Image Motion

The Information from Image Motion

- 3D motion between observer and scene + structure of the scene
  - Wallach O'Connell (1953): Kinetic depth effect
  - http://www.michaelbach.de/ot/mot-ske/index.html
  - Motion parallax: two static points close by in the image with different image motion; the larger translational motion corresponds to the point closer by (smaller depth)

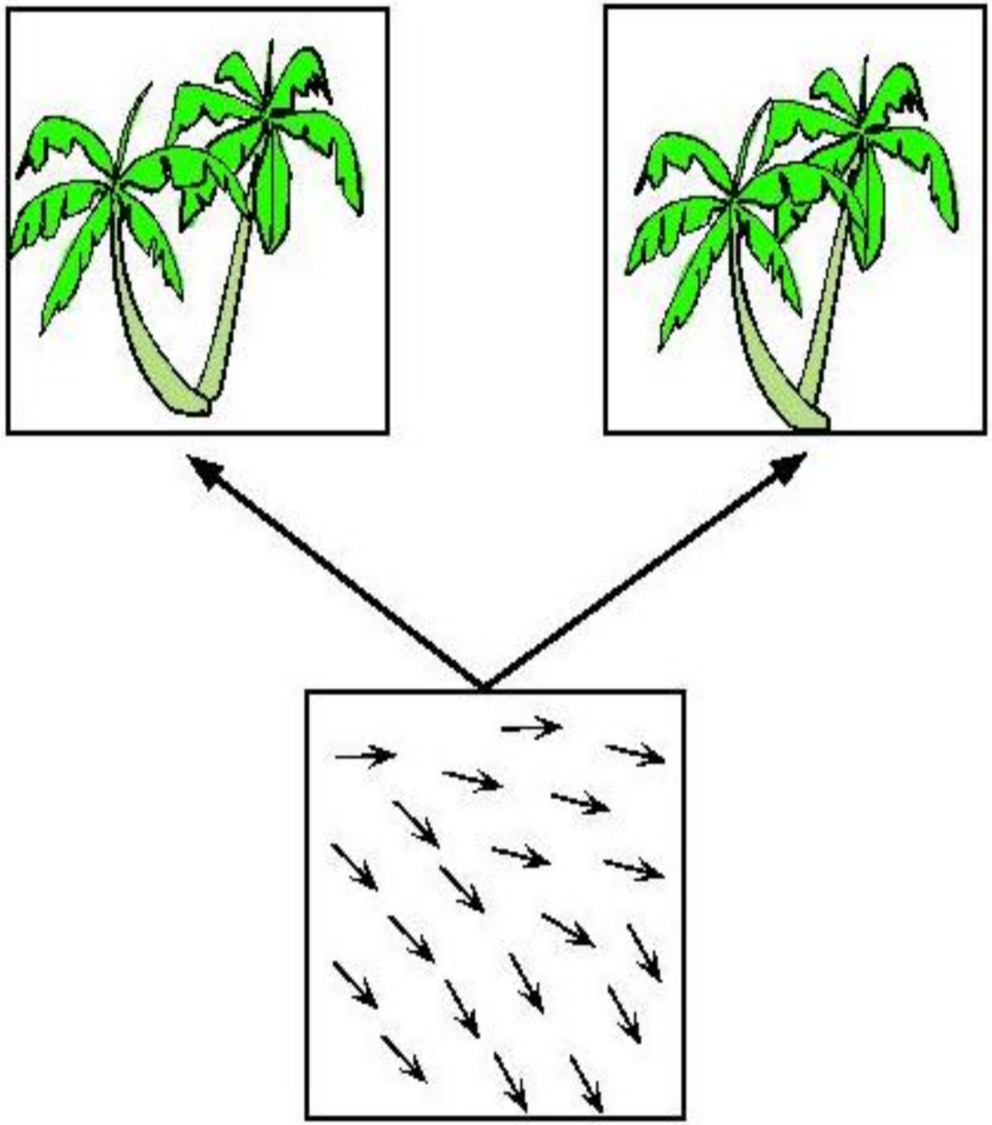
- Recognition
  - Johansson (1975): Light bulbs on joints
  - http://www.biomotionlab.ca/Demos/BMLwalker.html

Motion Field and Optic Flow

- Motion Field: Projection of 3D relative velocity vectors onto 2D image plane

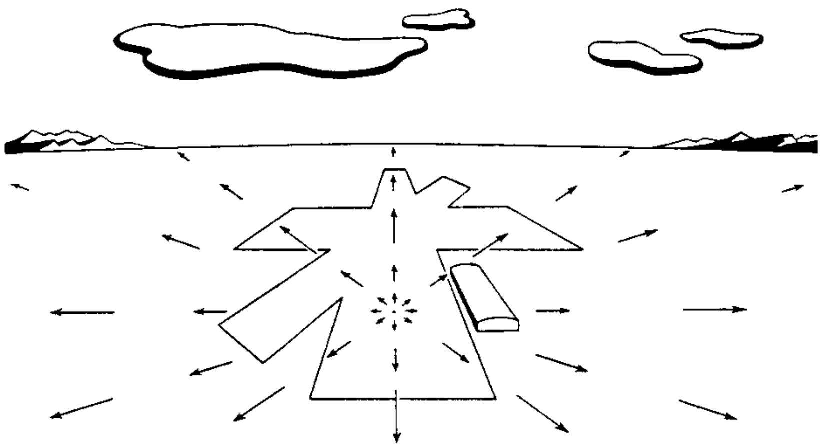
- Optic flow: Observed 2D displacements of intensity patterns in the image.

- We want to know Motion field, by estimating optic flow.

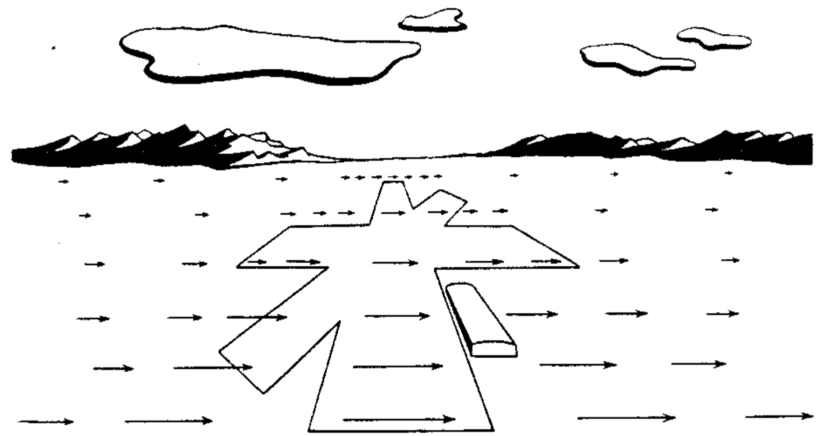


Optical flow estimation

Examples of Motion Fields I



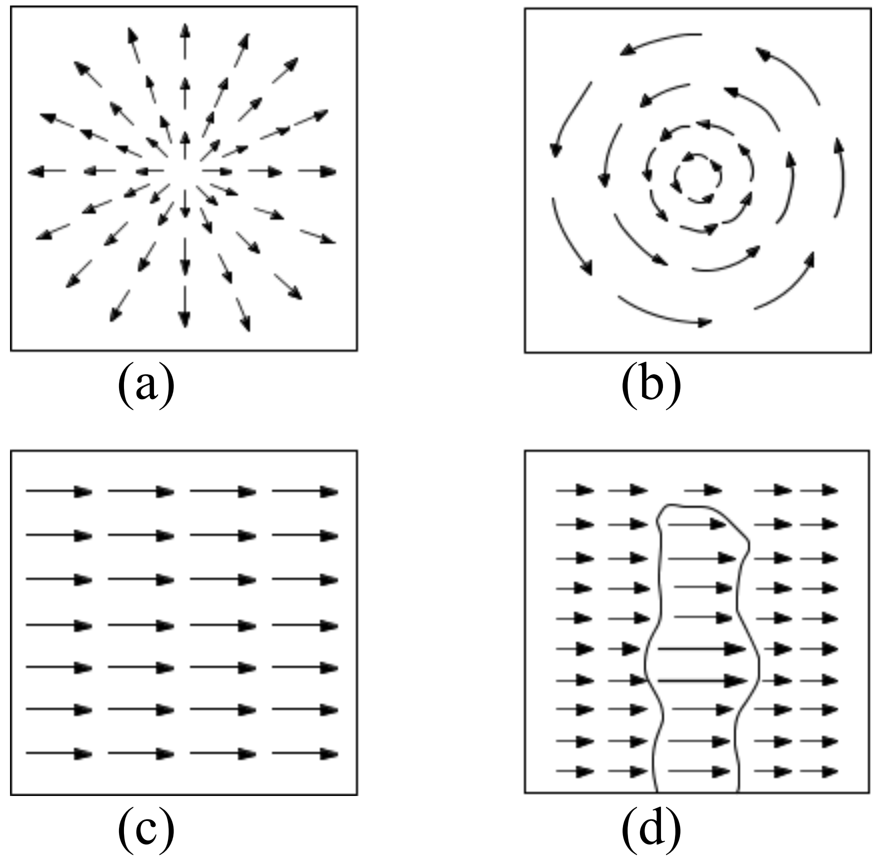
(a)



(b)

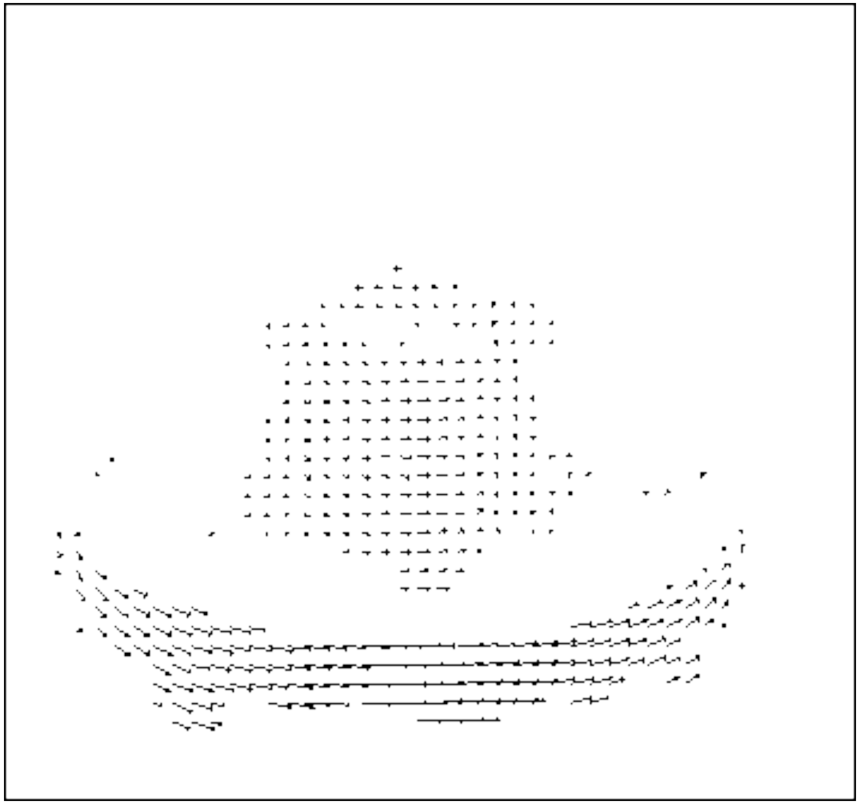
(a) Motion field of a pilot looking straight ahead while approaching a fixed point on a landing strip. (b) Pilot is looking to the right in level flight.

Examples of Motion Fields II



(a) Translation perpendicular to a surface. (b) Rotation about axis perpendicular to image plane. (c) Translation parallel to a surface at a constant distance. (d) Translation parallel to an obstacle in front of a more distant background.

Optical flow



Assuming that illumination does not change:

- Image changes are due to the RELATIVE MOTION between the scene and the camera.
- There are 3 possibilities:
  - Camera still, moving scene
  - Moving camera, still scene
  - Moving camera, moving scene

Motion Analysis Problems

- Correspondence Problem
  - Track corresponding elements across frames
- Reconstruction Problem
  - Given a number of corresponding elements, and camera parameters, what can we say about the 3D motion and structure of the observed scene?
- Segmentation Problem
  - What are the regions of the image plane which correspond to *different* moving objects?

Motion Field (MF)

- The MF assigns a velocity vector to each pixel in the image.
- These velocities are INDUCED by the RELATIVE MOTION btw the camera and the 3D scene
- The MF can be thought as the *projection* of the 3D velocities on the image plane.

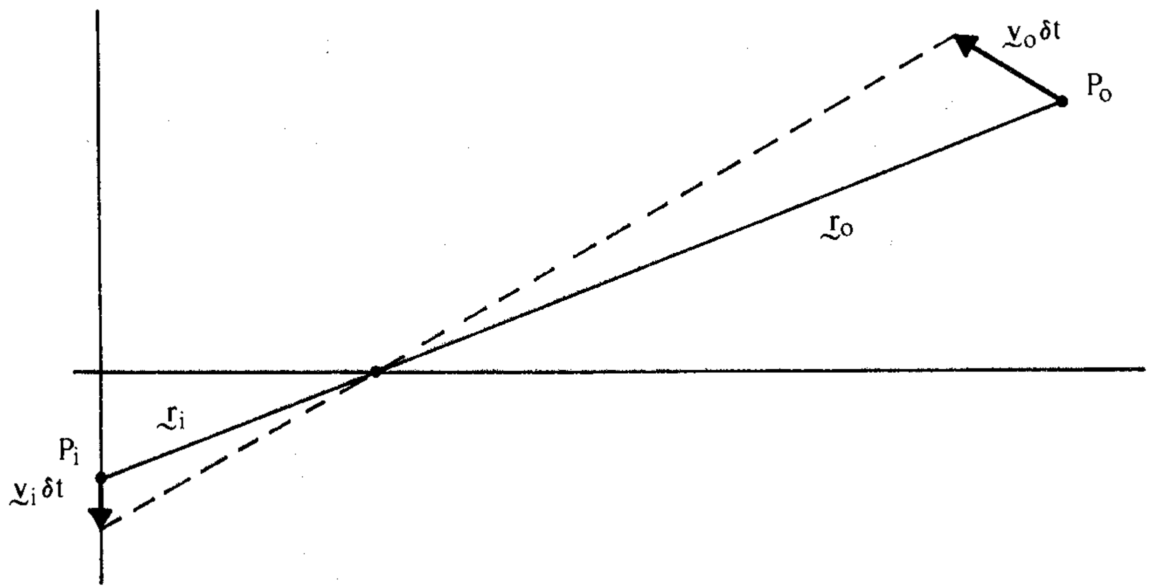
Motion Field and Optical Flow Field

- Motion field: projection of 3D motion vectors on image plane

Object point P_0 has velocity $\mathbf{v}_0$, induces $\mathbf{v}_i$ in image

$$\mathbf{v}_0 = \frac{d\mathbf{r}_0}{dt} \quad \mathbf{v}_1 = \frac{d\mathbf{r}_i}{dt}$$

$\mathbf{r}_0$ related to $\mathbf{r}_i$ by $\dfrac{\mathbf{r}_i}{f} = \dfrac{\mathbf{r}_0}{\mathbf{r}_0 \cdot \hat{\mathbf{z}}_0}$



- Optical flow field: apparent motion of brightness patterns
- We equate motion field with optical flow field

Motion Field and Optic Flow

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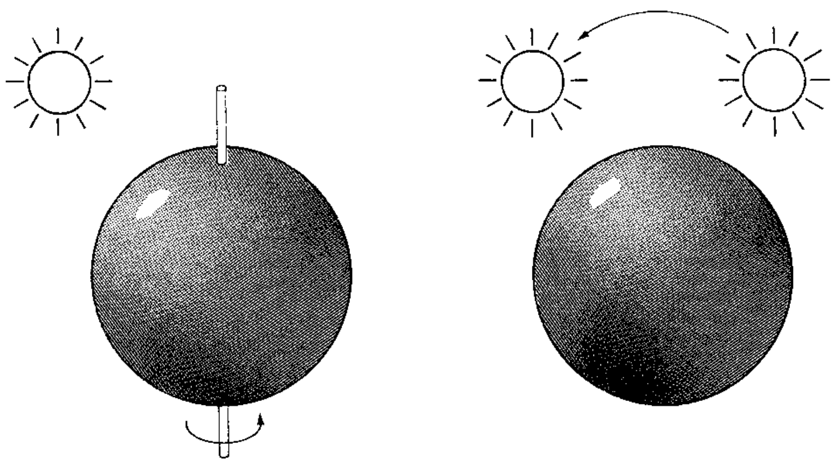
- We want to know Motion field, by estimating optic flow.

2 Cases Where this Assumption Clearly is not Valid

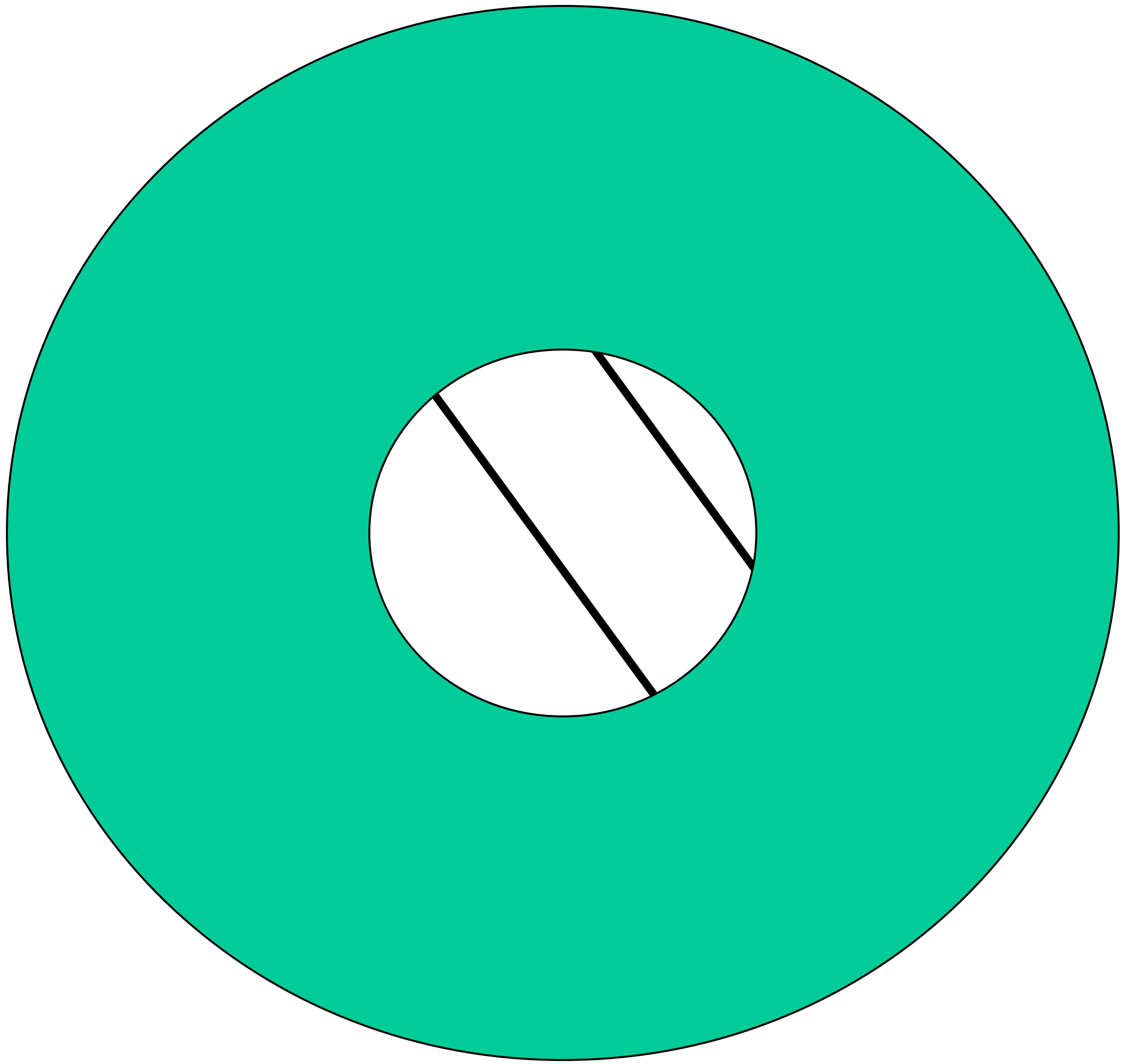


(a) (b)

(a) A smooth sphere is rotating under constant illumination. Thus the optical flow field is zero, but the motion field is not.

(b) A fixed sphere is illuminated by a moving source—the shading of the image changes. Thus the motion field is zero, but the optical flow field is not.

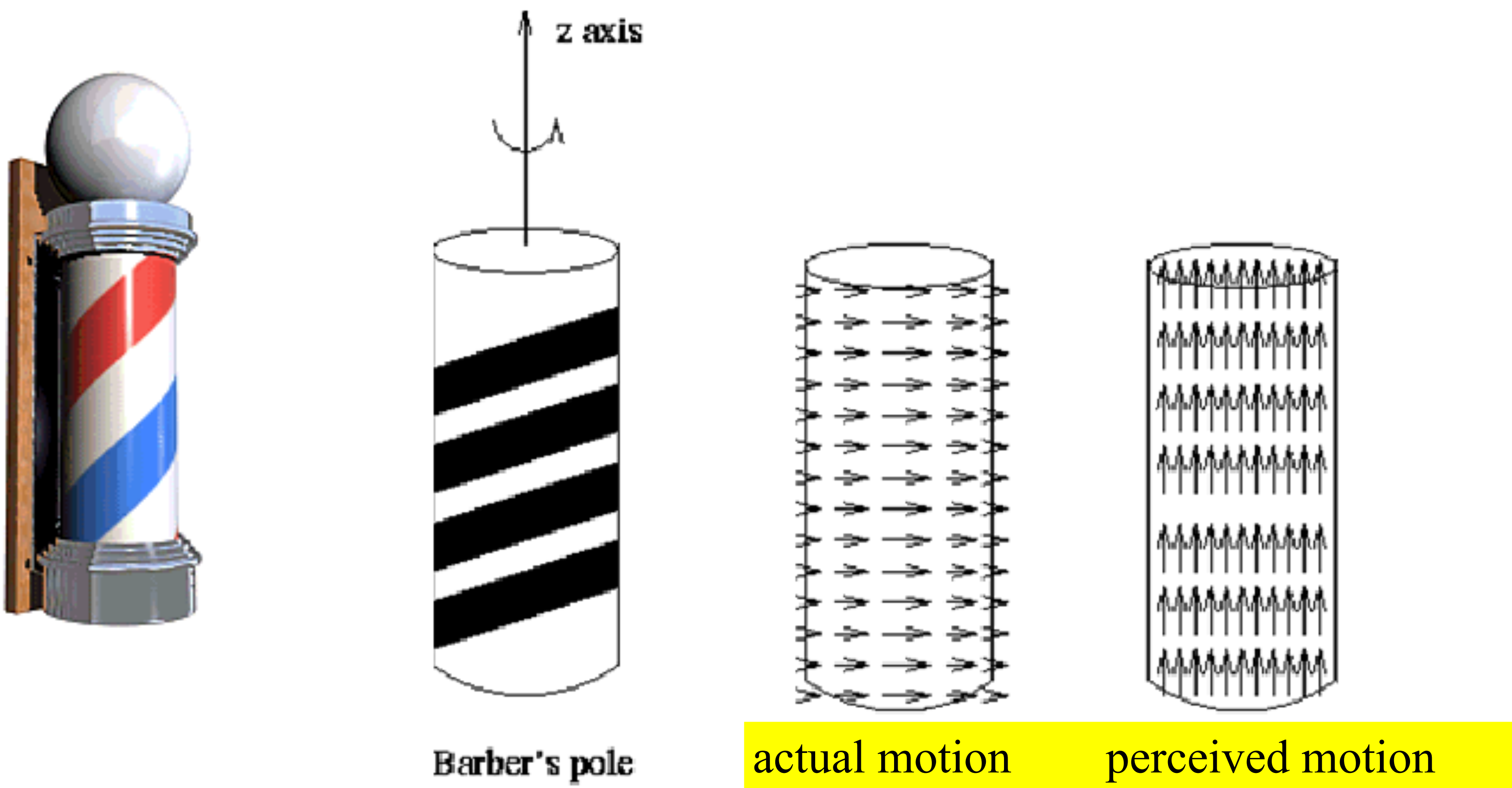
Aperture problem



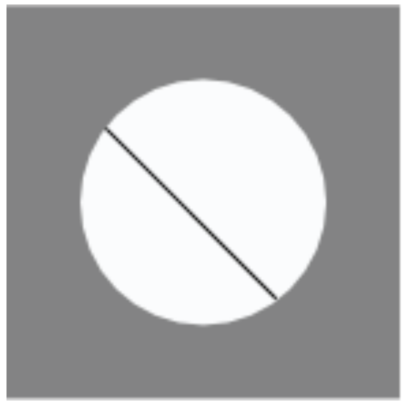
Aperture Problem in Real Life

Aperture Problem

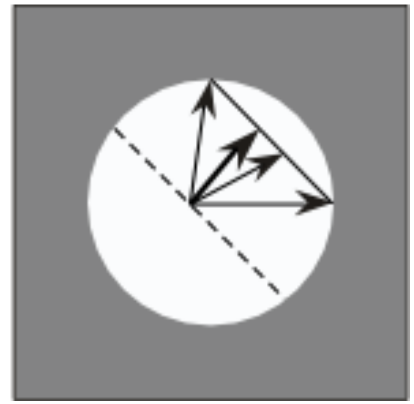
Barber pole illusion



Aperture Problem



(a)



(b)

(a) Line feature observed through a small aperture at time t.

(b) At time $t+\delta t$ the feature has moved to a new position. It is not possible to determine exactly where each point has moved. From local image measurements only the flow component perpendicular to the line feature can be computed.

Normal flow: Component of flow perpendicular to line feature.

Brightness Constancy Equation

Image intensity at Time = t, $I(x, y, t)$

Image intensity at Time = t + dt, $I(x + dx, y + dy, t + dt)$

Assuming $I(x, y, t) = I(x + dx, y + dy, t + dt)$

Taylor Series expansion

$$I(x + dx, y + dy, t + dt) \approx I(x, y, t) + \frac{\partial I}{\partial x}dx + \frac{\partial I}{\partial y}dy + \frac{\partial I}{\partial t}dt$$

Therefore,

$$I(x, y, t) = I(x, y, t) + \frac{\partial I}{\partial x}dx + \frac{\partial I}{\partial y}dy + \frac{\partial I}{\partial t}dt$$

$$\frac{\partial I}{\partial x}dx + \frac{\partial I}{\partial y}dy + \frac{\partial I}{\partial t}dt = 0$$

Brightness Constancy Equation

$$\frac{\partial I}{\partial x}dx + \frac{\partial I}{\partial y}dy + \frac{\partial I}{\partial t}dt = 0$$

Taking derivative wrt time:

$$\frac{\partial I}{\partial x}\frac{dx}{dt} + \frac{\partial I}{\partial y}\frac{dy}{dt} + \frac{\partial I}{\partial t} = 0$$

Let

$$\nabla I = \begin{bmatrix} \frac{\partial I}{\partial x} \\ \frac{\partial I}{\partial y} \end{bmatrix} \qquad d = \begin{bmatrix} \frac{dx}{dt} \\ \frac{dy}{dt} \end{bmatrix} \qquad I_t = \frac{\partial I}{\partial t}$$

(Frame spatial gradient) (optical flow) (derivative across frames)

Brightness Constancy Equation

$$\frac{\partial I}{\partial x}\frac{dx}{dt} + \frac{\partial I}{\partial y}\frac{dy}{dt} + \frac{\partial I}{\partial t} = 0$$

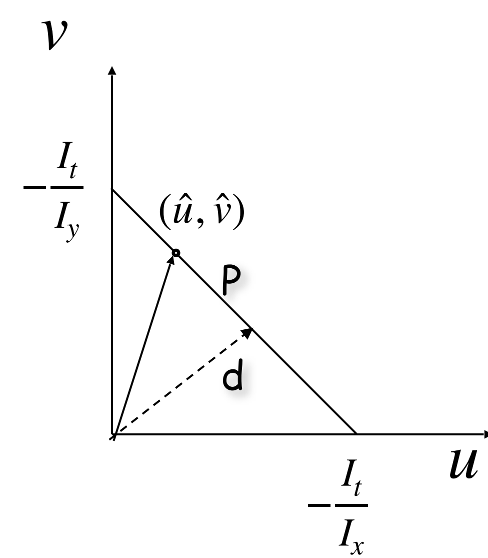
ax+by +c = 0

Becomes:

$$\boxed{I_x u + I_y v + I_t = 0}$$

where, $u = \frac{dx}{dt}; v = \frac{dy}{dt}$

Line equation $v = -\frac{I_x}{I_y}u - \frac{I_t}{I_y}$



$$d = \frac{I_t}{\sqrt{I_x^2 + I_y^2}}$$

The OF is CONSTRAINED to be on a line !

Brightness Constancy Equation

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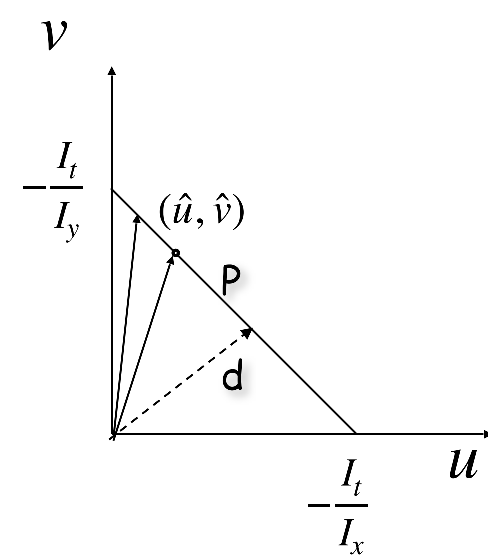

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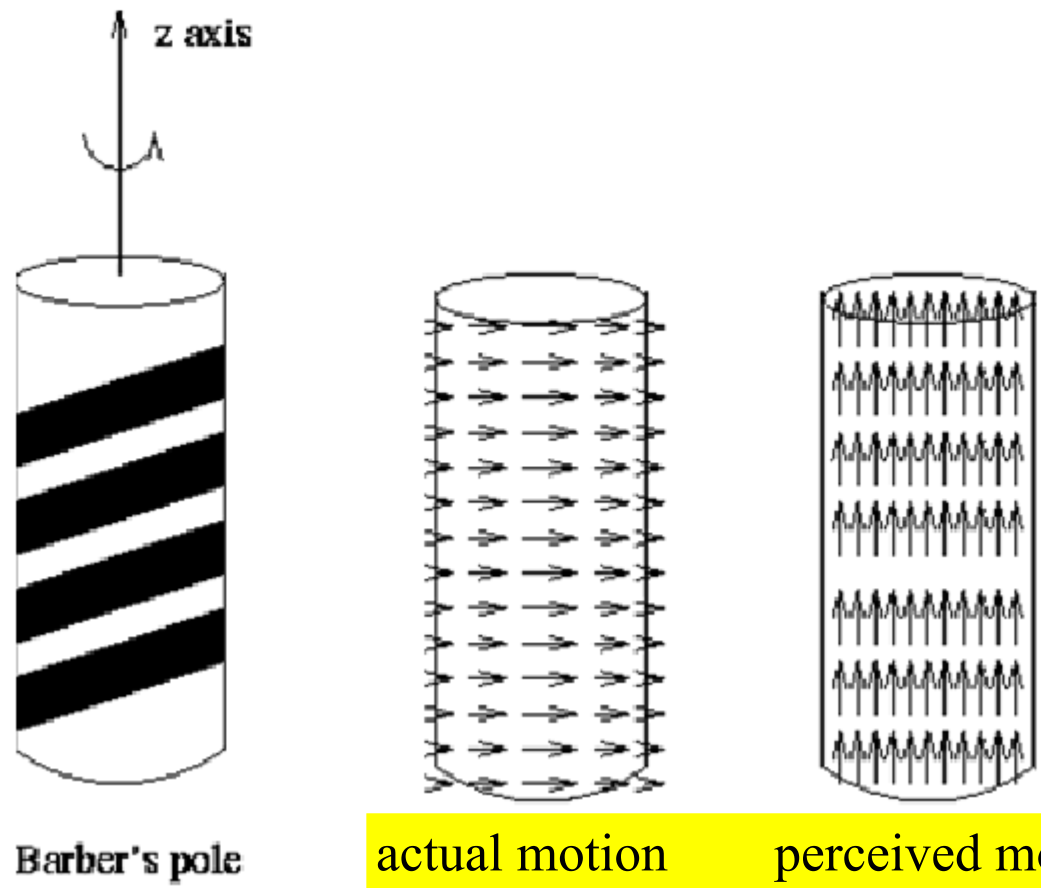


actual motion perceived motion

Solving the aperture problem

- How to get more equations for a pixel?
  - Basic idea: impose additional constraints
    - most common is to assume that the flow field is smooth locally
    - one method: pretend the pixel's neighbors have the same (u,v)
      - If we use a 5x5 window, that gives us 25 equations per pixel!

$$0 = I_t(\mathbf{p_i}) + \nabla I(\mathbf{p_i}) \cdot [u \ \ v]$$

$$\underset{25\times 2}{\underbrace{\begin{bmatrix} I_x(\mathbf{p_1}) & I_y(\mathbf{p_1}) \\ I_x(\mathbf{p_2}) & I_y(\mathbf{p_2}) \\ \vdots & \vdots \\ I_x(\mathbf{p_{25}}) & I_y(\mathbf{p_{25}}) \end{bmatrix}}_{A}} \underset{2\times 1}{\underbrace{\begin{bmatrix} u \\ v \end{bmatrix}}_{d}} = - \underset{25\times 1}{\underbrace{\begin{bmatrix} I_t(\mathbf{p_1}) \\ I_t(\mathbf{p_2}) \\ \vdots \\ I_t(\mathbf{p_{25}}) \end{bmatrix}}_{b}}$$

Lukas-Kanade flow

- Prob: we have more equations than unknowns

$$\underset{25\text{x}2}{A}\ \underset{2\text{x}1}{d} = \underset{25\text{x}1}{b} \longrightarrow \text{minimize } \|Ad - b\|^2$$

- Solution: solve least squares problem
  - minimum least squares solution given by solution (in d) of: $\underset{2\text{x}2}{(A^T A)}\ \underset{2\text{x}1}{d} = \underset{2\text{x}1}{A^T b}$

$$\underbrace{\begin{bmatrix} \sum I_x I_x & \sum I_x I_y \\ \sum I_x I_y & \sum I_y I_y \end{bmatrix}}_{A^T A} \begin{bmatrix} u \\ v \end{bmatrix} = - \underbrace{\begin{bmatrix} \sum I_x I_t \\ \sum I_y I_t \end{bmatrix}}_{A^T b}$$

  - The summations are over all pixels in the K x K window
  - This technique was first proposed by Lukas & Kanade (1981)

Taking a closer look at ($A^T A$)

$$A = \begin{bmatrix} I_x(p_1) & I_y(p_1) \\ I_x(p_2) & I_y(p_2) \\ \dots & \dots \\ I_x(p_{N^2}) & I_y(p_{N^2}) \end{bmatrix}$$

$$A^T = \begin{bmatrix} I_x(p_1) & I_x(p_2) & \dots & I_x(p_{N^2}) \\ I_y(p_1) & I_y(p_2) & \dots & I_y(p_{N^2}) \end{bmatrix}$$

$$A^T A = \begin{bmatrix} \sum I_x^2 & \sum I_x I_y \\ \sum I_x I_y & \sum I_y^2 \end{bmatrix}$$

This is the same matrix we used for corner detection!

Taking a closer look at ($A^T A$)

The matrix for corner detection:

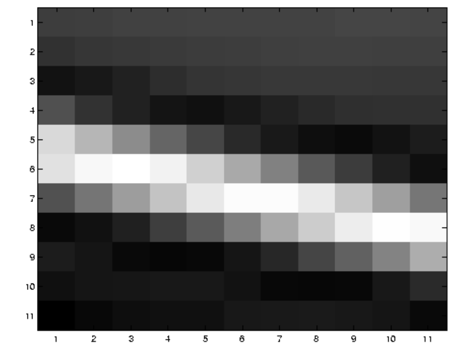
$$A^T A = \begin{bmatrix} \sum I_x^2 & \sum I_x I_y \\ \sum I_x I_y & \sum I_y^2 \end{bmatrix}$$

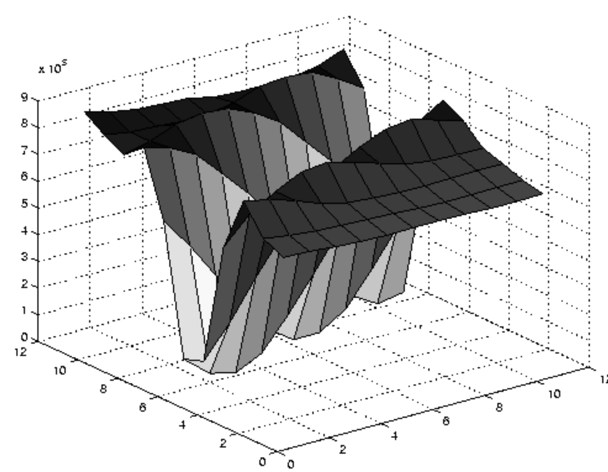
is singular (not invertible) when det($A^T A$) = 0

But det($A^T A$) = $\prod \lambda_i$ = 0 -> one or both e.v. are 0

One e.v. = 0 -> no corner, just an edge
Two e.v. = 0 -> no corner, homogeneous region

Aperture Problem !

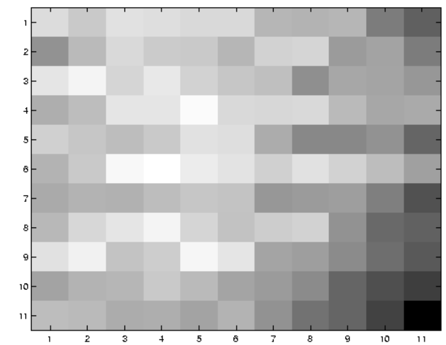


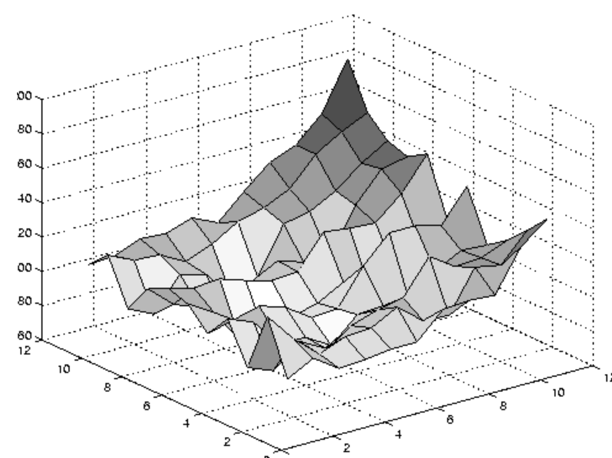


$$\sum \nabla I (\nabla I)^T$$

- large gradients, all the same
- large λ_1, small λ_2

Low texture region

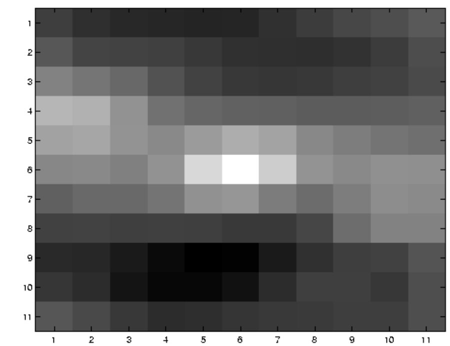


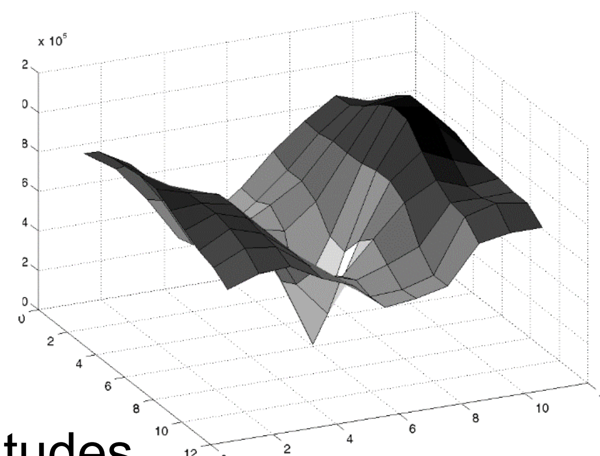


$$\sum \nabla I (\nabla I)^T$$

– gradients have small magnitude
– small λ_1, small λ_2

High textured region





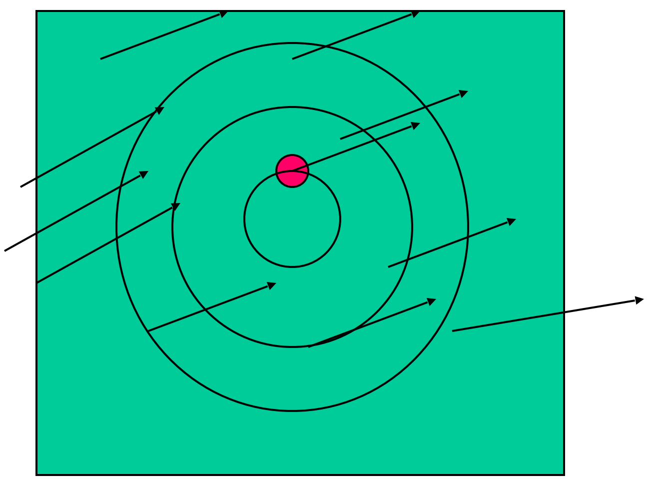
$$\sum \nabla I (\nabla I)^T$$

– gradients are different, large magnitudes

– large λ_1, large λ_2

An improvement …

- NOTE:
  - The assumption of constant OF is more likely to be wrong as we move away from the point of interest (the center point of Q)



Use weights to control the influence of the points: the farther from p, the less weight

Solving for v with weights:

- Let W be a diagonal matrix with weights
- Multiply both sides of $Av = b$ by W:

$$W A v = W b$$

- Multiply both sides of $WAv = Wb$ by $(WA)^T$:

$$A^T WWA v = A^T WWb$$

- $A^T W^2 A$ is square (2x2):
  - $(A^T W^2 A)^{-1}$ exists if $\det(A^T W^2 A) \neq 0$
- Assuming that $(A^T W^2 A)^{-1}$ does exists:

$$(A^T W^2 A)^{-1} (A^T W^2 A) v = (A^T W^2 A)^{-1} A^T W^2 b$$

$$v = (A^T W^2 A)^{-1} A^T W^2 b$$

Observation

- This is a problem involving two images BUT
  - Can measure sensitivity by just looking at one of the images!
  - This tells us which pixels are easy to track, which are hard
    - very useful later on when we do feature tracking...

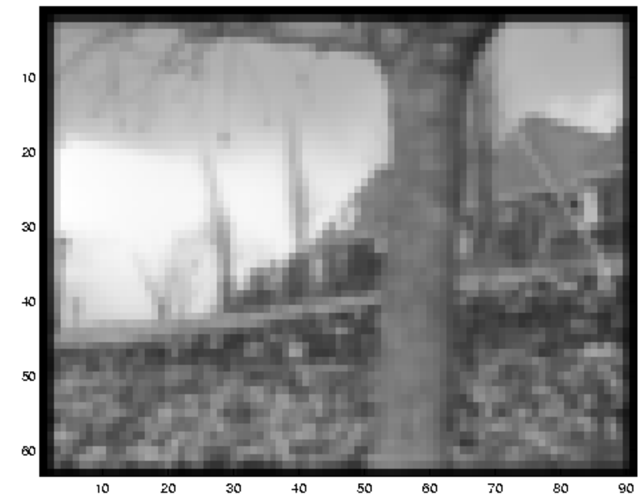
Revisiting the small motion assumption

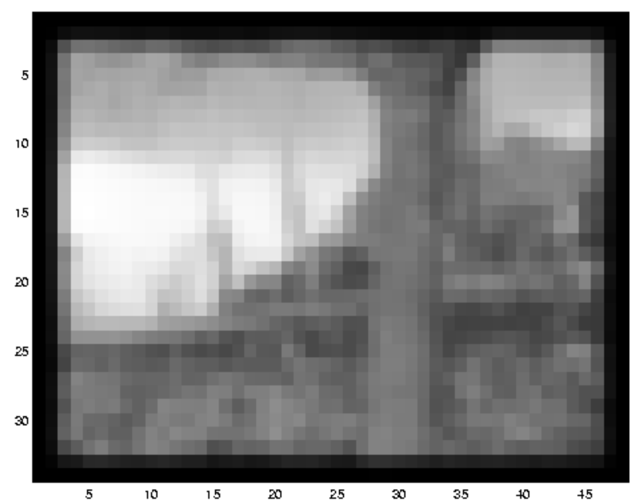
- Is this motion small enough?
  - Probably not—it's much larger than one pixel (2nd order terms dominate)
  - How might we solve this problem?

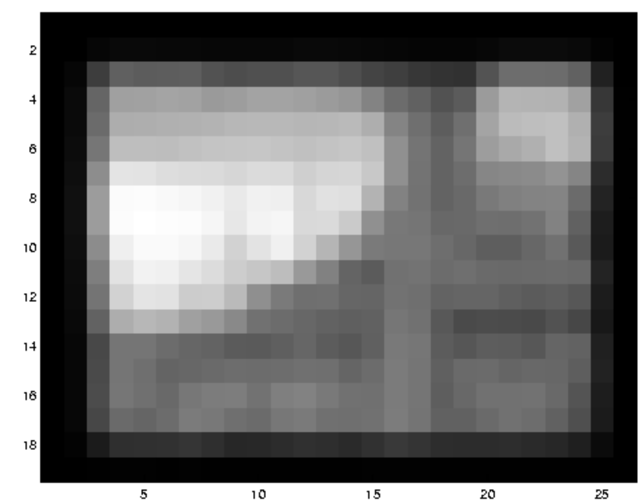
Iterative Refinement

- Iterative Lukas-Kanade Algorithm
  1. Estimate velocity at each pixel by solving Lucas-Kanade equations
  2. Warp I(t-1) towards I(t) using the estimated flow field
     - *use image warping techniques*
  3. Repeat until convergence

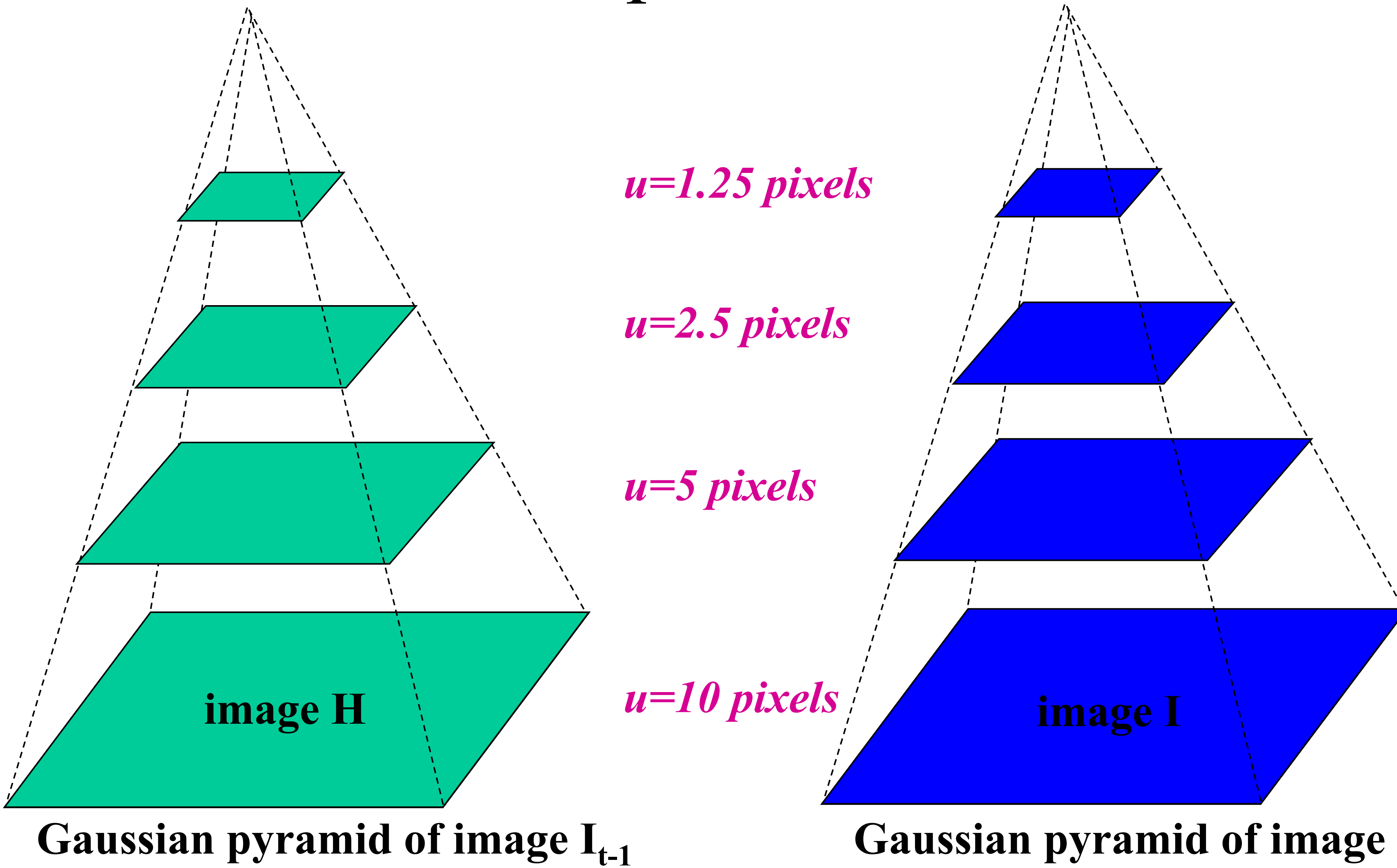
Reduce the resolution!



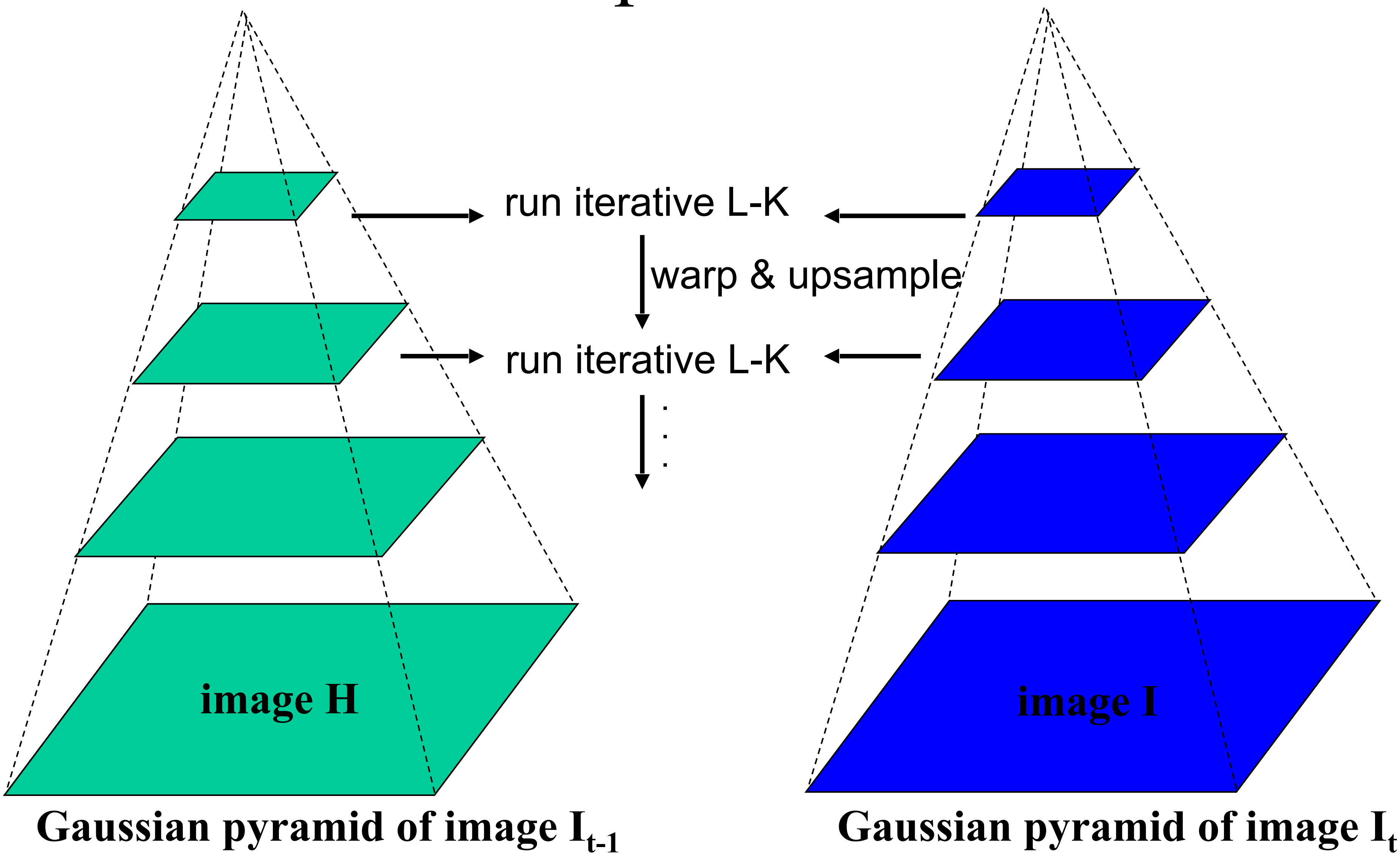




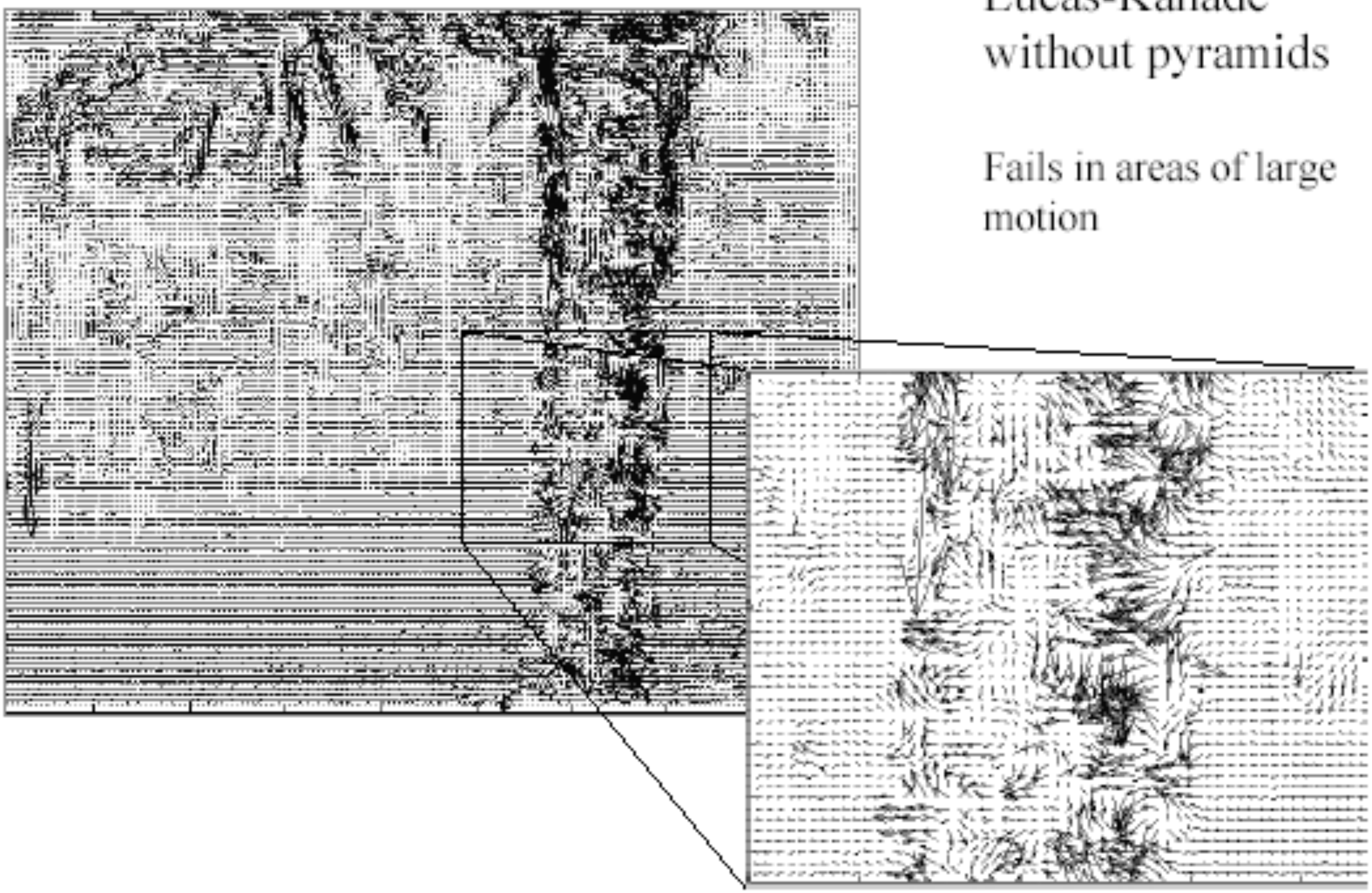
Coarse-to-fine optical flow estimation



Coarse-to-fine optical flow estimation



Optical Flow Results

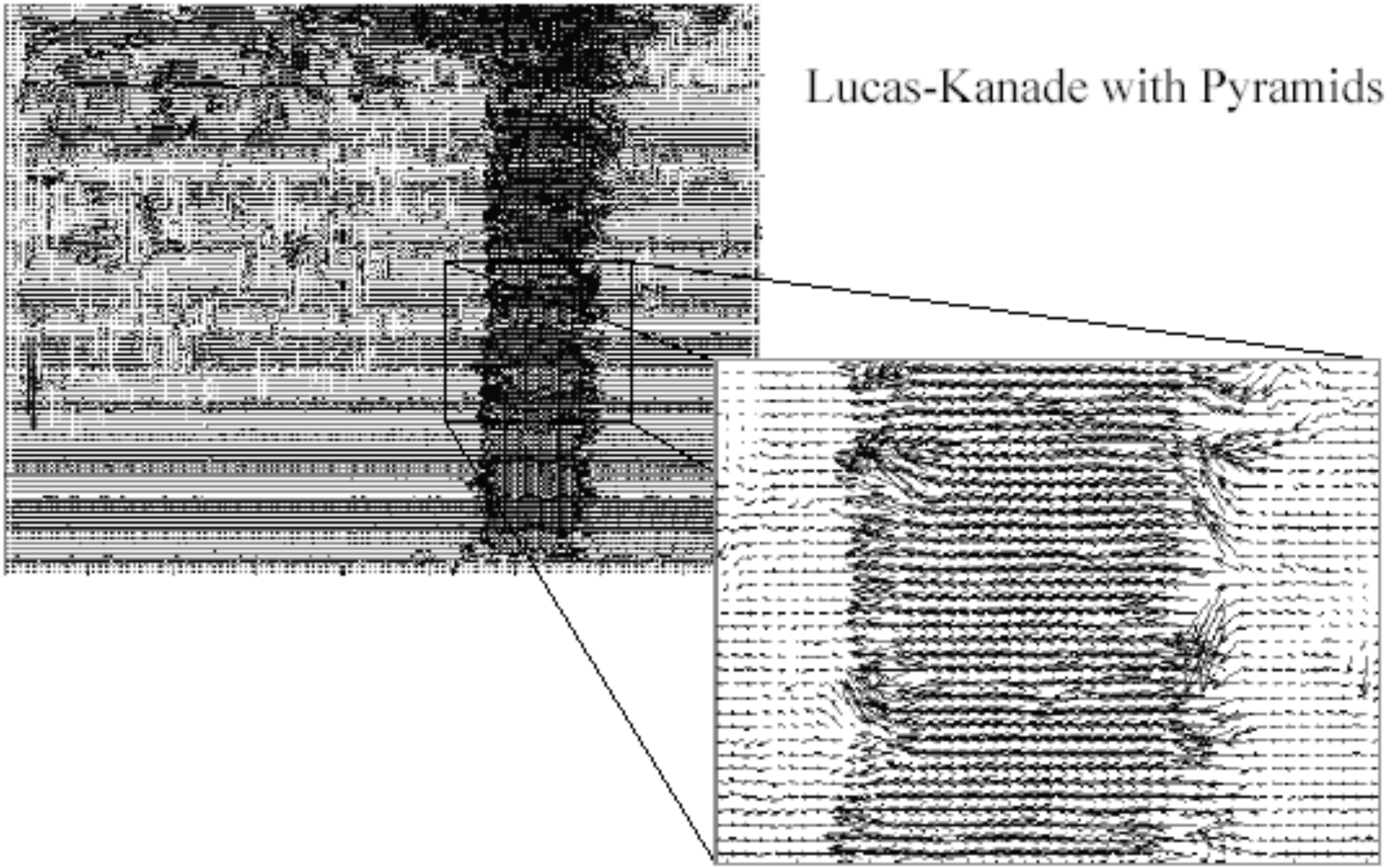


Lucas-Kanade
without pyramids

Fails in areas of large
motion

* From Khurram Hassan-Shafique CAP5415 Computer Vision 2003

Optical Flow Results



Lucas-Kanade with Pyramids

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