Geometry and Geometric Programming III

CMSC425.01 Spring 2019

Still at tables …

Administrivia

- Project 1a grades released tonight
- Final project introduction this week
- Hw1 posted to web site, due next Sunday

Final project proposals

Include

- Team members
- Game title
- General description
- Platform and resources
- Coordination

Final project proposals

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- General description
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Advice

- Teams of 2-3 best, > 4 ask
- Demoable at end of semester
- Do one thing well
- Involve entire team
- Design in layers
- K.I.S.S. (look it up …)

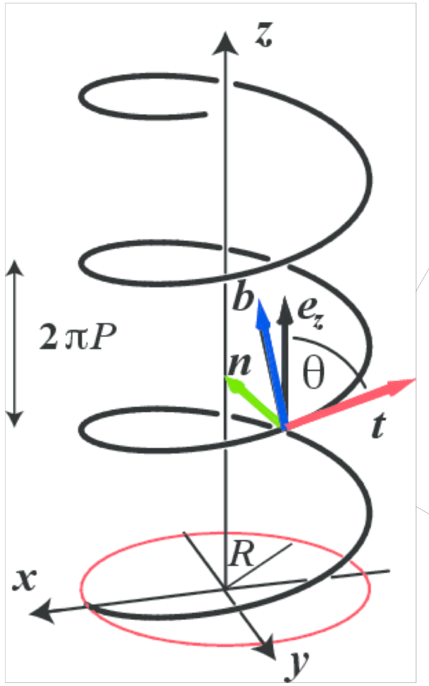
Today’s question

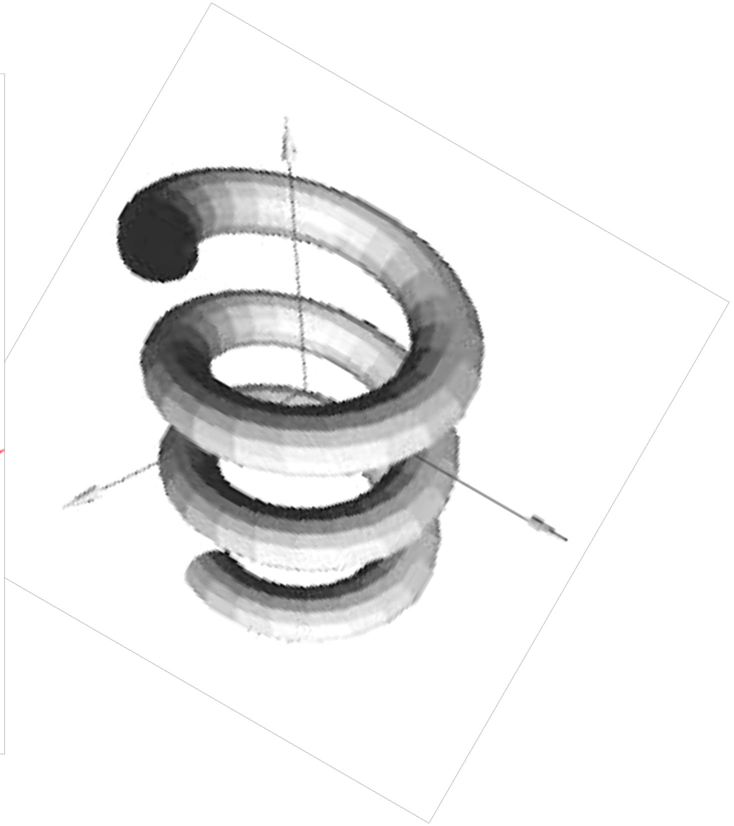
How do we move and orient shapes?

Examples

- Rotate moon around Earth around sun (multiple motions)

- Orient cylinder sections of 3D helix

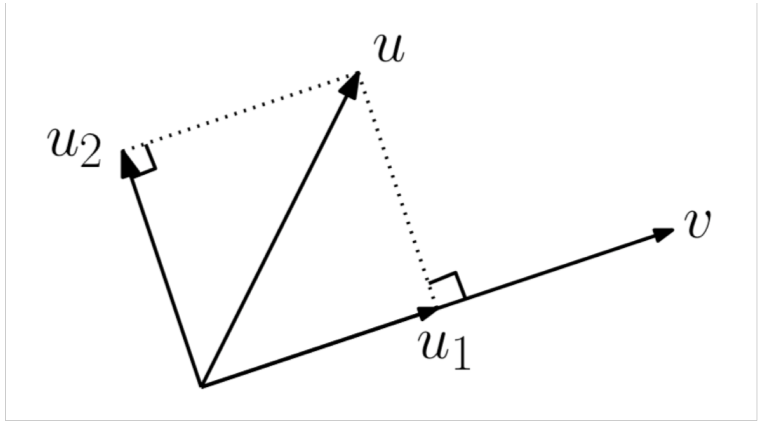




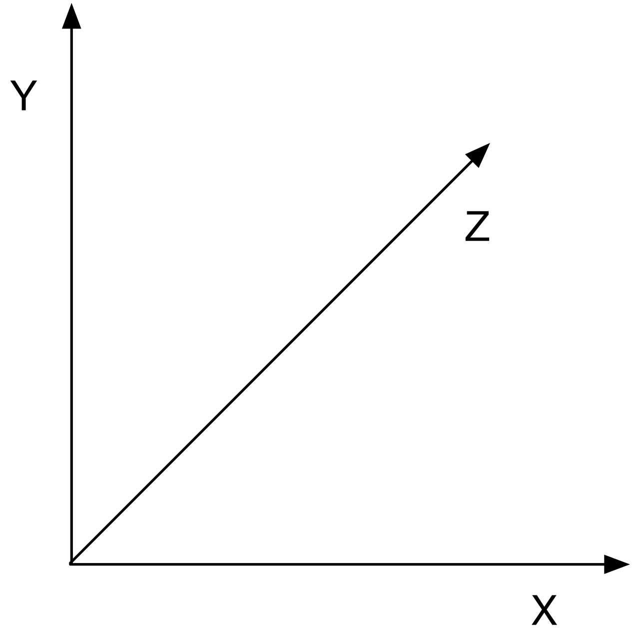
Start with frame of references

Global or local coordinate system in which to define pts and vectors

- 2D

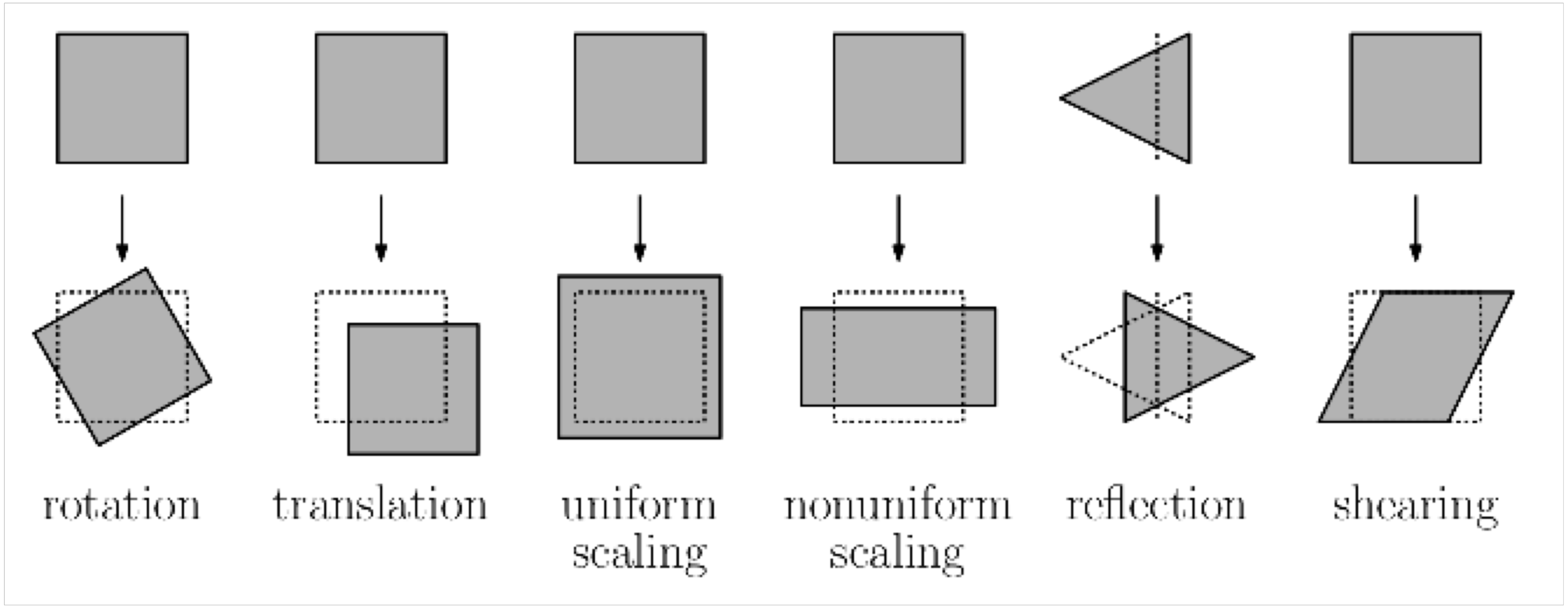


- 3D



Affine transformations

- Key: translation, rotation, scale



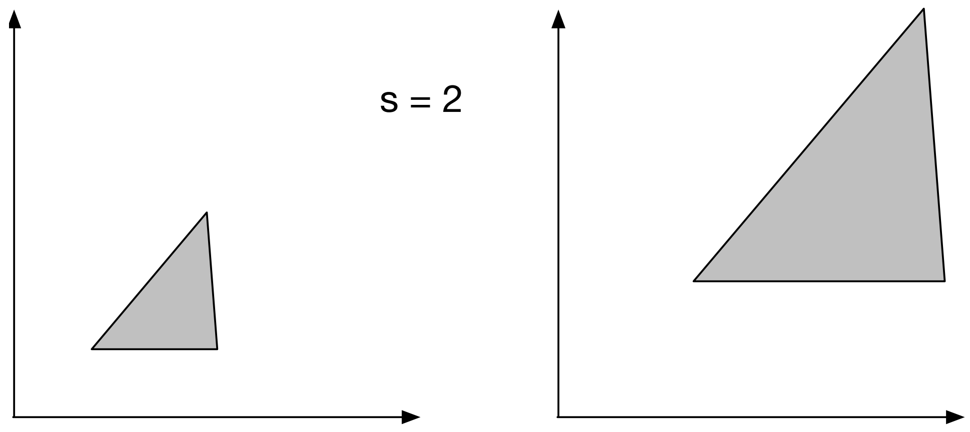
Scaling

- Coordinate free - uniform scale s

$$v = su$$

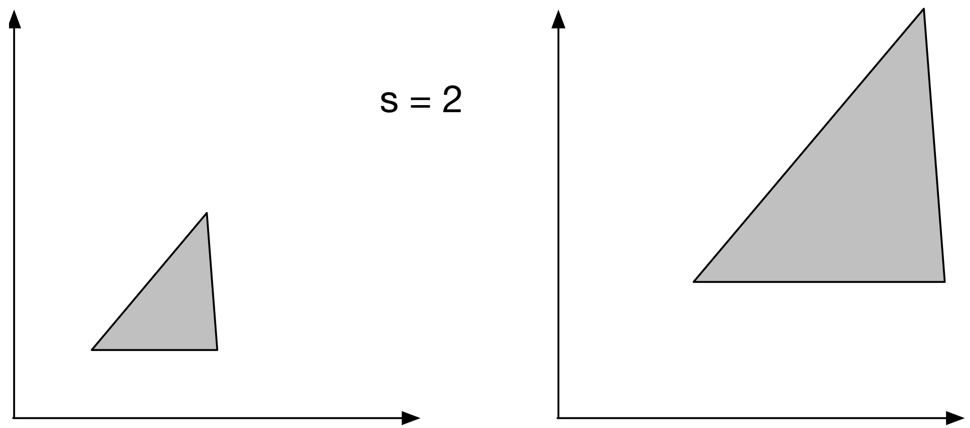
- Coordinate based

$$< v_x, v_y, v_z > = < su_x, su_y, su_z >$$



- Scaling sizes and moves

Scaling



- Coordinate free – uniform scale s

$$v = su$$

- Coordinate based

$$< v_x, v_y, v_z > = < su_x, su_y, su_z >$$

- Scaling sizes and moves

- Homogeneous coordinates – vector

$$< v_x, v_y, v_z , 0 > = < su_x, su_y, su_z , 0 >$$

- Homogeneous coordinates – points (simple scalar * doesn't work)

$$(v_x, v_y, v_z , 1) = (su_x, su_y, su_z , s)$$

Scaling

- Matrix form 2D

$$v^t = M_s u^t$$

$$M_s = \begin{bmatrix} s & 0 & 0 \\ 0 & s & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

- Vector

$$< v_x, v_y, 0 > =< su_x, su_y, 1 * 0 >$$

- Point

$$(q_x, q_y, 1) \ =< sp_x, sp_y, 1 * 1 >$$

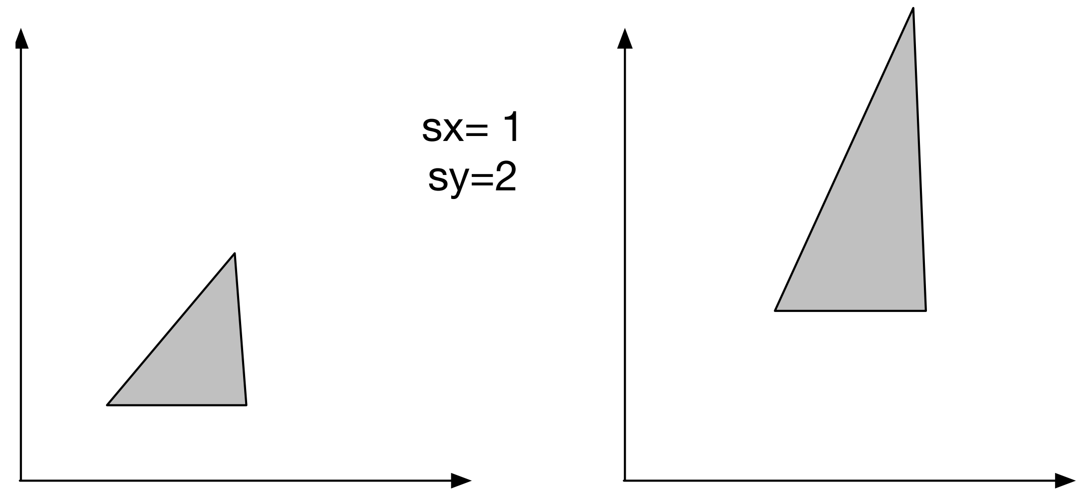
- Matrix multiplication on the right with transpose of vector v^t
- Works for vectors and points
- Maintains homogeneous coordinate w

Scaling – non-uniform

- Matrix form 2D
-

$$v = M_s u$$

$$M_s = \begin{bmatrix} s_x & 0 & 0 \\ 0 & s_y & 0 \\ 0 & 0 & 1 \end{bmatrix}$$



Translation

- Matrix form 2D
-

$$v = M_t u$$

$$M_t = \begin{bmatrix} 1 & 0 & t_x \\ 0 & 1 & t_y \\ 0 & 0 & 1 \end{bmatrix}$$

- Translate point

$$(q_x, q_y, 1) = \begin{bmatrix} 1 & 0 & t_x \\ 0 & 1 & t_y \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} p_x \\ p_y \\ 1 \end{bmatrix}$$

$$(q_x, q_y, 1) = \left(p_x + t_x, p_y + t_y, 1\right)$$

First version: coordinate based equations

- Translation by v: $q = p + T(v)$ Add vector v
- Scale by a: $q = a\ p$ Multiply by scalar a
- Rotate by t: $(qx,qy) = \langle px*\cos(t) – py*\sin(t),\ px*\sin(t) + py*\cos(t)\rangle$

- Repeated scalings and translations:

- $q = a\ (\ p + T(V)\ ) = a\ (\ (a\ p +T(V)) + T(v)) =$ and so on …

- Complex

Second version: Homogeneous coordinates

- Unify all transformations in matrix notation

$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

Identity Matrix

$$\begin{pmatrix} 1 & 0 & 0 & tx \\ 0 & 1 & 0 & ty \\ 0 & 0 & 1 & tz \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

glTranslatef(tx,ty,tz)

$$\begin{pmatrix} sx & 0 & 0 & 0 \\ 0 & sy & 0 & 0 \\ 0 & 0 & sz & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

glScalef(sx,sy,sz)

$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos(d) & -\sin(d) & 0 \\ 0 & \sin(d) & \cos(d) & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

glRotatef(d,1,0,0)

$$\begin{pmatrix} \cos(d) & 0 & \sin(d) & 0 \\ 0 & 1 & 0 & 0 \\ -\sin(d) & 0 & \cos(d) & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

glRotatef(d,0,1,0)

$$\begin{pmatrix} \cos(d) & -\sin(d) & 0 & 0 \\ \sin(d) & \cos(d) & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

glRotatef(d,0,0,1)

Chalkboard – review all transformations

Defining rotations

- Euler angles
- Angle Axis
- Quaternions

Roll – around forward direction

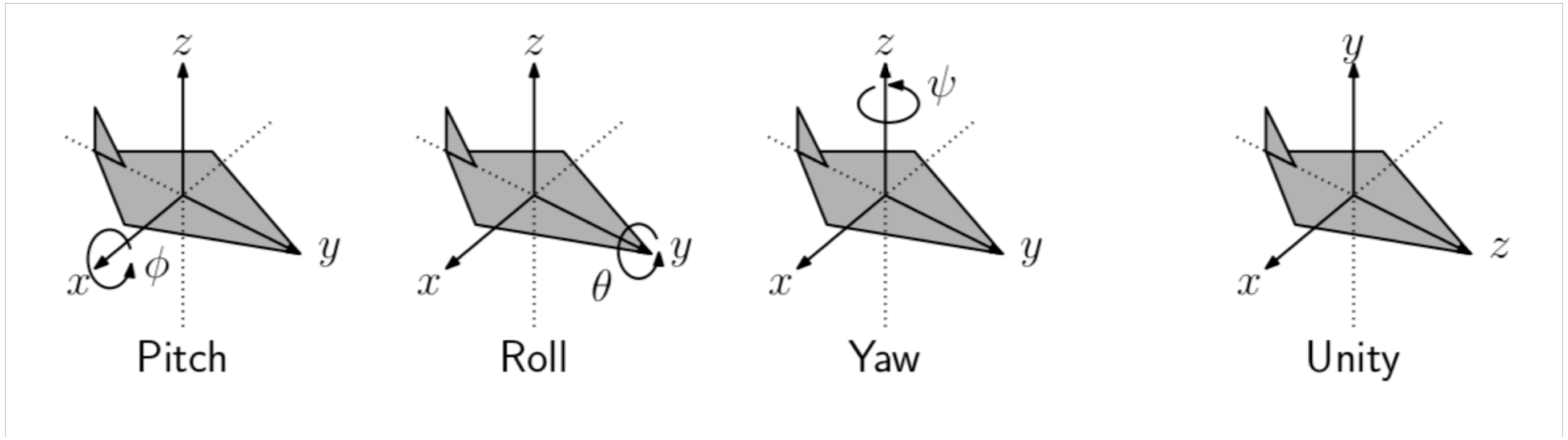
Pitch – around right direction

Yaw – around up direction

- In Unity

    transform.Rotate(x, y, z))

- Euler angles in order z, x, y



Defining rotations

- Angle Axis

Quaternion.AngleAxis

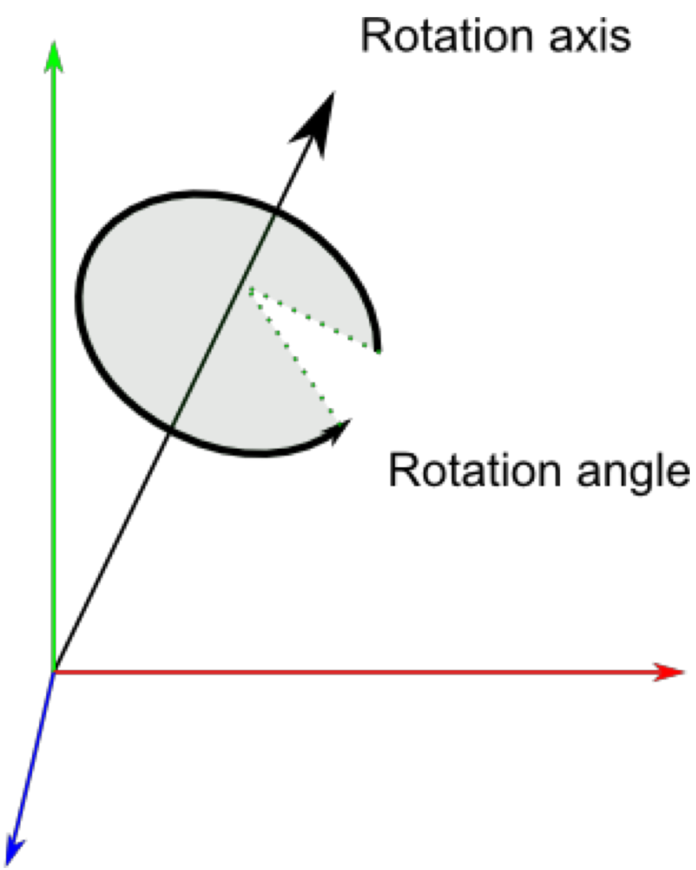
public static Quaternion **AngleAxis**(float **angle**, Vector3 **axis**);

Description

Creates a rotation which rotates `angle` degrees around `axis`.

```
using UnityEngine;

public class Example : MonoBehaviour
{
    void Start()
    {
        // Sets the transforms rotation to rotate 30 degrees around the y-axis
        transform.rotation = Quaternion.AngleAxis(30, Vector3.up);
    }
}
```



Interpolating transformations

- Translation. Easy – move v*dt each frame
- Scale. Easy – scale by s*dt each frame

- Interpolating rotations? Harder
  - Interpolate Euler angles? Doesn’t work well
  - Interpolate Axis Angle? Better
  - Interpolate Quaternions? Best Why Unity uses them.

Quaternion.Slerp

public static Quaternion **Slerp**(Quaternion **a**, Quaternion **b**, float **t**);

Description

Spherically interpolates between `a` and `b` by t. The parameter `t` is clamped to the range [0, 1].

```
// Interpolates rotation between the rotations "from" and "to"
// (Choose from and to not to be the same as
// the object you attach this script to)

using UnityEngine;
using System.Collections;

public class ExampleClass : MonoBehaviour
{
    public Transform from;
    public Transform to;

    private float timeCount = 0.0f;

    void Update()
    {
        transform.rotation = Quaternion.Slerp(from.rotation, to.rotation, timeCount);
        timeCount = timeCount + Time.deltaTime;
    }
}
```

Activity 4b: Build a computer game

- At each table plan out a game for your team. Answer these questions (quickly!)

- What platform(s)?
- Any special hardware or peripherals needed?
- What software elements needed?
- Build from scratch or use engine? Which language or engine?
- What assets will you need? How will you make or get them?

Given vectors u, v, and w, all of type Vector3, the following operators are supported:

```
u = v + w; // vector addition
u = v - w; // vector subtraction
if (u == v || u != w) { ... } // vector comparison
u = v * 2.0f; // scalar multiplication
v = w / 2.0f; // scalar division
```

You can access the components of a Vector3 using as either using axis names, such as, u.x, u.y, and u.z, or through indexing, such as u[0], u[1], and u[2].

The Vector3 class also has the following members and static functions.

```
float x = v.magnitude; // length of v
Vector3 u = v.normalize; // unit vector in v's direction
float a = Vector3.Angle (u, v); // angle (degrees) between u and v
float b = Vector3.Dot (u, v); // dot product between u and v
Vector3 u1 = Vector3.Project (u, v); // orthog proj of u onto v
Vector3 u2 = Vector3.ProjectOnPlane (u, v); // orthogonal complement
```

Some of the Vector3 functions apply when the objects are interpreted as points. Let p and q be points declared to be of type Vector3. The function Vector3.Lerp is short for *linear interpolation*. It is essentially a two-point special case of a convex combination. (The combination parameter is assumed to lie between 0 and 1.)

```
float b = Vector3.Distance (p, q); // distance between p and q
Vector3 midpoint = Vector3.Lerp(p, q, 0.5f); // convex combination
```

Readings

- David Mount's lectures on Geometry and Geometric Programming