals (variables)
- ω Start symbol or axiom
- P Production rules
- Generative – forward derivations
- Deterministic – no choice
- **variables** : A B
- **constants** : none
- **axiom** : A
- **rules** : $(A \rightarrow AB), (B \rightarrow A)$
- Derivation
- $A \rightarrow AB \rightarrow ABB \rightarrow ABA \rightarrow \dots$
- ABAABABAABABAABABA

Anabaena catenula algae



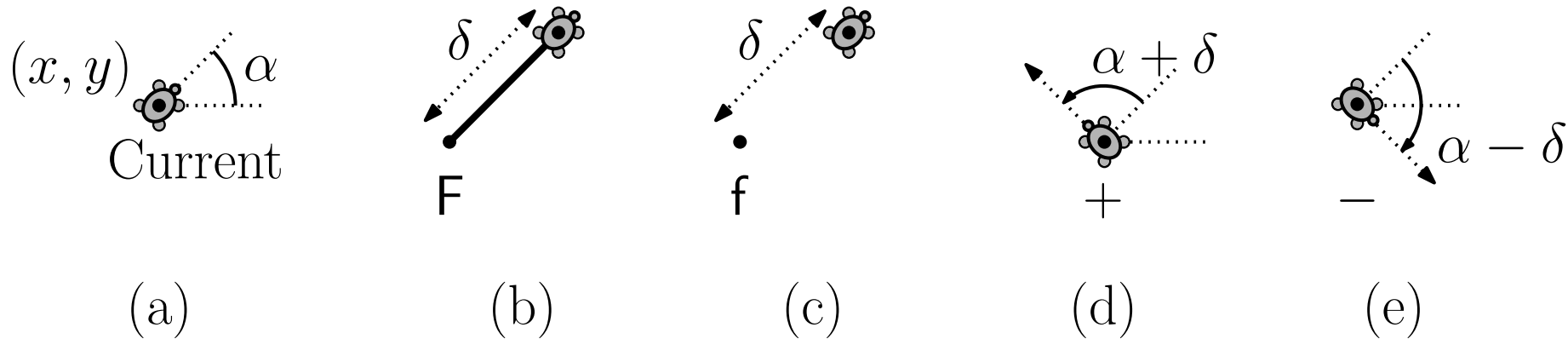
(a)



(b)

Interpreter: Turtle graphics: state (x, y, α)

- “F”: Draw line of length d forward
- “+”: Increase the turn angle by δ
- “f”: Move forward no line
- “-”: Decrease the turn angle by δ



Turtle has: length d , turn angle δ

Example

$d = 1, \delta = 90$

Initial state $(0,0,90)$ (facing up)

Draw: F+F+F+F

- Shape?

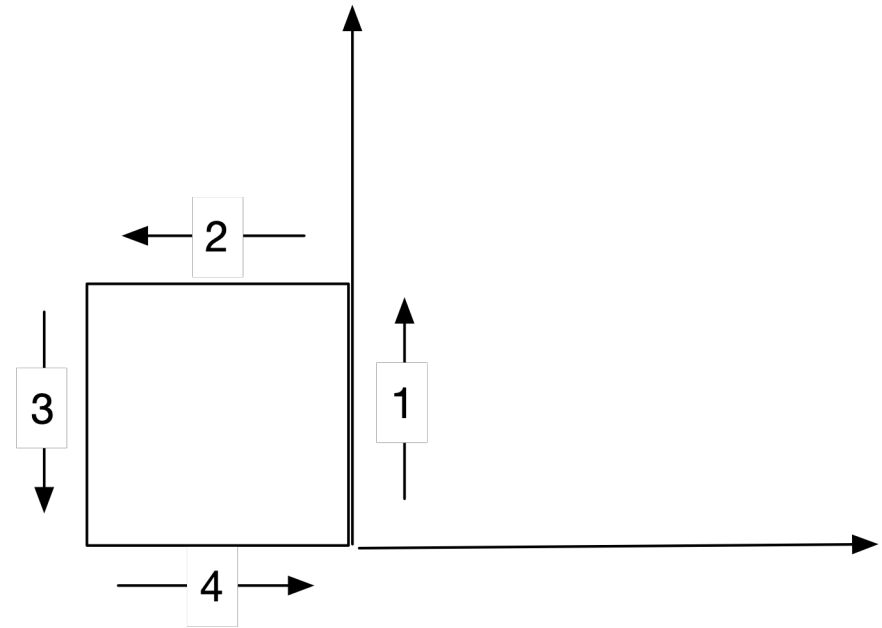
Example

$d = 1, \delta = 90$

Initial state $(0,0,90)$ (facing up)

Draw: $F+F+F+F$

- Shape?

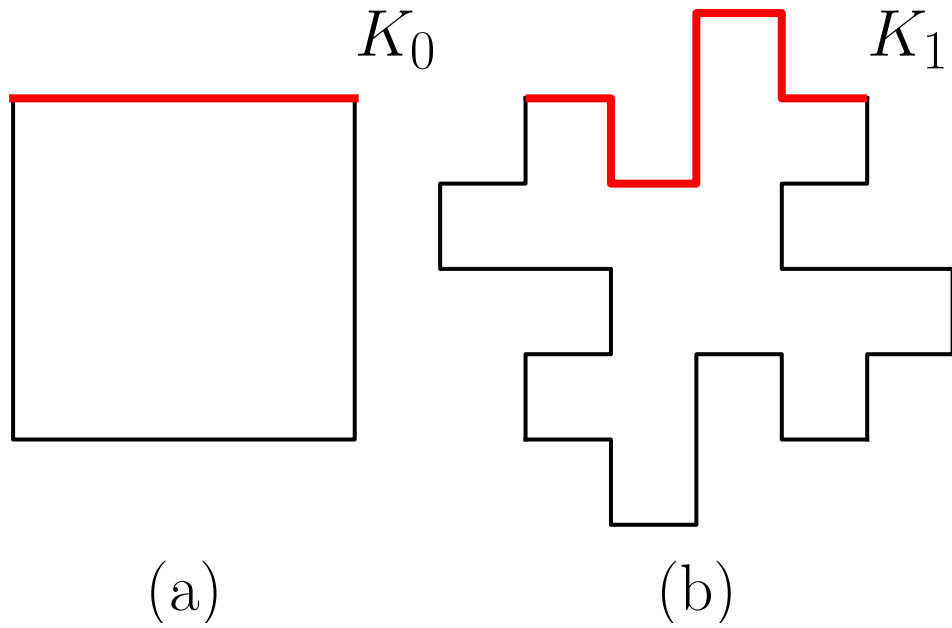


Practice



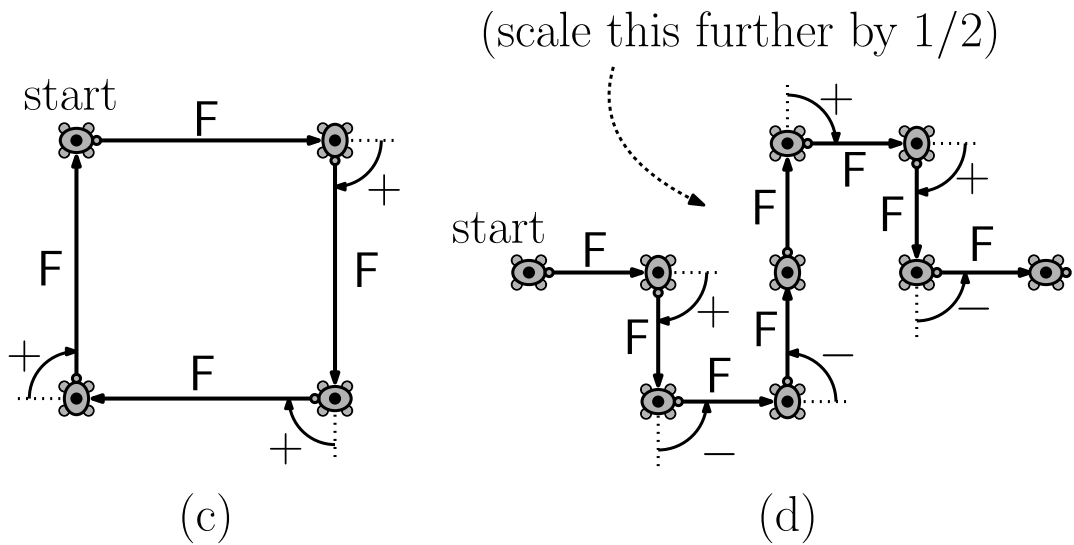
- Turtle commands and angle for
- Lower case a?
- Lower case s?
- Lower case w?
- Lower case e?
- You can start the turtle wherever you want, and at any angle. Just need to make clear.

Practice



- Turtle commands for Koch island
- Axiom $\omega = F+F+F+F$
- $d = 1$
- $\delta = ??$
- $F \rightarrow ?$

Practice



- Turtle commands for Koch island
- Axiom $\omega = F+F+F+F$
- $d = 1$
- $\delta = ??$
- $F \rightarrow F+F-F-FF+F+F-F$

Step 1



Step 2



Step 3



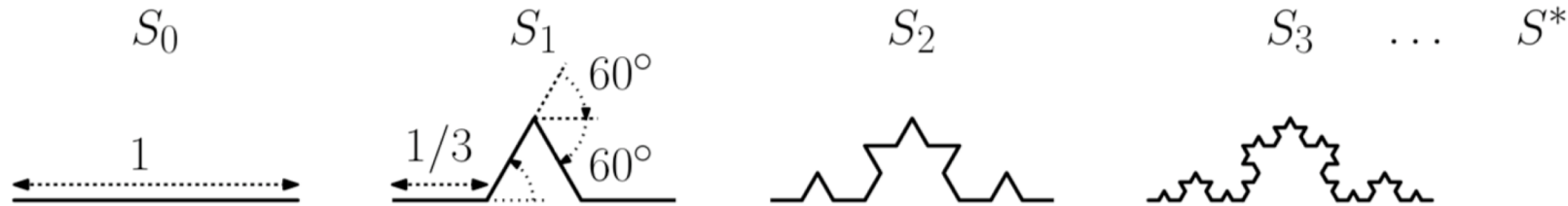
Step 4



- Note: Scaling down with each generation to keep same size

In-Class Exercise 1

Consider the series of shapes S_0, S_1, \dots shown in the figure below. With each step, each line segment of length x is replaced by four segments each of length $x/3$. Let $S^* = \lim_{i \rightarrow \infty} S_i$.



Derive an L-system to generate this shape. (For the sake of consistency, please start at the left endpoint.) Be sure to include the step size, angle increment (in degrees), and the initial angle. Explain your symbols and their meanings in the drawing process. Give the start string and all the production rule(s). **Hint:** All angles are multiples of 60° .

Turtle stack

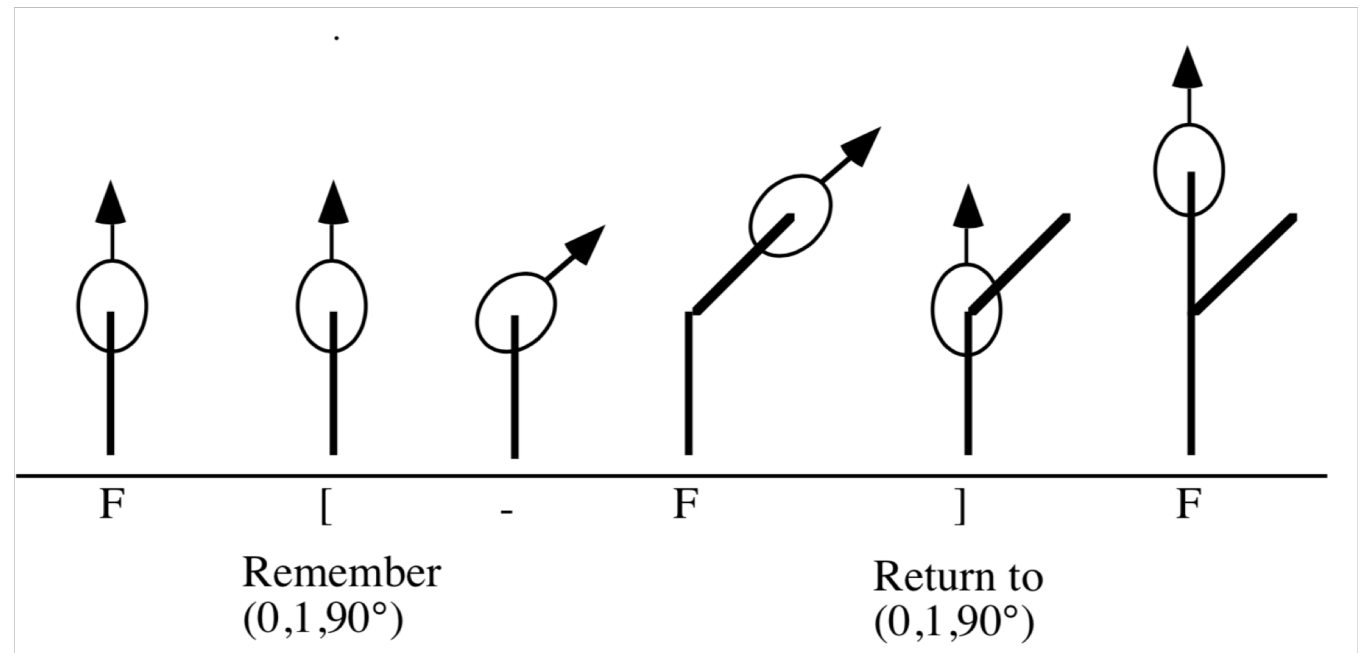
- No ...



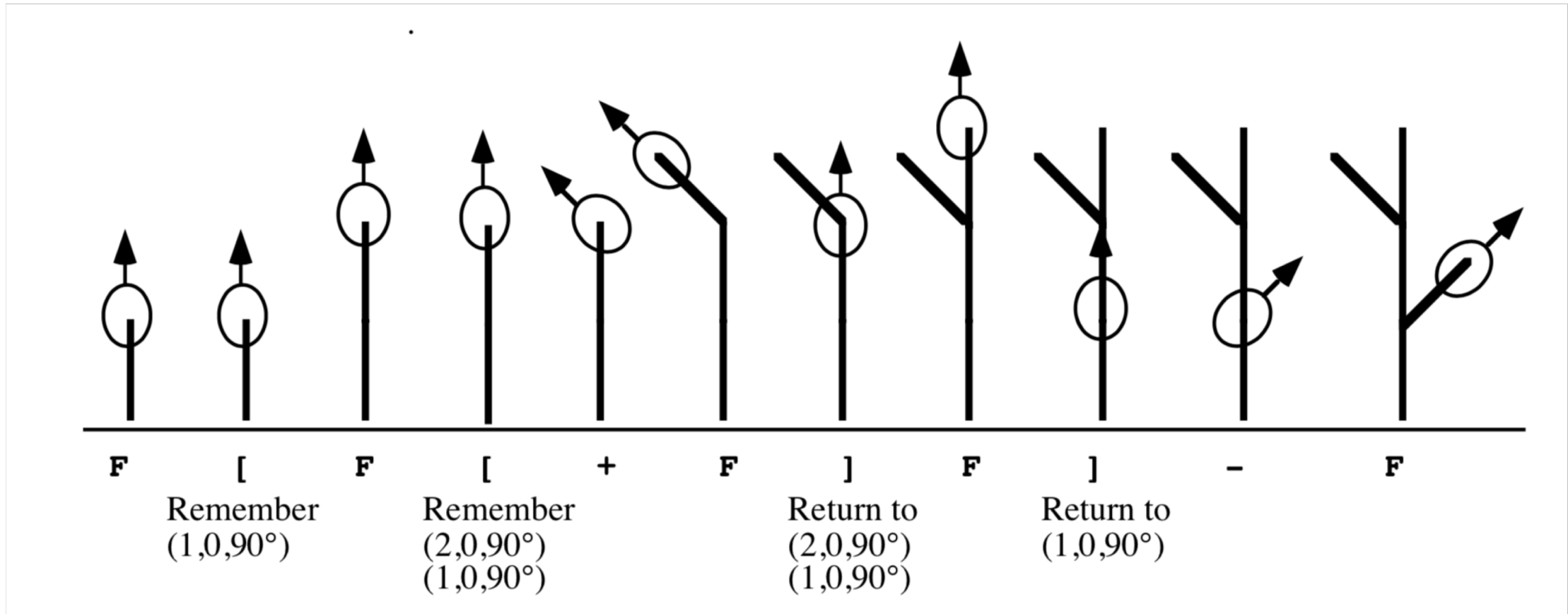
Turtle stack

- Add to vocabulary V :
- “[” Push state of turtle
- “]” Pop state of turtle

- Example: **F[-F]F**
- Angle 45, initial 90

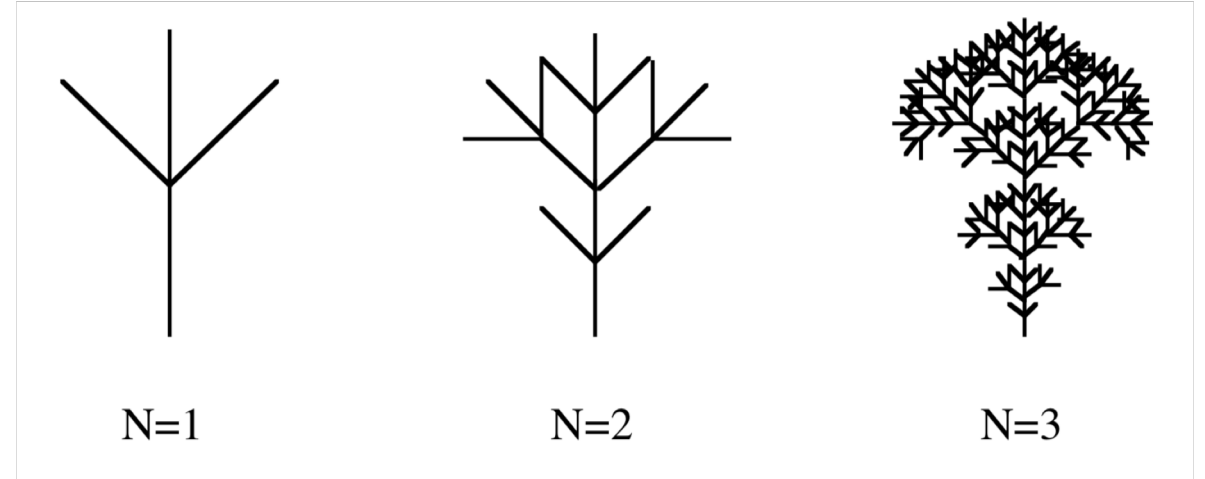


F[F[+F]F]-F



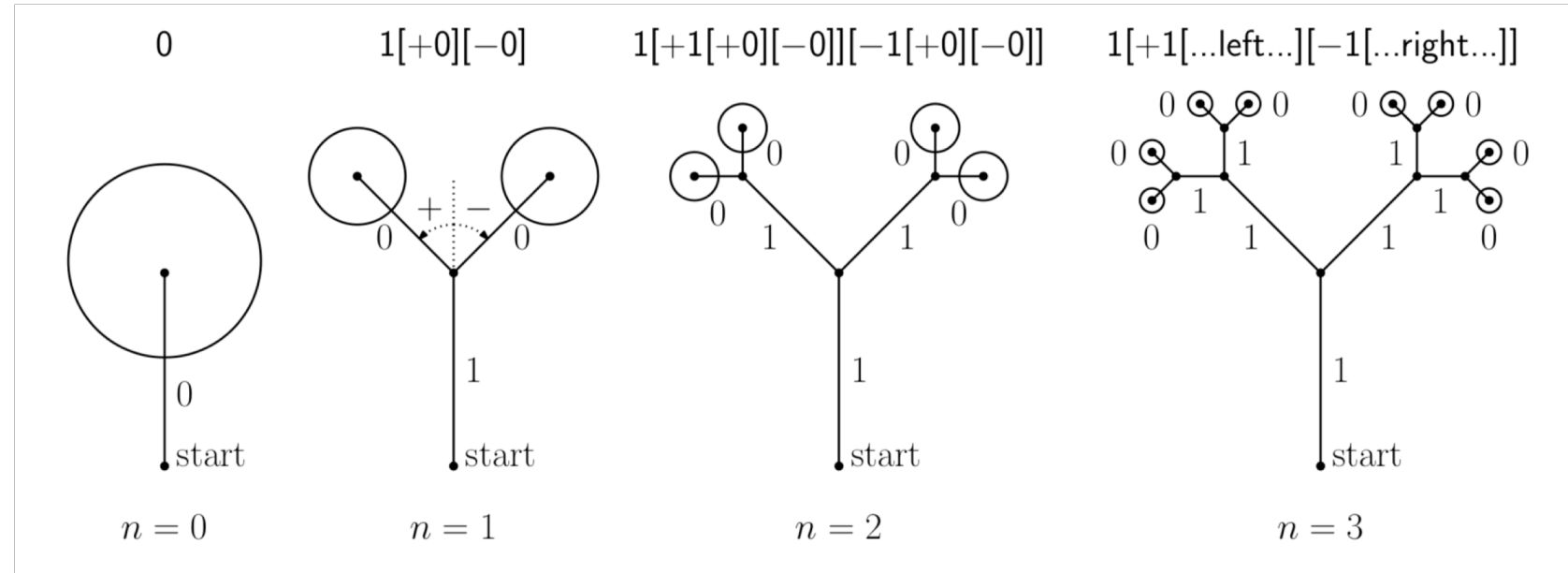
Symmetric bush 1

- $V = \{ F, +, -, [,] \}$
- $\omega = F$
- $\delta = 45$
- $F \rightarrow F[+F]F[-F]F$



Symmetric bush 2

- $V = \{0, 1, +, -, [,]\}$
- $\omega = 0$
- $\delta = 45$
- $P = \{0 \rightarrow 1[+0][-0]\}$



$n = 0 : 0$

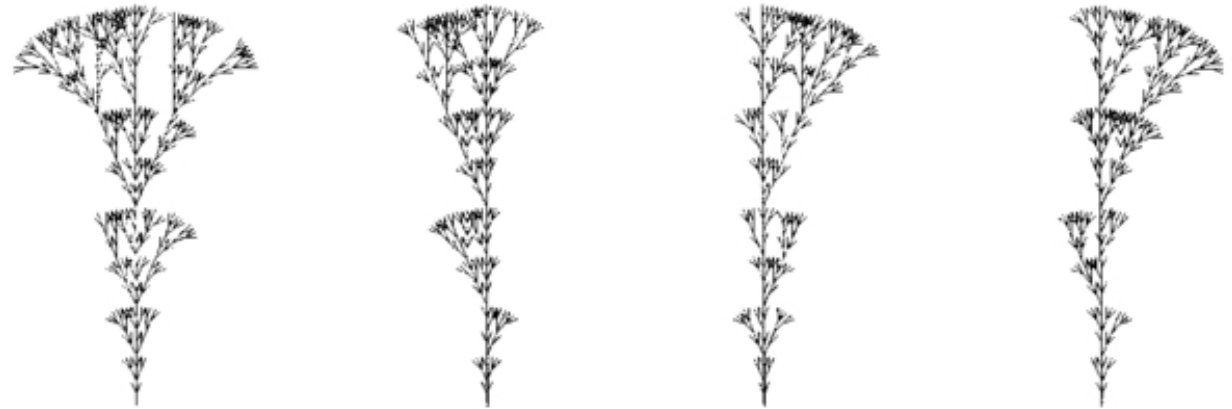
$n = 1 : 1[+0][-0]$

$n = 2 : 1[+1[+0][-0]][-1[+0][-0]]$

$n = 3 : 1[+1[+1[+0][-0]][-1[+0][-0]][-1[+1[+0][-0]][-1[+0][-0]]]$

Stochastic L-systems: Randomize

- Randomize production selection
- Choose rule 1 with $p=0.4$, rule 2 with $p=0.6$
- $a \rightarrow_{[0.4]} a [b]$
- $a \rightarrow_{[0.6]} b [a] b$
- Randomize parametric vocabulary
- $F \rightarrow F(d) +(\delta) F(d')$



Axiom F
Angle (δ) 22.5°
 $F^{.50} \rightarrow F[+F][-F]F;$
 $F^{.30} \rightarrow F[-F]F;$
 $F^{.20} \rightarrow F[+F]F$

3D L-system

- Add to vocabulary 3D turns

$$\mathbf{R}_U(\alpha) = \begin{bmatrix} \cos \alpha & \sin \alpha & 0 \\ -\sin \alpha & \cos \alpha & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\mathbf{R}_L(\alpha) = \begin{bmatrix} \cos \alpha & 0 & -\sin \alpha \\ 0 & 1 & 0 \\ \sin \alpha & 0 & \cos \alpha \end{bmatrix}$$

$$\mathbf{R}_H(\alpha) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos \alpha & -\sin \alpha \\ 0 & \sin \alpha & \cos \alpha \end{bmatrix}$$

- + Turn left by angle δ , using rotation matrix $\mathbf{R}_U(\delta)$.
- - Turn right by angle δ , using rotation matrix $\mathbf{R}_U(-\delta)$.
- & Pitch down by angle δ , using rotation matrix $\mathbf{R}_L(\delta)$.
- ^ Pitch up by angle δ , using rotation matrix $\mathbf{R}_L(-\delta)$.
- \ Roll left by angle δ , using rotation matrix $\mathbf{R}_H(\delta)$.
- / Roll right by angle δ , using rotation matrix $\mathbf{R}_H(-\delta)$.
- | Turn around, using rotation matrix $\mathbf{R}_U(180^\circ)$.

$n=7, \delta=22.5^\circ$

ω : A
 p_1 : A \rightarrow [&FL!A]/////' [&FL!A]/////' [&FL!A]
 p_2 : F \rightarrow S ///// F
 p_3 : S \rightarrow F L
 p_4 : L \rightarrow [',',^{\wedge}\{-f+f+f-|-f+f+f\}]



Making "real" trees

- Model real trees
- Manage self-intersections
- Add animation features
 - Sway, bend correctly
 - Grow from sapling

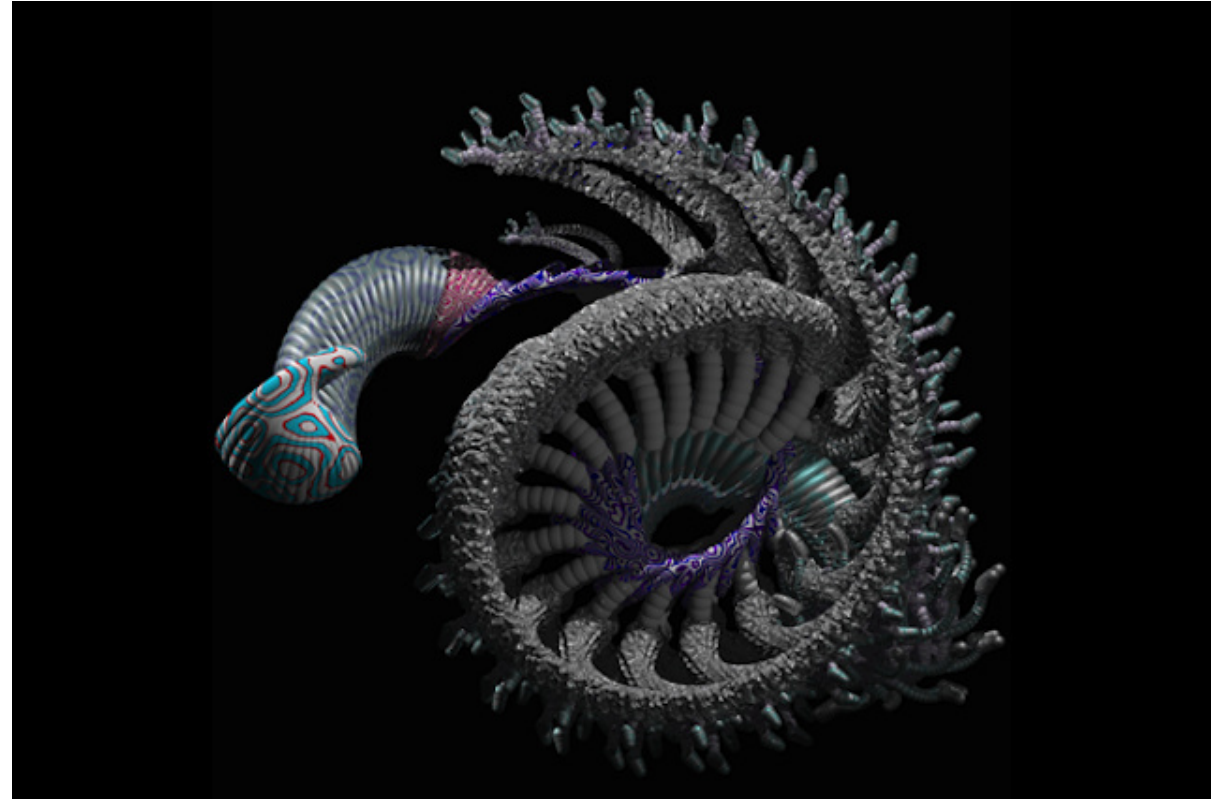


Not just trees

- Gian Carlo Mingati



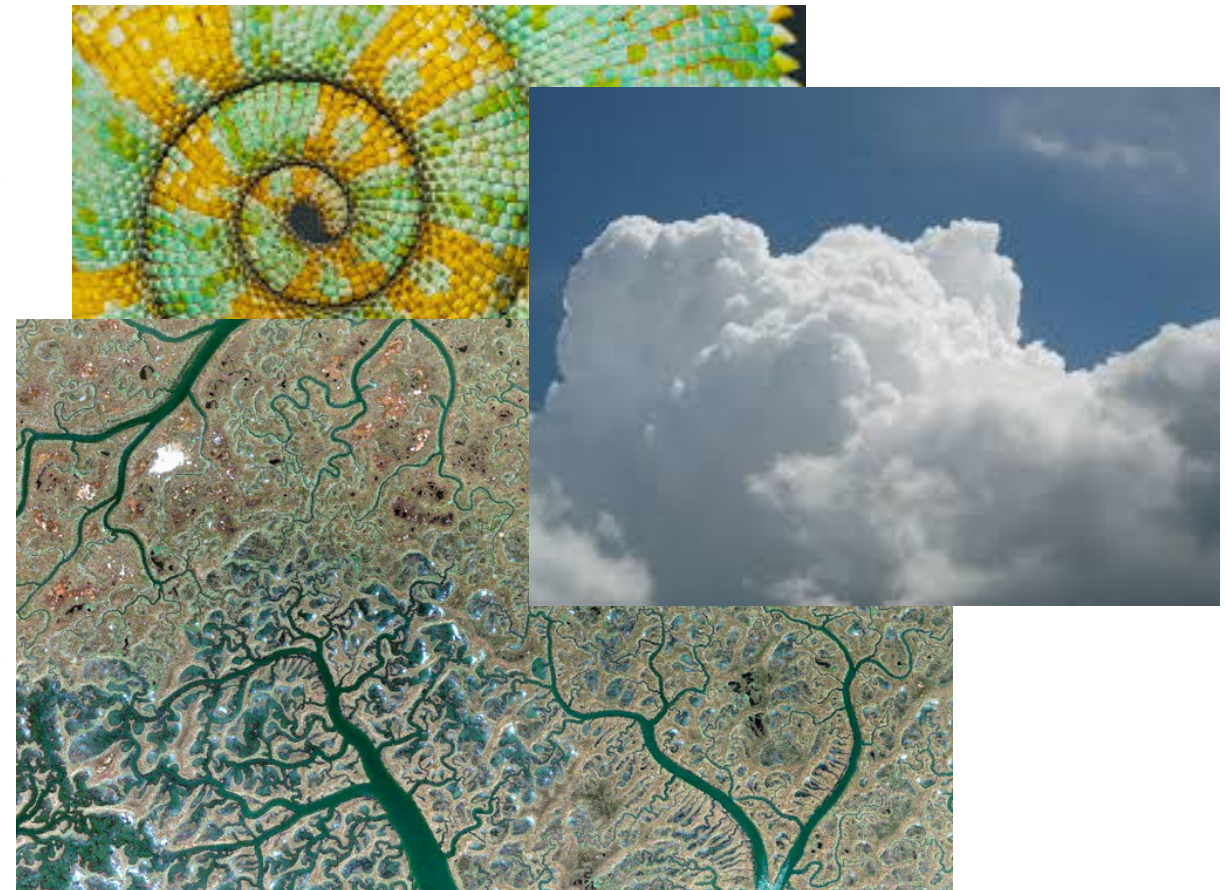
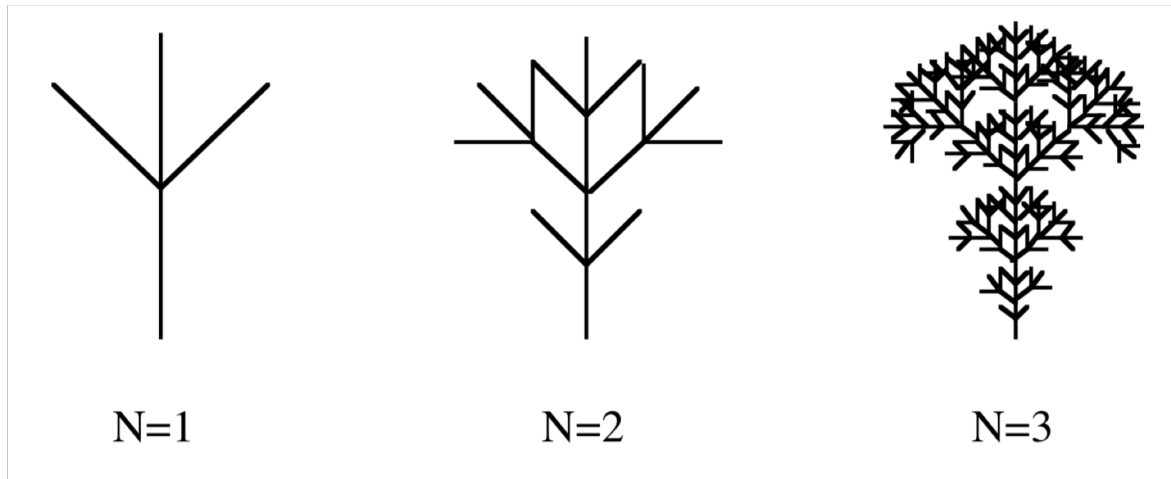
- William Latham/Stephen Todd



Self-similarity and fractal dimension

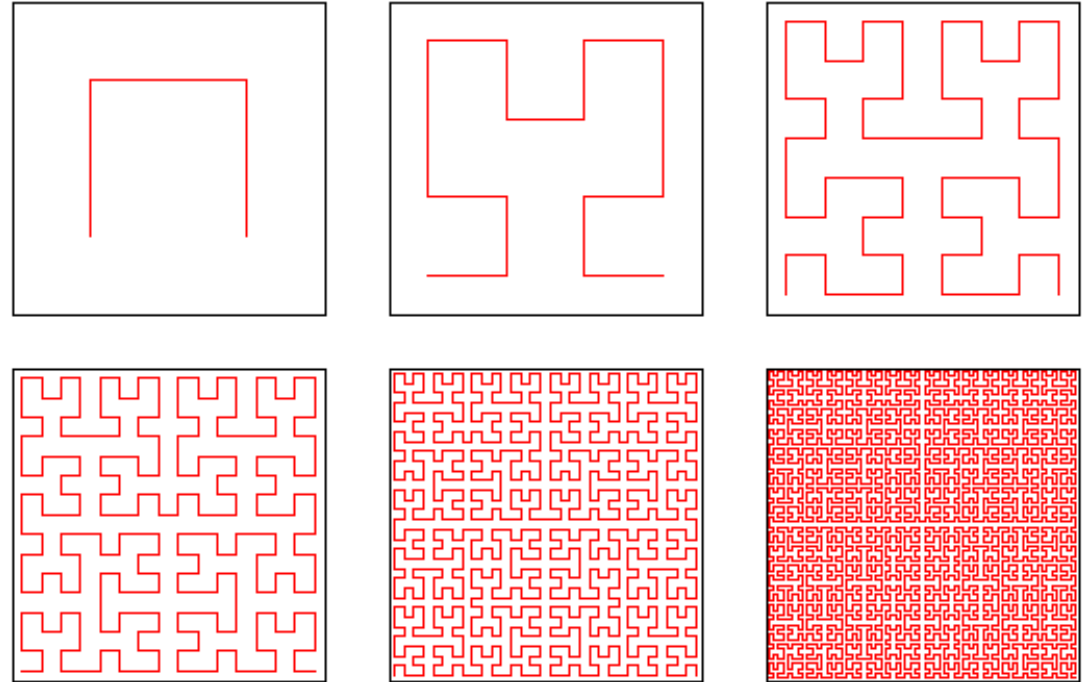
- L-system shapes are *self-similar*
- Part looks like whole

- Typical of many natural objects



Space filling curves

- Hilbert curves



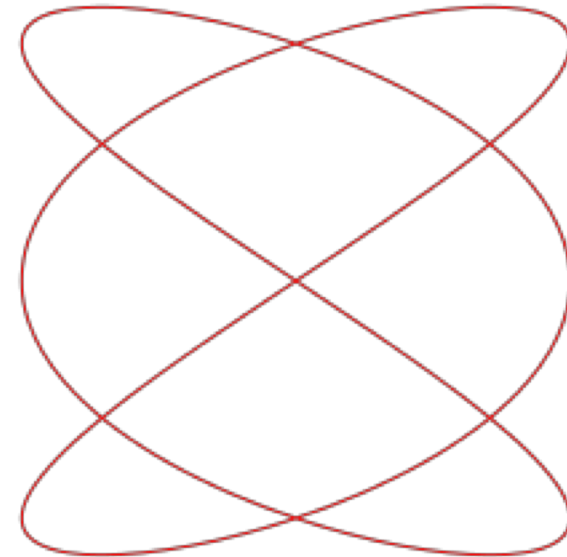
Dimension

- What is the dimension of a set of points?
- Point – zero dimensions
- Line – 1 dimension
- Plane – 2 dimensions
- Box – 3 dimensions

Dimension

- What is the dimension of a set of points?
- Point – zero dimensions
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- Curve?
 - Lives in 2D
 - Only takes 1D to trace out



Fractal dimension

- Measure of complexity of curve or surface
- How many parameters needed to describe?
- Related to self-similarity
- *Hausdorff* measure

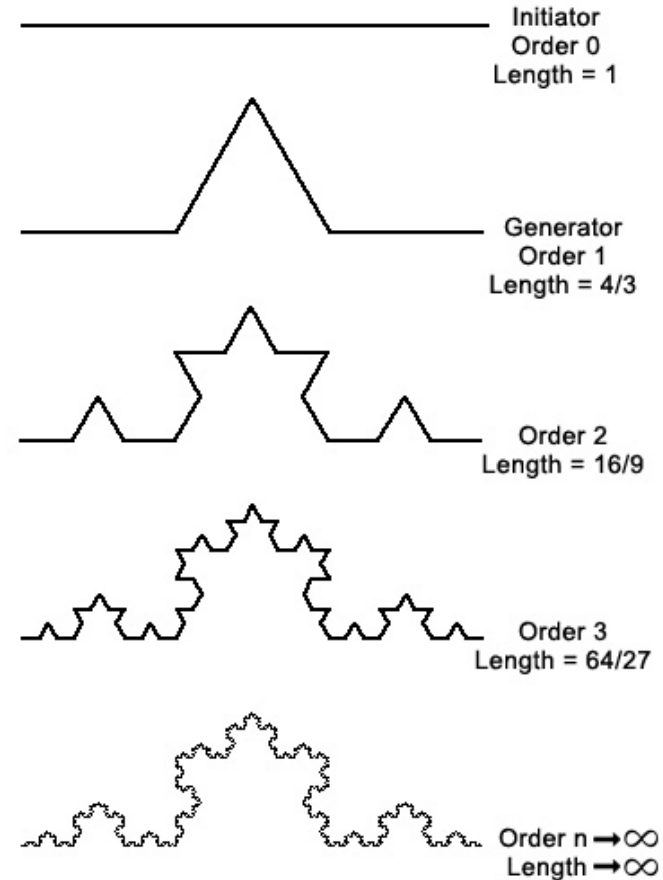


An example of the coastline paradox. If the coastline of [Great Britain](#) is measured using units 100 km (62 mi) long, then the length of the coastline is approximately 2,800 km (1,700 mi). With 50 km (31 mi) units, the total length is approximately 3,400 km (2,100 mi), approximately 600 km (370 mi) longer.

Length of fractal curve as $n \rightarrow \infty$

- Koch curve
- Each generation
 - 4 new parts
 - Each length $1/3$
 - New length $4/3 * \text{old}$

- $\lim_{n \rightarrow \infty} \left(\frac{4}{3}\right)^n = \infty$

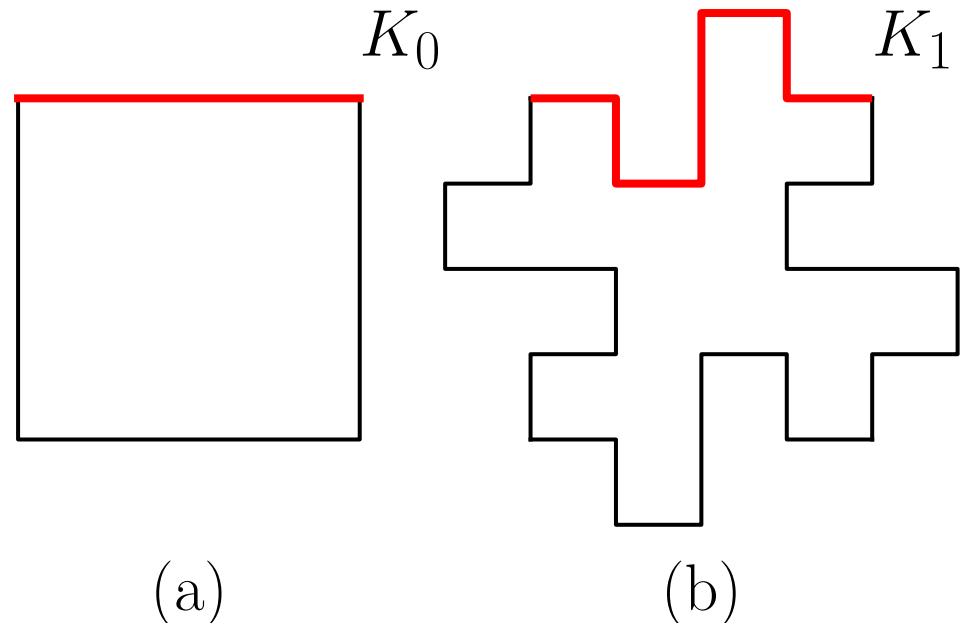


Koch island fractal dimension

- $b = \#$ of parts in recursive rule
- $a =$ scale factor

- Here

$$b = 8$$
$$s = \frac{1}{4} = 1/a \text{ so } a = 4$$



Koch island fractal dimension

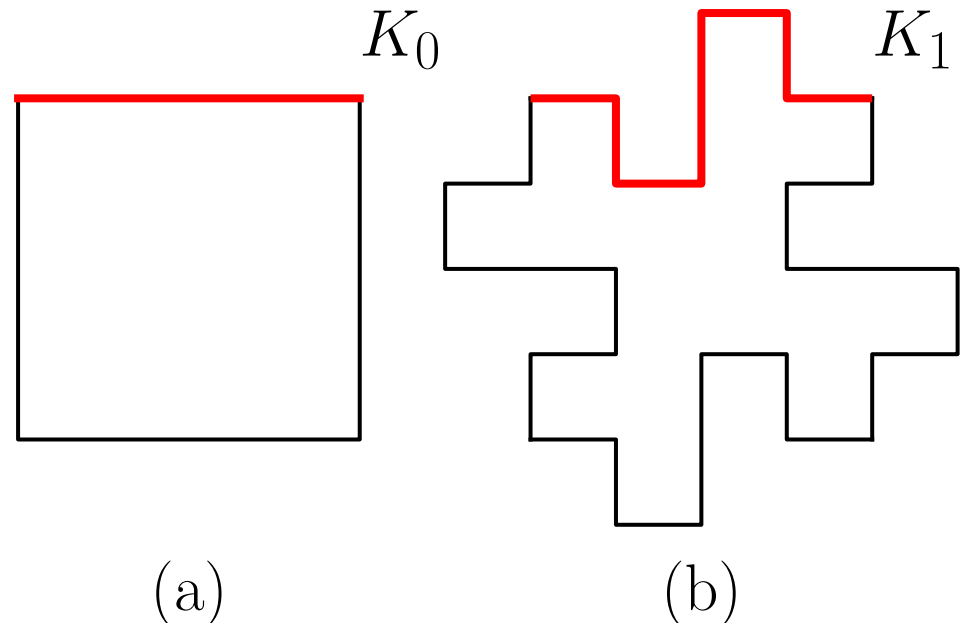
- $b = \#$ of parts in recursive rule
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- Here

$$b = 8$$

$$s = \frac{1}{4} = 1/a \text{ so } a = 4$$

- $D = \frac{\log(b)}{\log(a)} = \frac{\log(8)}{\log(4)} = \frac{3}{2} = 1.5$

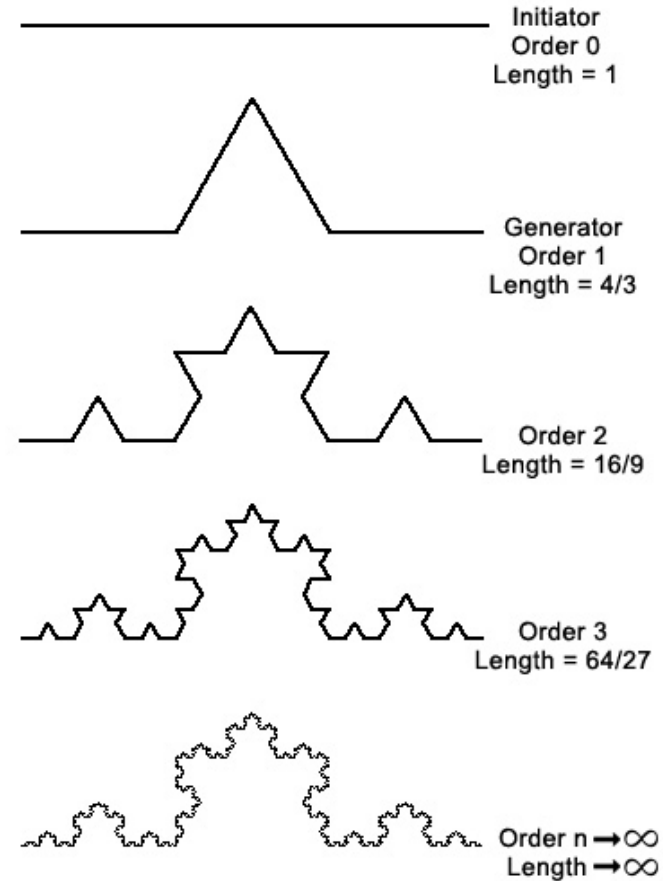


Koch curve dimension?

- $b = ??$

- $a = ??$

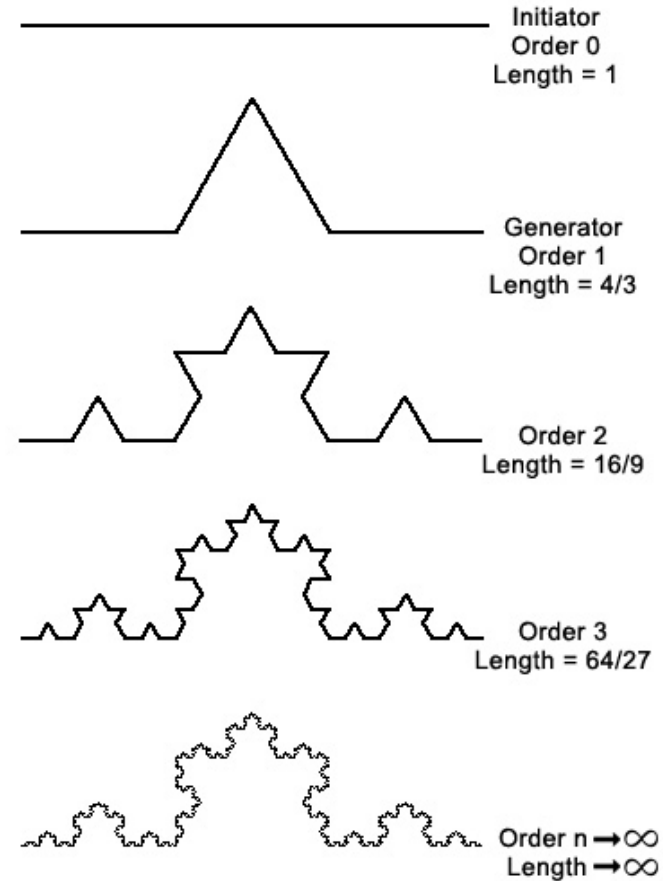
- $D = \frac{\log(b)}{\log(a)} = ??$



Koch curve dimension?

- $b = 4$
- $a = 3$

- $D = \frac{\log(4)}{\log(3)} \approx 1.26186$



Fractal dimension of line, plane, box?

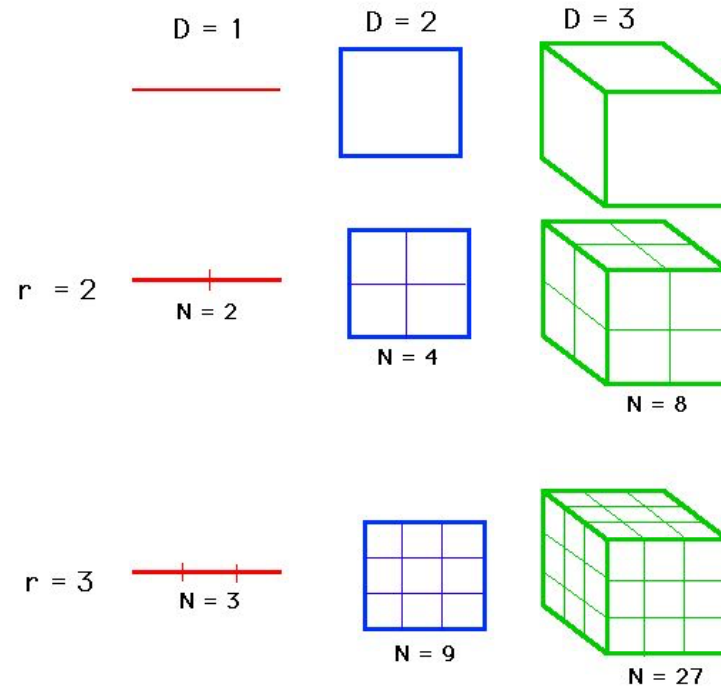


11.9

Hausdorff dimension

- **Hausdorff dimension** is defined as:

$$D \equiv \frac{\ln N}{\ln r}$$



$$\ln N = D \ln r$$

$$\ln N = \ln r^D$$

$$N = r^D$$

$$N = r^D$$

Purpose for fractal dimension

- First, fun. Interest calculation
- Second, can measure fractal dimension of real objects, match in your simulation
- Third, watch out for $d = 2$ or 3 !
- (Space filling ...)