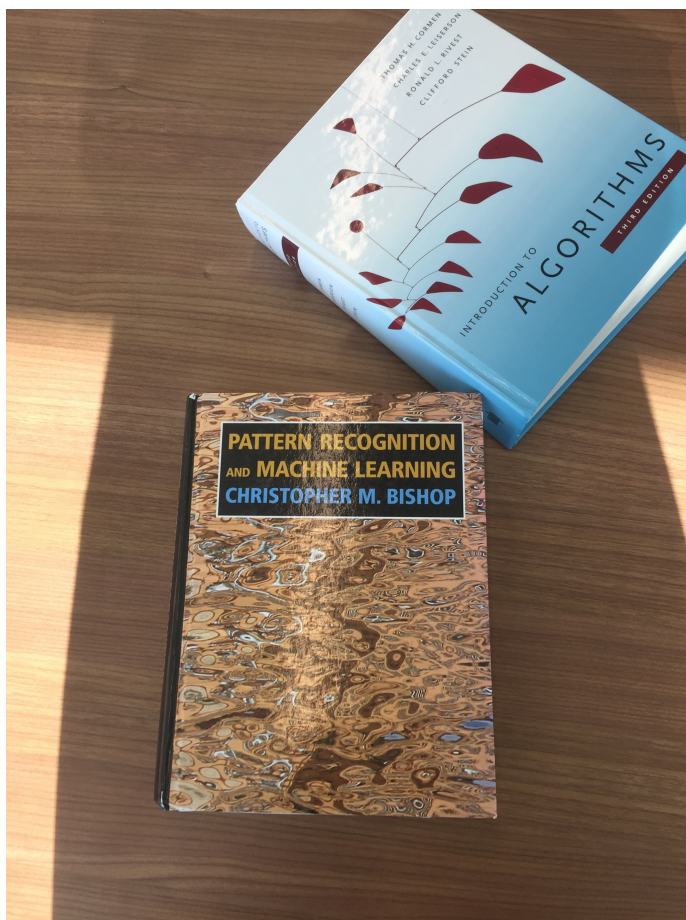
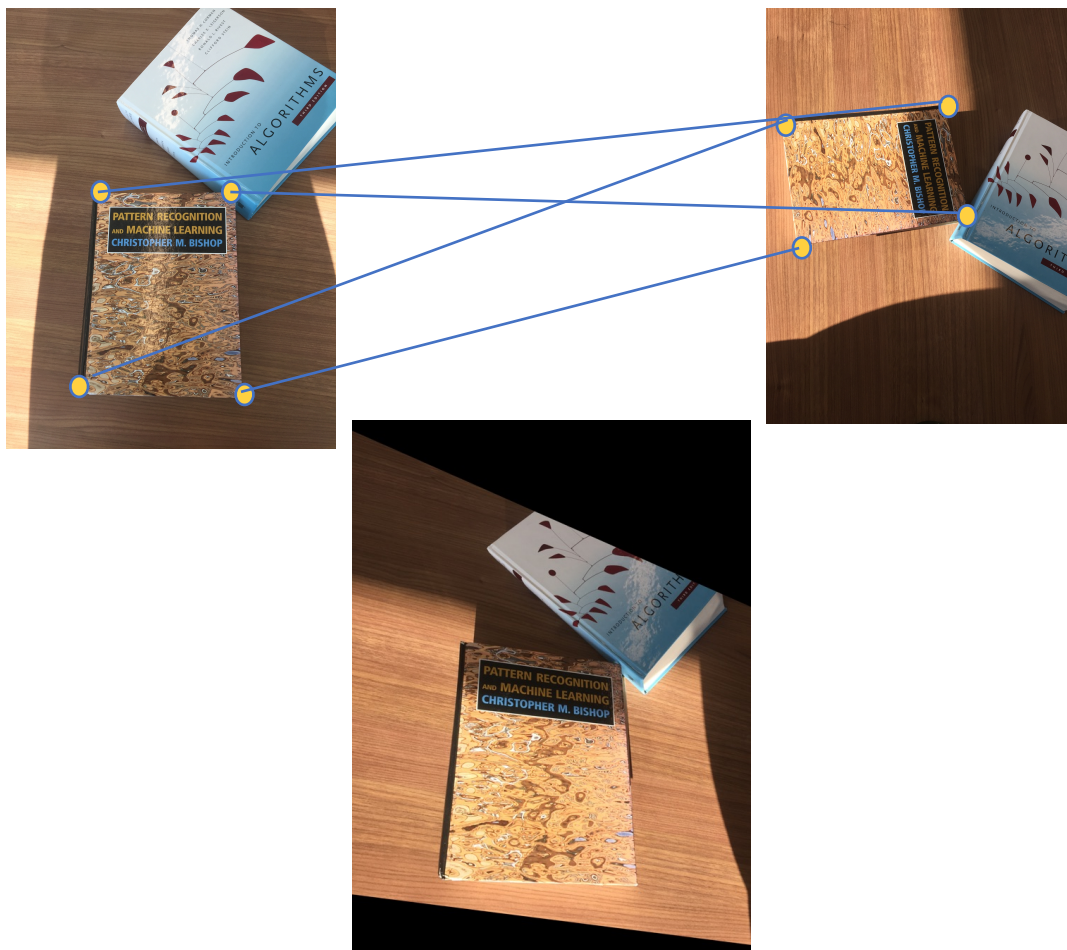


Homography

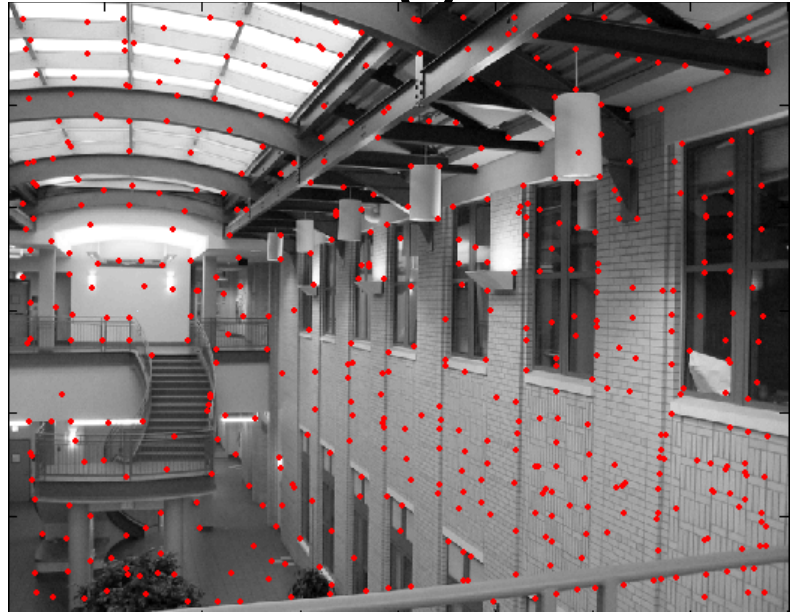
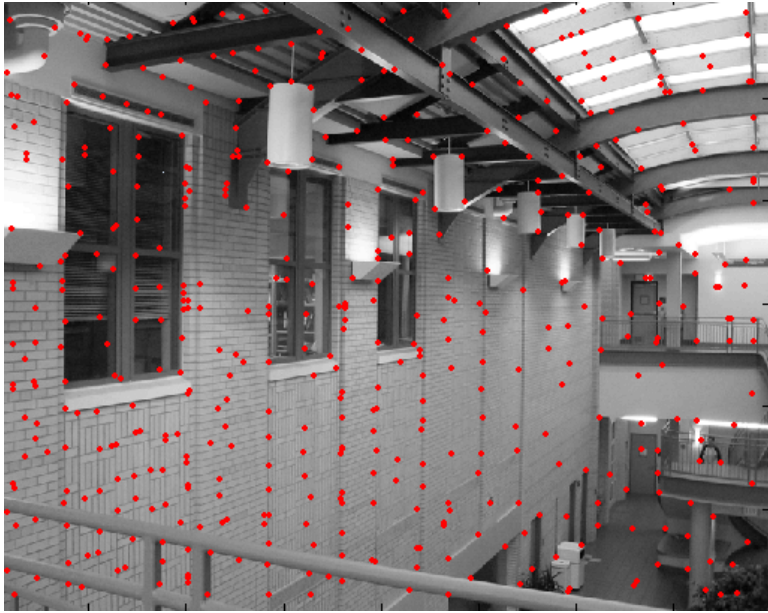
Homography



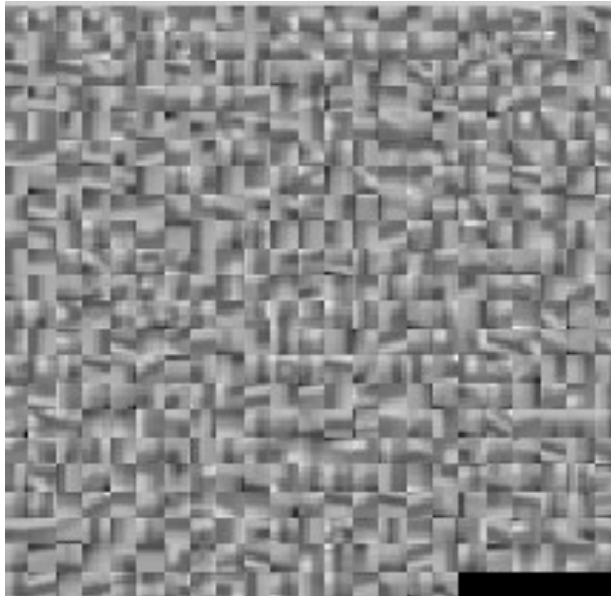
Homography



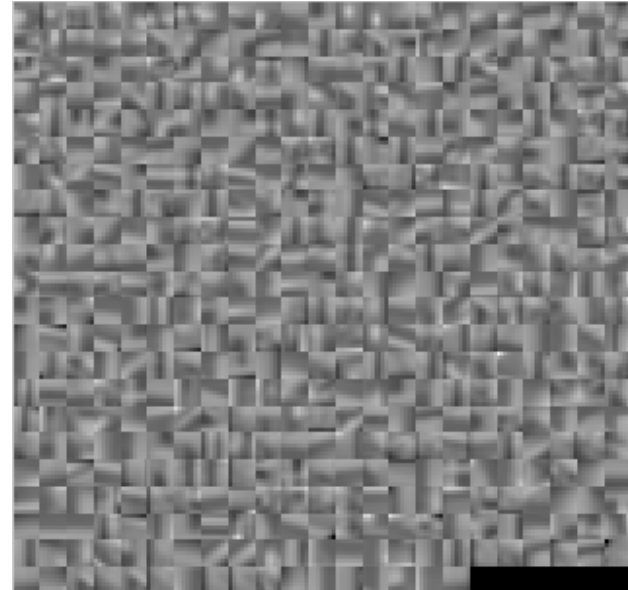
Feature matching



descriptors for left image feature points

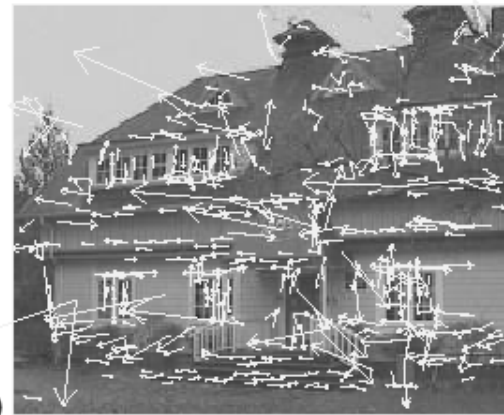


descriptors for right image feature points



SIFT features

- Example



- (a) 233x189 image
- (b) 832 DOG extrema
- (c) 729 left after peak value threshold
- (d) 536 left after testing ratio of principle curvatures

Strategies to match images robustly

(a) Working with individual features: For each feature point, find most similar point in other image (SIFT distance)

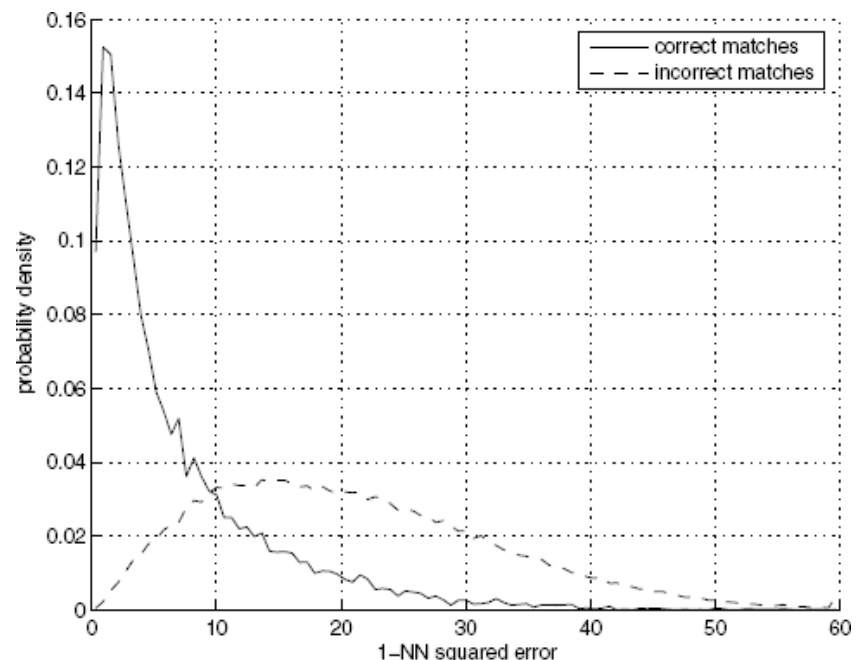
Reject ambiguous matches where there are too many similar points

(b) Working with all the features: Given some good feature matches, look for possible homographies relating the two images

Reject homographies that don't have many feature matches.

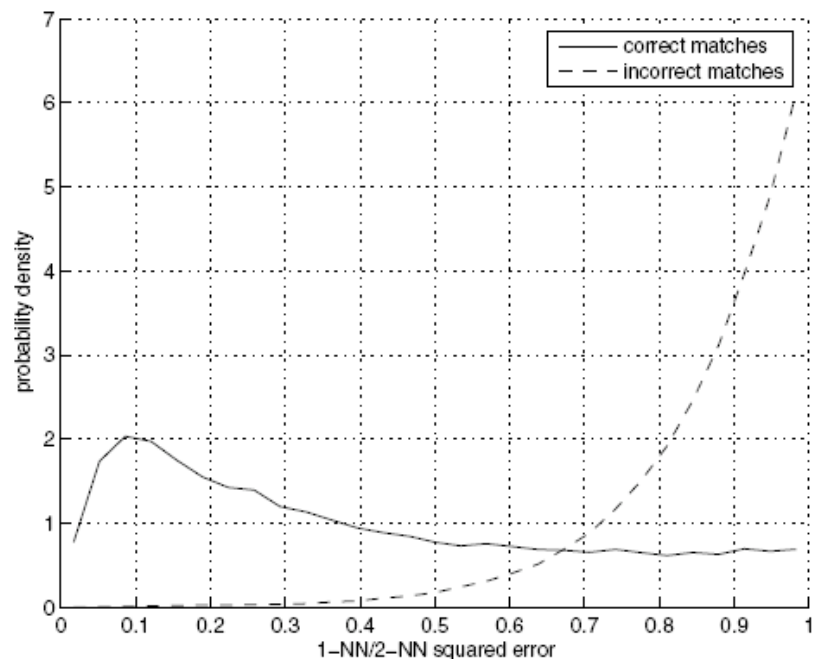
(a) Feature-space outlier rejection

- Let's not match all features, but only these that have "similar enough" matches?
- How can we do it?
 - $SSD(patch1, patch2) < threshold$
 - How to set threshold?
Not so easy.

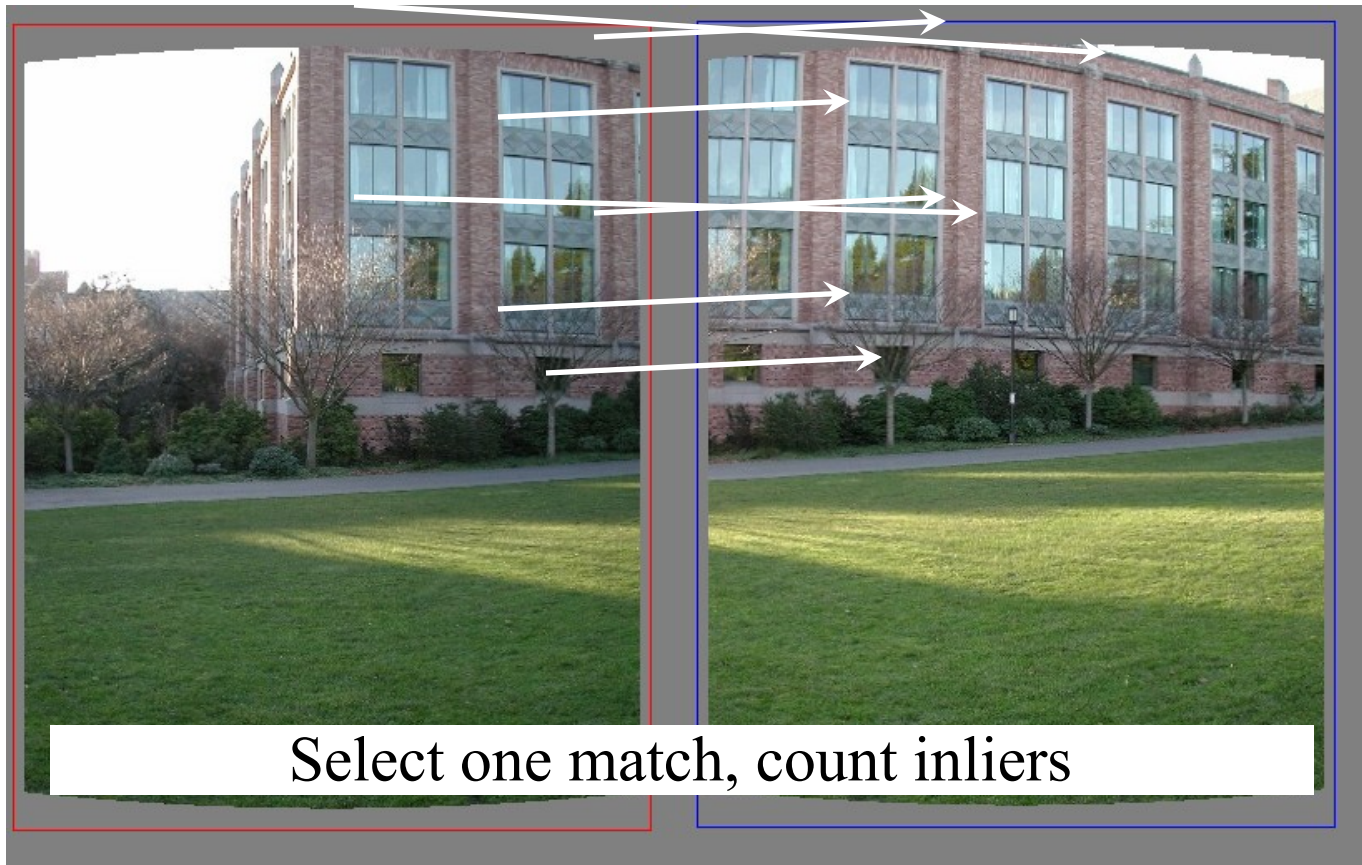


Feature-space outlier rejection

- A better way [Lowe, 1999]:
 - 1-NN: SSD of the closest match
 - 2-NN: SSD of the second-closest match
 - Look at how much better 1-NN is than 2-NN, e.g. $1\text{-NN}/2\text{-NN}$
 - That is, is our best match so much better than the rest?



Random Sample Consensus



RANSAC for estimating homography

RANSAC loop:

Select four feature pairs (at random)

Compute homography H (exact)

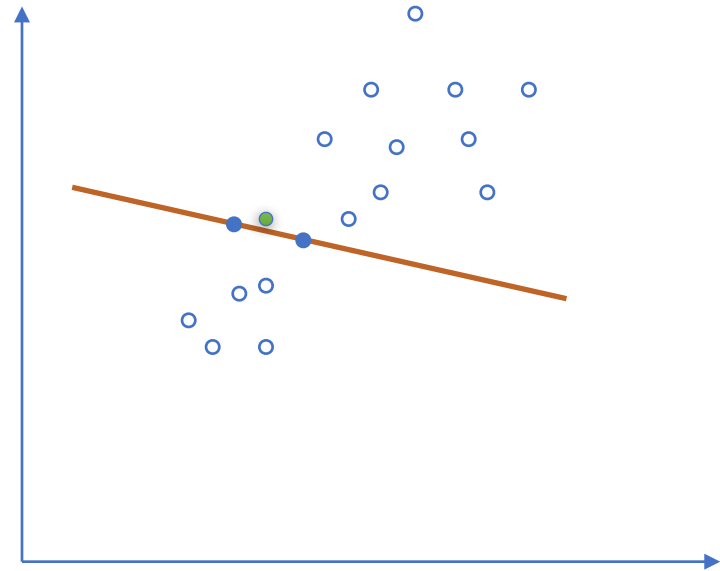
Compute inliers where $\|p_i', H p_i\| < \epsilon$

Keep largest set of inliers

Re-compute least-squares H estimate using all of the inliers

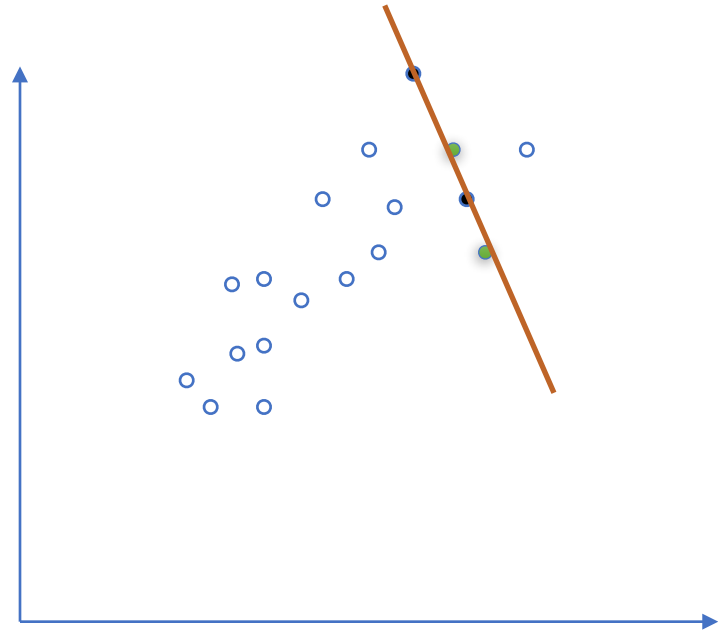
Fit a line

- Pick 2 points
- Fit line
- Count inliers (3 inliers)



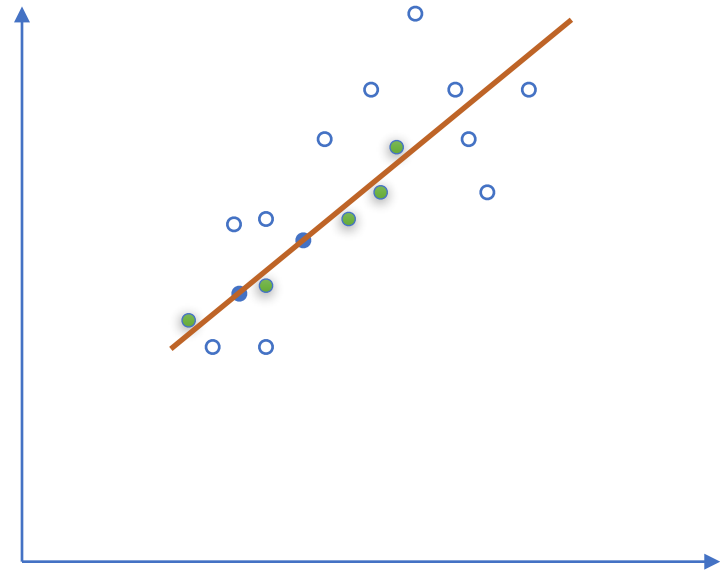
Fit a line

- Pick 2 points
- Fit line
- Count inliers (4 inliers)



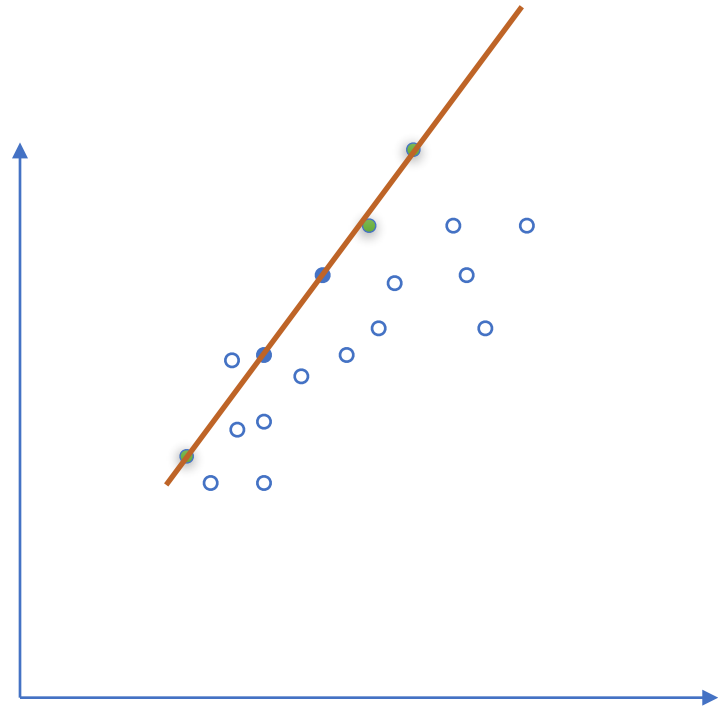
Fit a line

- Pick 2 points
- Fit line
- Count inliers (7 inliers)



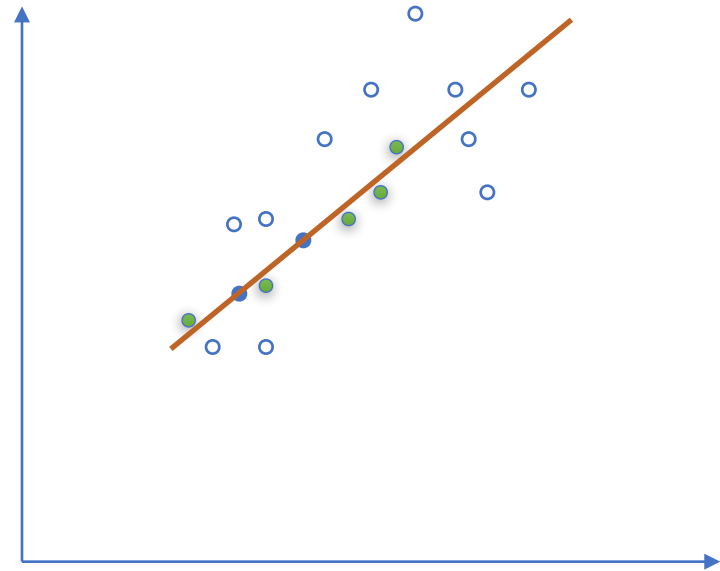
Fit a line

- Pick 2 points
- Fit line
- Count inliers (5 inliers)

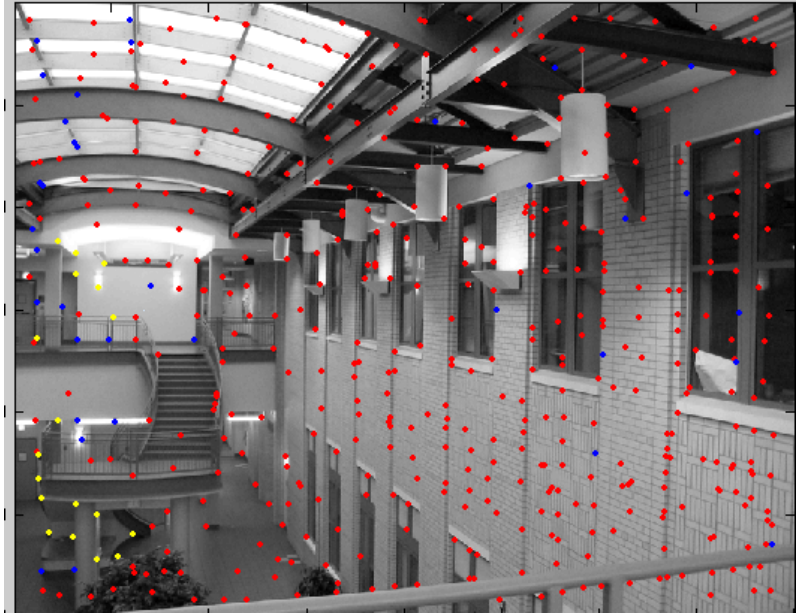
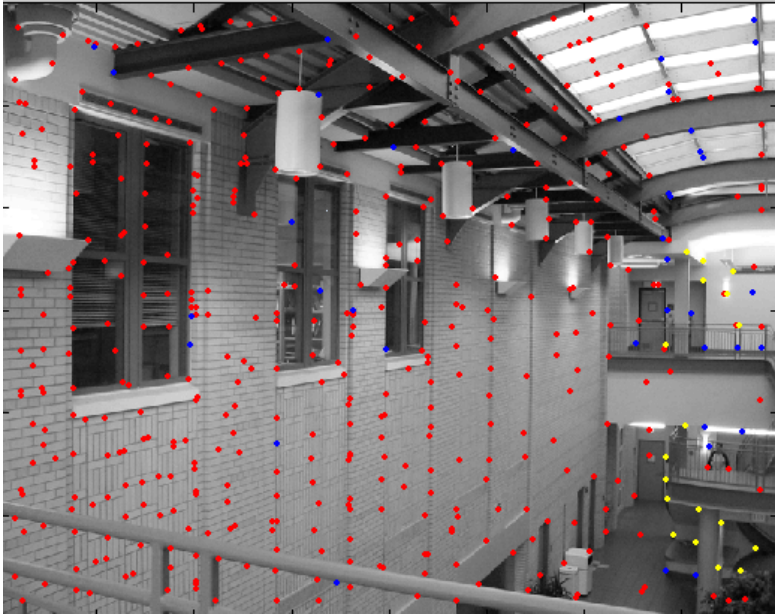


Fit a line

- Use biggest set of inliers
- Least squares fit



RANSAC



red:
rejected by 2nd nearest
neighbor criterion
blue:
Ransac outliers
yellow:
inliers



RANSAC parameters

Decision Threshold: $|X' - HX| < \epsilon$

Number of trials: N

Size of the inlier set: M

Robustness

- Proportion of inliers in our pairs is g (for “good”)
- Our model needs P pairs
 - $P=4$ for homography
- Probability that we pick P inliers?
 g^P
- Probability that after N RANSAC iterations we have not picked a set of inliers?
 - $(1 - g^P)^N$

Robustness: example

- Proportion of inliers $G=0.5$
- Probability that we pick $P=4$ inliers?
 - $0.5^4 = 0.0625$ (6 % chance)
- Probability that we have not picked a set of inliers?

$N = 100$ iterations

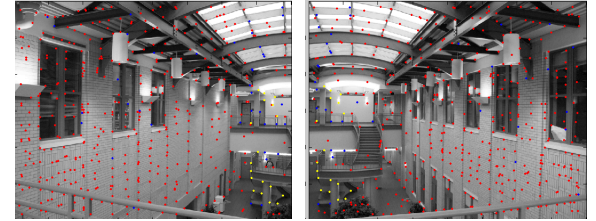
$$(1 - 0.5^4)^{100} = 0.00157 \text{ (1 chance in 600)}$$

$N = 1000$ iterations

$$(1 - 0.5^4)^{1000} = 0.00157 \text{ (1 chance in } 10^{28}\text{)}$$

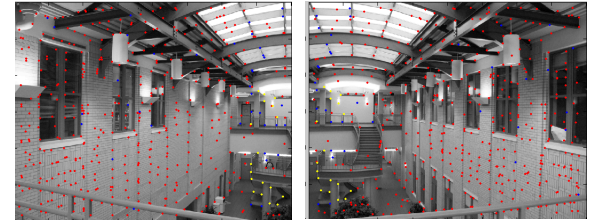
1 chance in 1e28

Robustness: example



- Proportion of inliers $G=0.3$
- Probability that we pick $P=4$ inliers?
 - $0.3^4=0.0081$ (0.8% chance)
- Probability that we have not picked a set of inliers?
 - $N=100$ iterations:
 $(1-0.3^4)^{100}=0.44$ (1 chance in 2)
 - $N=1000$ iterations:
1 chance in 3400

Robustness: example



- Proportion of inliers $G=0.1$
- Probability that we pick $P=4$ inliers?
 - $0.1^4=0.0001$ (0.01% chances, 1 in 10,000)
- Probability that we have not picked a set of inliers?
 - $N=100$ iterations: $(1-0.1^4)^{100}=0.99$
 - $N=1000$ iterations: 90%
 - $N=10,000$: 36%
 - $N=100,000$: 1 in 22,000

Robustness: conclusions

- Effect of number of parameters of model/
number of necessary pairs
 - Bad exponential
- Effect of percentage of inliers
 - Base of the exponential
- Effect of number of iterations
 - Good exponential

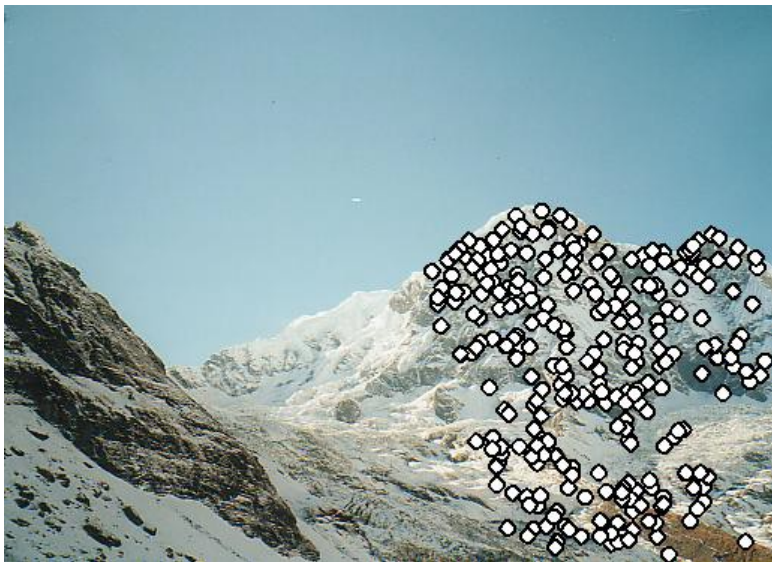
RANSAC recap

- For fitting a model with low number P of parameters (8 for homographies)
- Loop
 - Select P random data points
 - Fit model
 - Count inliers
(other data points well fit by this model)
- Keep model with largest number of inliers

RANSAC for Homography



RANSAC for Homography



RANSAC for Homography

