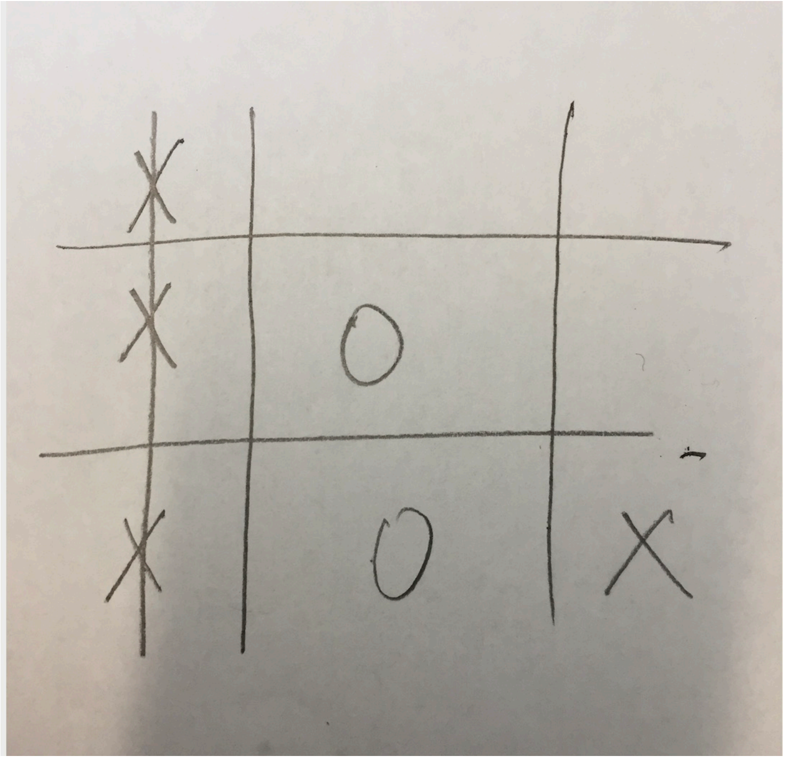
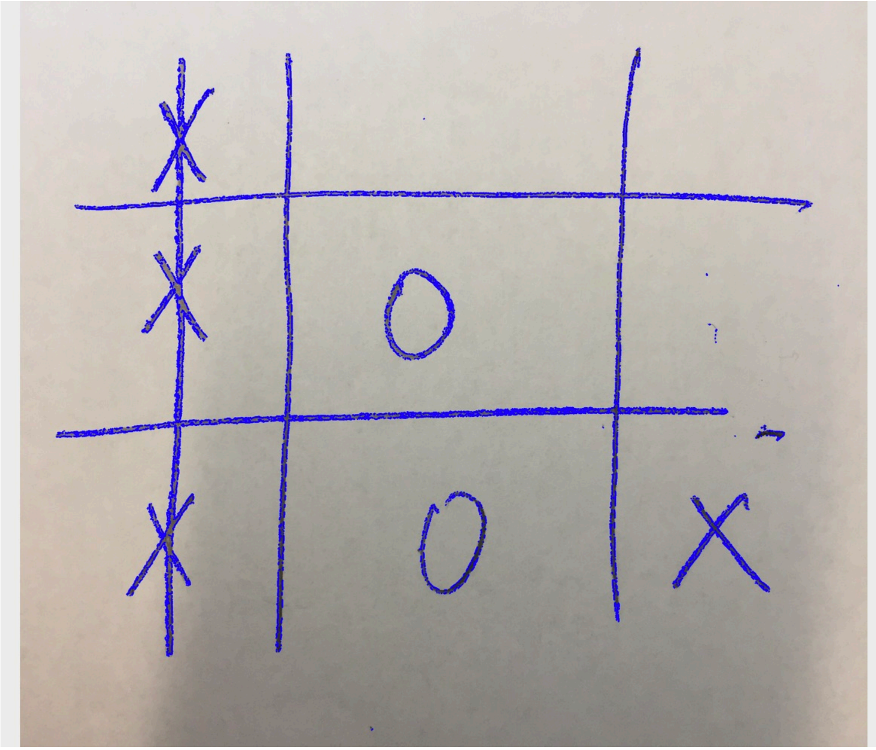
Hough Transform





Finding lines in an image

Option 1:

- Search for the line at every possible position/orientation
- What is the cost of this operation?

Option 2:

- Use a voting scheme: Hough transform

Finding lines in an image

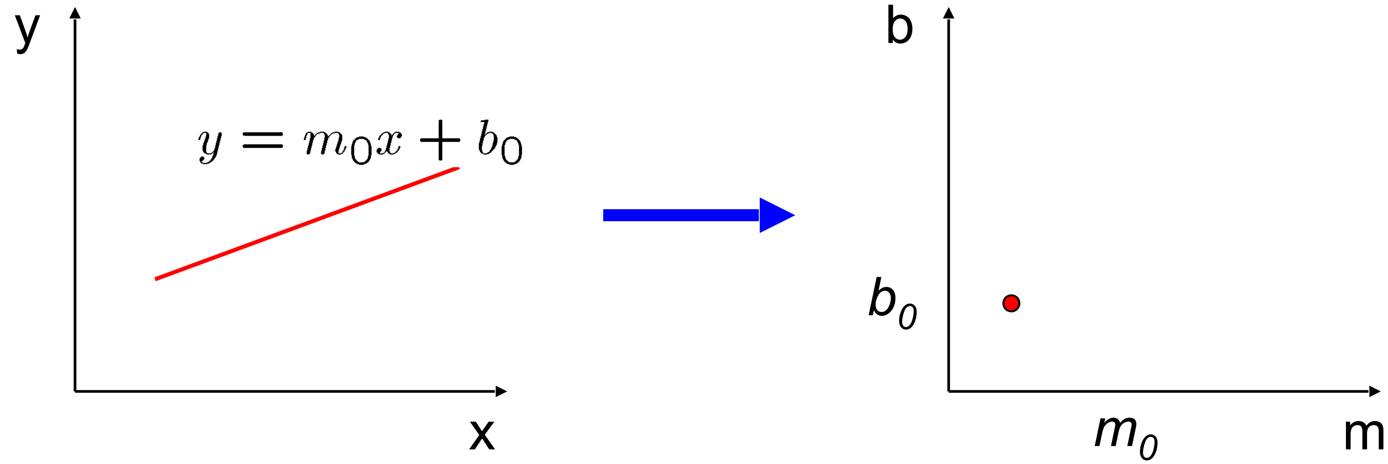


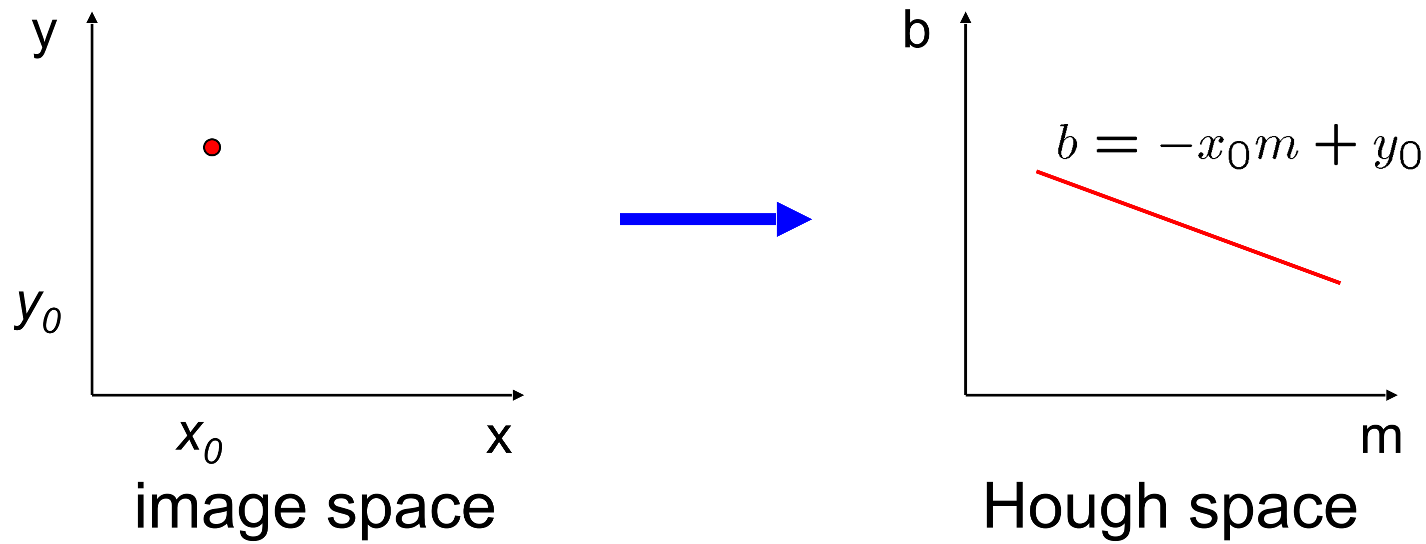
image space

Hough space

Connection between image (x,y) and Hough (m,b) spaces

- A line in the image corresponds to a point in Hough space
- To go from image space to Hough space:
  – given a set of points (x,y), find all (m,b) such that y = mx + b

Finding lines in an image



Connection between image (x,y) and Hough (m,b) spaces

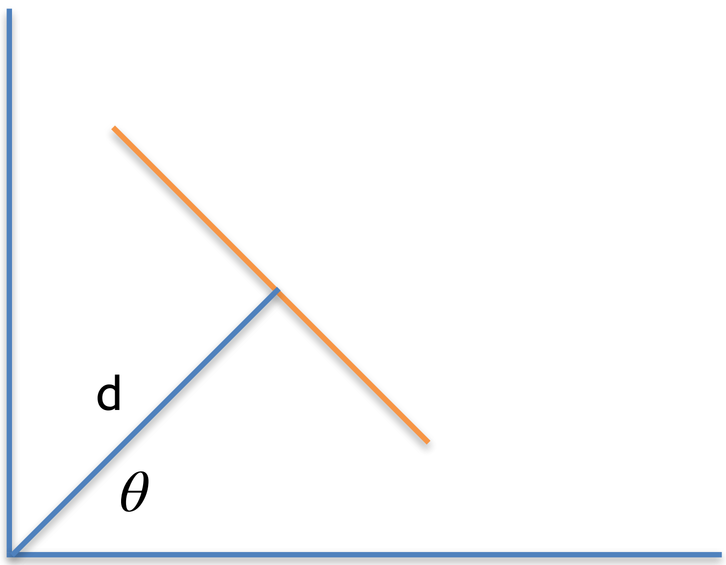
- A line in the image corresponds to a point in Hough space
- To go from image space to Hough space:
  - given a set of points (x,y), find all (m,b) such that y = mx + b
- What does a point (x_0, y_0) in the image space map to?
  - A: the solutions of $b = -x_0 m + y_0$
  - this is a line in Hough space

Hough transform algorithm

Typically use a different parameterization

$$d = x cos\theta + y sin\theta$$

- d is the perpendicular distance from the line to the origin
- θ is the angle this perpendicular makes with the x axis
- Why?



Hough transform algorithm

Typically use a different parameterization

$$d = x cos\theta + y sin\theta$$

- d is the perpendicular distance from the line to the origin
- θ is the angle this perpendicular makes with the x axis
- Why?

Basic Hough transform algorithm

1. Initialize H[d, θ]=0
2. for each edge point I[x,y] in the image

   for θ = 0 to 180

   $$d = x cos\theta + y sin\theta$$

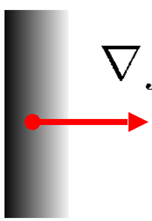
   H[d, θ] += 1
3. Find the value(s) of (d, θ) where H[d, θ] is maximum
4. The detected line in the image is given by $d = x cos\theta + y sin\theta$

Image gradient

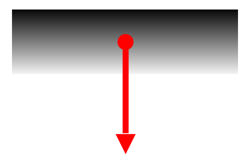
The gradient of an image:

$$\nabla f = \left[\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}\right]$$

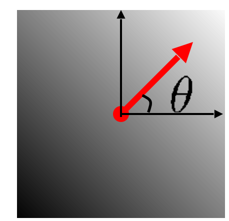
The gradient points in the direction of most rapid change in intensity



$$\nabla f = \left[\frac{\partial f}{\partial x}, 0\right]$$



$$\nabla f = \left[0, \frac{\partial f}{\partial y}\right]$$



$$\nabla f = \left[\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}\right]$$

The gradient direction is given by:

$$\theta = \tan^{-1}\left(\frac{\partial f}{\partial y} \Big/ \frac{\partial f}{\partial x}\right)$$

- How does this relate to the direction of the edge?

The *edge strength* is given by the gradient magnitude

$$\|\nabla f\| = \sqrt{\left(\frac{\partial f}{\partial x}\right)^2 + \left(\frac{\partial f}{\partial y}\right)^2}$$

Extensions

Extension 1: Use the image gradient

1. Initialize H[d, θ]=0
2. for each edge point I[x,y] in the image

   compute unique (d, θ) based on image gradient at (x,y)

   H[d, θ] += 1
3. Find the value(s) of (d, θ) where H[d, θ] is maximum
4. The detected line in the image is given by $d = xcos\theta + ysin\theta$

Hough Transform for Curves

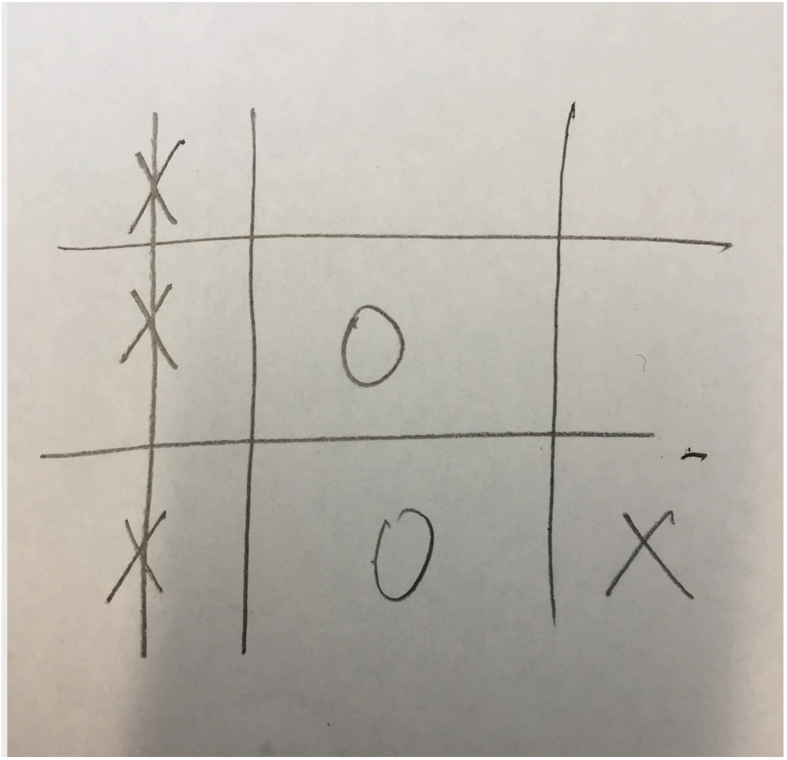
The H.T. can be generalized to detect any curve that can be expressed in parametric form:

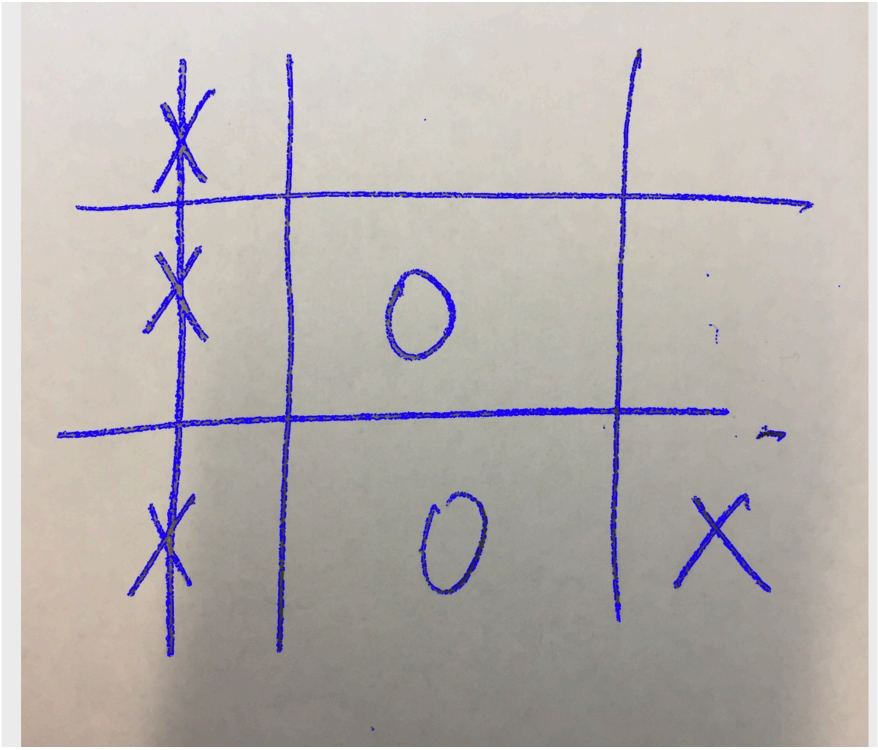
- Y = f(x, a1,a2,…ap)
- a1, a2, … ap are the parameters
- The parameter space is p-dimensional
- The accumulating array is LARGE!

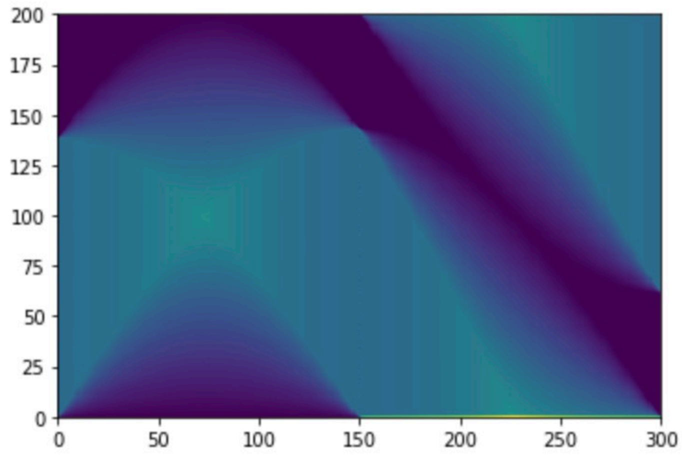
For circle: vote on x_0, y_0, r

$$(x - x_0)^2 + (y - y_0)^{=} r^2$$

Hough Transform







H.T. Summary

H.T. is a “voting” scheme

- points vote for a set of parameters describing a line or curve.

The more votes for a particular set

- the more evidence that the corresponding curve is present in the image.

Can detect MULTIPLE curves in one shot.

Computational cost increases with the number of parameters describing the curve.