Problem 1. Assume that each word of your machine has 32 bits. Assume that you can multiply two n-word numbers in time $2n^2$ with a standard algorithm. Assume that you can multiply two n-word numbers in time $10n^{\lg 3}$ with a "fancy" algorithm. For each part *briefly justify* and *show your work*.

(a) Approximately how large does n have to be for the fancy algorithm to be better?

(b) Approximately how many bits is that?

(c) Approximately how many decimal digits is that?

Problem 2. Assume that you multiply the two two-digit numbers 36 and 52 using the method that does only three atomic multiplies. Show that steps of the algorithm on this example.

Problem 3. Consider the following recurrence, defined for n a power of 4 (for the time of some algorithm):

$$T(n) = \begin{cases} 3 & \text{if } n = 1 \\ 2T(n/4) + 4n + 1 & \text{otherwise} \end{cases}$$

(a) Calculate $T(16)$ by hand. Show your work.

(b) Use the tree method to solve the recurrence exactly, assuming n is a power of 4.

(c) Use the formula to calculate $T(1)$. Show your work.

(d) Use the formula to calculate $T(4)$. Show your work.

(e) Use the formula to calculate $T(16)$. Show your work.

(f) Use Mathematical Induction to prove that your formula is correct in general.