There are eight problems. Within reason, you should show your work.

Problem 1. Evaluate the following sums (by hand).

(a)
$$\sum_{i=1}^{4} i(i+1)$$

(b)
$$\sum_{i=0}^{4} 2^i$$

Problem 2. Write
$$3\sum_{i=1}^{n}(5i^2 - 4) \; - \; 2\sum_{i=1}^{n}(3i^2 - 1)$$
as a single summation.

Problem 3. Use mathematical induction to show the following:

(a)
$$\sum_{i=1}^{n} i(i+1) \; = \; \frac{n(n+1)(n+2)}{3}$$

(b)
$$\sum_{i=0}^{n} 2^i = 2^{n+1} - 1$$

Problem 4.   Assume that you guess that
$$\sum_{i=0}^{n} 2^i \; = \; a2^n + b$$

for constants a and b. Use constructive induction to verify the formula and derive a and b.

Problem 5.

(a) Assume $b^x = a$. What is x (in terms of a and b)?

(b) Using only part (a), show that $\log_c(ab) = \log_c a + \log_c b$.

(c) Show that $a^{\log_b n} = n^{\log_b a}$.

Problem 6. Differentiate the following functions:

(a) $\ln(x^2 + 5)$

(b) $\lg(x^2 + 5)$ [NOTE: In Computer Science we use $\lg x$ to mean $\log_2 x$.]

(c) $\frac{1}{\ln(x^2+5)}$

Problem 7. Integrate the following functions:

(a) $\frac{1}{x}$

(b) $\frac{1}{7x+3}$

(c) $\ln x$ [HINT: Use integration by parts.]

(d) $x \ln x$ [HINT: Use integration by parts.]

(e) $x \lg x$

Problem 8. Consider the formula $3n^4 + 7n^3 \log n + 2n^2$.

(a) What is the high order term?

(b) What is the second order term?

(c) Write the formula in Θ notation (in simplest form).