Curves and Motion

CMSC425.01 Spring 2019
Administrivia

• Final project
  • Update for Monday (need to verify group membership for Elms)
  • Rubric

• Final homework (Hw3)

• Final midterm
  • Thursday May 8th
Final project rubric – from proposal handout

• A quality of planning and execution that can't be achieved in the last week.

• Work by all members of the team, documented by some record of your work schedule and individual contributions.

• Some innovation beyond copying an existing game, although it's not easy to be fully new in this space.

• Achievement relative to ambition. Try for something ambitious, and lack a little polish, ok. Try for less ambitious results, then make it look good.

• Non-trivial scripting, and scripts that aren't just copied as assets. Shapes and animations can be assets (although adding your own terrain or animation script would good.)
## Final project rubric

<table>
<thead>
<tr>
<th>Topic</th>
<th>Scoring 5/5</th>
<th>Weights</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Concept – Clear, consistent, not just copy</strong></td>
<td>/5</td>
<td>15</td>
</tr>
<tr>
<td><strong>Artistic – Consistent, good look (not mixed assets)</strong></td>
<td>/5</td>
<td>15</td>
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<tr>
<td><strong>Algorithmic – Non-trivial scripting somewhere</strong></td>
<td>/5</td>
<td>15</td>
</tr>
<tr>
<td><strong>Team work – everyone contributed, documented</strong></td>
<td>/5</td>
<td>15</td>
</tr>
<tr>
<td><strong>Completeness – All of it works, relative to ambition</strong></td>
<td>/5</td>
<td>20</td>
</tr>
<tr>
<td><strong>Group size – more people, higher expectations</strong></td>
<td>/5</td>
<td>10</td>
</tr>
<tr>
<td><strong>Video – video is submitted, clear</strong></td>
<td>/5</td>
<td>5</td>
</tr>
<tr>
<td><strong>Report – report is submitted, clear and complete</strong></td>
<td>/5</td>
<td>5</td>
</tr>
<tr>
<td><strong>Intangibles – instructor overall opinion</strong></td>
<td>/5</td>
<td>5</td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td></td>
<td>100</td>
</tr>
</tbody>
</table>
Asterisk *

• In your report you can spell out that
  
  • We did something ambitious and it didn’t quite work
  
  • We intentionally copied this game and here’s what we did a bit new
  
  • Anything else the grader should take into consideration
Today’s question

Curves for shapes and motion
Cubic interpolation

• \( P(t) = ax^3 + bx^2 + cx + d \)

• Can match tangents at ends
• Good enough for human eye
Bicubic surface patch

• Cubic curve in both directions
Polyline of control points
Piecewise interpolation vs. approximation

Interpolating – through points

Approximating – controlled by points
Piecewise continuity

• Continuity gives smoothness

• Applies to shape and motion

• Eg, Navmesh path

• We care about C1 continuity
  • Cubic curve enables
Calculating tangent at each point?

Problem – if we use vector to next point we don’t get C1 continuity
Solution: use vector $P(i-1)$ to $P(i+1)$

Use same vector at $P2$ for segments $P1$ to $P2$, and $P2$ to $P3$
Solution: use vector $P(i-1)$ to $P(i+1)$

Use same vector at $P2$ for segments $P1$ to $P2$, and $P2$ to $P3$
Scaling tangent => tightness of curve
Finding cubic coefficients a, b, c, d for segment

- \( P_x(t) = at^3 + bt^2 + ct + d \)
- \( P'_x(t) = 4at^3 + 3bt + c \)
Finding cubic coefficients $a, b, c, d$ for segment

\begin{align*}
\bullet \quad P_x(t) &= at^3 + bt^2 + ct + d \\
\bullet \quad P'_x(t) &= 3at^2 + 2bt + c \\
\bullet \quad P_x(0) &= x_1 = d \\
\bullet \quad P'_x(0) &= x_0 = c \\
\bullet \quad P_x(1) &= x_2 = a + b + c + d \\
\bullet \quad P'_x(1) &= x_3 = 3a + 2b + c
\end{align*}
System of equations in four unknowns

- $x_1 = d$
- $x_0 = c$
- $x_2 = a + b + c + d$
- $x_3 = 3a + 2b + c$

- Solve?

$$\begin{bmatrix} x_0 \\ x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 1 & 1 & 1 \\ 3 & 2 & 1 & 0 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix}$$
System of equations in four unknowns

\[ \begin{aligned}
\bullet & \ x_1 = d \\
\bullet & \ x_0 = c \\
\bullet & \ x_2 = a + b + c + d \\
\bullet & \ x_3 = 3a + 2b + c \\
\end{aligned} \]

\[
\begin{bmatrix}
  x_0 \\
  x_1 \\
  x_2 \\
  x_3 \\
\end{bmatrix} =
\begin{bmatrix}
  0 & 0 & 1 & 0 \\
  0 & 0 & 0 & 1 \\
  1 & 1 & 1 & 1 \\
  3 & 2 & 1 & 0 \\
\end{bmatrix} \begin{bmatrix}
  a \\
  b \\
  c \\
  d \\
\end{bmatrix}
\]

\[ \text{Solve? M inverse} \]

\[
\begin{bmatrix}
  a \\
  b \\
  c \\
  d \\
\end{bmatrix} =
\begin{bmatrix}
  1 & 2 & -2 & 1 \\
  -2 & -3 & 3 & -1 \\
  1 & 0 & 0 & 0 \\
  0 & 1 & 0 & 0 \\
\end{bmatrix} \begin{bmatrix}
  x_0 \\
  x_1 \\
  x_2 \\
  x_3 \\
\end{bmatrix}
\]
Using with Quadratic form  $\mathbf{q}^\top \mathbf{M} \mathbf{p}$

- $P_x(t) = at^3 + bt^2 + ct + d$

- $P_x(t) = [t^3 \ t^2 \ t \ 1] \begin{bmatrix} 1 & 2 & -2 & 1 \\ -2 & -3 & 3 & -1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_0 \\ x_1 \\ x_2 \\ x_3 \end{bmatrix}$

- $P(t) = [t^3 \ t^2 \ t \ 1] \begin{bmatrix} 1 & 2 & -2 & 1 \\ -2 & -3 & 3 & -1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} T_0 \\ P_1 \\ P_2 \\ T_3 \end{bmatrix}$
Spline equations

- Different constraints on points and tangents => different matrices

- Which one?

\[
P(t) = \sum_{i=0}^{3} P_i B_i(t)
= (1 - t)^3 P_0 + 3t(1 - t)^2 P_1 + 3t^2(1 - t)P_2 + t^3 P_3
= \begin{bmatrix}
(1 - t)^3 & 3t(1 - t)^2 & 3t^2(1 - t) & t^3
\end{bmatrix}
\begin{bmatrix}
P_0 \\
P_1 \\
P_2 \\
P_3
\end{bmatrix}
= \begin{bmatrix}
1 & 0 & 0 & 0 \\
-3 & 3 & 0 & 0 \\
3 & -6 & 3 & 0 \\
-1 & 3 & -3 & 1
\end{bmatrix}
\begin{bmatrix}
P_0 \\
P_1 \\
P_2 \\
P_3
\end{bmatrix}
\]

Beziers, Cardinal Spline, and B-Spline Curves
Summary

- Take control points
- Compute curve/surface coefficients
  - Represent as matrix
- Draw parametrized curve/surface

\[
P(u,v) = \sum_{j=0}^{3} \sum_{i=0}^{3} P_{i,j} B_{i,3}(u) B_{j,3}(v)
\]

\[
= \sum_{j=0}^{3} \left[ \begin{array}{ccc}
1 & u & u^2 & u^3
\end{array} \right] \left[ \begin{array}{cccc}
1 & 0 & 0 & 0 \\
-3 & 3 & 0 & 0 \\
3 & -6 & 3 & 0 \\
-1 & 3 & -3 & 1
\end{array} \right] B_{j,3}(v)
\]

- Bezier patch 16 control points
Crowd motion

- How do multiple agents move without colliding?
- Detect collisions in advance
Crowd motion

• How do multiple agents move without colliding?
• Detect collisions in advance

• Dynamic obstacles – anticipate where someone will be
Crowd motion

- Model each agent with
- Current position $P_i(0)$
- Current velocity $\vec{v}_i(0)$
- Target velocity $\vec{v}_i^0(0)$
  - Towards goal

- Forces $\vec{F}_i(0)$ push on agent
Possible forces

• Like boid flocking?
Possible forces

• Like boid flocking?
• Separation
• Obstacle Avoidance
• Attraction
• Traffic signals and social conventions
• Individual variations
Velocity obstacles

• Compute forbidden velocities
  • That would lead to collision

• Example
  • Agent $\mathbf{a}$ walking towards obstacle $\mathbf{b}$

  • What velocities at time $i$ cause collision?
Velocity obstacles

• Compute forbidden velocities
  • That would lead to collision

• Example
  • Agent \( a \) walking towards obstacle \( b \)

  • What velocities at time \( i \) cause collision?

  • Velocity \( v = (pb - pa) \) causes immediate collision

  • Others?
Velocity obstacles

• Region $V_0^{(a|b)}$ of forbidden velocities

• Cone around Ball $B(p_b-p_a, r_a+r_b)$
Velocity obstacles

• Region $V_{0(a|b)}$ of forbidden velocities

• Cone around Ball $B(p_b - p_a, ra + rb)$

• Apex of cone is what?
Velocity obstacles

• Region $V_0(a|b)$ of forbidden velocities

• Apex of cone is very slow velocities

• Limit length of future time to $(0,\tau)$

• Limiting time truncates cone – why?
Obstacle \( \mathbf{b} \) moving?

\[
\begin{align*}
\text{(a)} & \quad \text{VO}_a^\tau \mathbf{b} + \mathbf{v}_b \\
\text{(b)} & \quad \text{VO}_a^\tau \mathbf{b} \oplus \mathbf{V}_b
\end{align*}
\]

\[
\begin{align*}
\text{CA}_a^\tau (\mathbf{V}_b) & \quad \mathbf{V}_b
\end{align*}
\]
Who is responsible for avoiding collision?

- Both agents fully responsible
- Oscillating motion

- Other avoids
- Your path becomes clear
- You resume original path
- Collision!
Avoiding multiple agents

- Simplify each agent’s forbidden velocity region to half plane
- Intersect acceptable velocity regions to get polygon
- Take velocity $v’_a$ nearest to target velocity $v^*_a$
Lin and Manocha

• https://www.youtube.com/watch?v=lyyyEcy_9so

• https://www.youtube.com/watch?v=xme4pRelwJ0
Problem 3. (20 points) Consider the collection of shaded rectangular obstacles shown in the figure below, all contained within a large enclosing rectangle. Also, consider the triangular robot, whose reference point is located at a point $s$. (You may take $s$ to be the origin.)

![Figure 2: Problem 3.](image)

(a) Draw the C-obstacles for the three rectangular obstacles, including the C-obstacle from region lying outside the large enclosing rectangle.

(b) Either draw an obstacle-avoiding path for the robot from $s$ to $t$, or explain why it doesn’t exist.