## Notes on line and plane representations

Line representations		
Two points	Point + vector	Point + normal
B	v=B-A A	n = v_perp

*Readings*: Hill, chapter 4 minus 4.5.1, 4.4.2, 4.5.4, 4.6.1, all of 4.8

If t is in [0,1], segment; in [0,inf], ray; in [-inf,inf], line Linear, affine, convex combo

P(t) = A + tv

 $n \bullet (A - P) = 0$ 

Implicit equation

## Applications

- Distance of point to line (use n from pt+normal form)
- Resolve vector to linear combination of two vectors (use pt+vector form, plus n)

Vector parametric

- Reflection vector (use pt+normal form)
- Tweening (use blending parametric)

P(t) = (1-t)A + tB

Blending parametric

- Perpendicular bisector (use pt+normal form) => midpoint displacement algorithm
- Intersection of two lines (use pt-vector & pt-vector, or pt-vector & pt-normal)

Plane representations			
Three points	Point + two vectors	Point + normal	
C B A	C v2=B-A v1=B-A A	$n = v1 \times v2$	
P(s,t) = sA + tB + (1-s-t)C	P(t) = A + sv1 + tv2	$n \bullet (A - P) = 0$	
Blending parametric	Vector parametric	Implicit equation	
If s, t are in [0,1], patch; in [0,inf], half-plane; in [-inf,inf], plane			

## Applications

- Distance of point to plane (use n from pt-normal form)
- Intersection of ray with plane (use vector parametric line with pt-normal plane)

## Other applications of vector operations

- Angle between two vectors
- Sign of angle between two vectors
- Simplicity of polygon (use line-line intersection)
- Winding direction (use area formula for polygon if area > 0, clockwise)
- Convexity of polygon (use cross product for adjacent lines)

• Normalize vector (find magnitude)