

CMSC425 Transformation notes

References:

Mount, Lecture 6: Affine Transformations and Rotations plus Rotation Matrix handout

Outline:

I. Representations

- We represent graphical objects using points, vectors, lines, polylines, patches.
- We transform them using homogeneous coordinates

II. Two-d transforms

A. Representation using matrices and homogenous coordinates

Form: $M \cdot P$, P column vector - left multiplication

B. Basic affine transforms

- Translation
- Scaling
- Rotation
- Shear
- Reflection

C. Inverse of a transform

D. Composition of transforms

- Rotation about an arbitrary pivot point
- Scaling and shearing about same
- Other examples

E. Properties of affine transformations

- Lines into lines, planes into planes
- Ratios are preserved
- Areas go by $\det M$

II. Three-d transforms

A. Elementary transforms

- Translation
- Scaling
- Shearing
- Rotations
 - Euler angles: elementary rotations about x, y and z
Roll, pitch, yaw
 - Rotations around an arbitrary axis – angle axis
 - Finding the rotation matrix from the new axes directly (Handout)

B. Quaternions

- Why use quaternions. Smooth interpolation, no gimbal lock
- Review of imaginary numbers $a + bi$
Operations: product, conjugate $a - bi$
- Quaternions $\mathbf{q} = q_0 + iq_1 + jq_2 + kq_3$. i, j, k
 $\mathbf{q} = (s, \mathbf{u})$ s scalar, \mathbf{u} vector
- Operations: conjugate $(s, -\mathbf{u})$, modulus, multiplication
- Rotating vector by quaternion