Problem 1. Use mathematical induction to show the following:

(a) \[\sum_{i=1}^{n} i(i + 1) = \frac{n(n + 1)(n + 2)}{3}\]

(b) \[\sum_{i=0}^{n} 2^i = 2^{n+1} - 1\]

Problem 2.

(a) Assume \(b^x = a\). What is \(x\) (in terms of \(a\) and \(b\))?  
(b) Using only part (a), show that \(\log_c(ab) = \log_c a + \log_c b\).  
(c) Show that \(a^{\log_b n} = n^{\log_b a}\).

Problem 3. Differentiate the following functions:

(a) \(\ln(x^2 + 5)\)  
(b) \(\lg(x^2 + 5)\) [NOTE: In Computer Science we use \(\lg x\) to mean \(\log_2 x\).]  
(c) \(\frac{1}{\ln(x^2 + 5)}\)

Problem 4. Integrate the following functions:

(a) \(\frac{1}{x}\)  
(b) \(\frac{1}{x+3}\)  
(c) \(\ln x\) [HINT: Use integration by parts.]  
(d) \(x \ln x\) [HINT: Use integration by parts.]  
(e) \(x \lg x\)

Problem 5. Consider the formula \(3n^4 + 7n^3 \log n + 2n^2\).

(a) What is the high order term?  
(b) What is the second order term?  
(c) Write the formula in \(\Theta\) notation (in simplest form).