CMSC 132: OBJECT-ORIENTED PROGRAMMING II

Advanced Tree Structures

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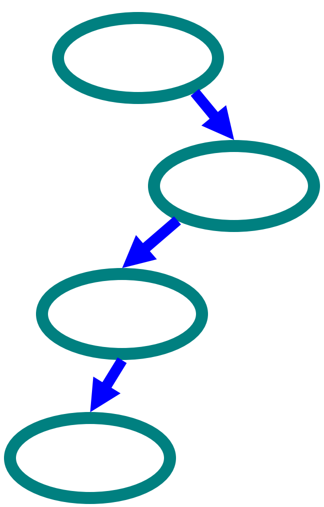
Overview

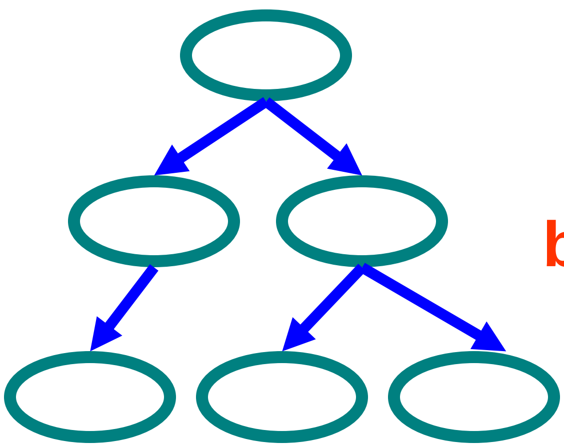
- **Binary trees**
  - Balance
  - Rotation
- **Multi-way trees**
  - Search
  - Insert
- **Indexed tries**

Tree Balance

- **Degenerate**
  - Worst case
  - Search in O(n) time
- **Balanced**
  - Average case
  - Search in O(log(n)) time



Degenerate binary tree



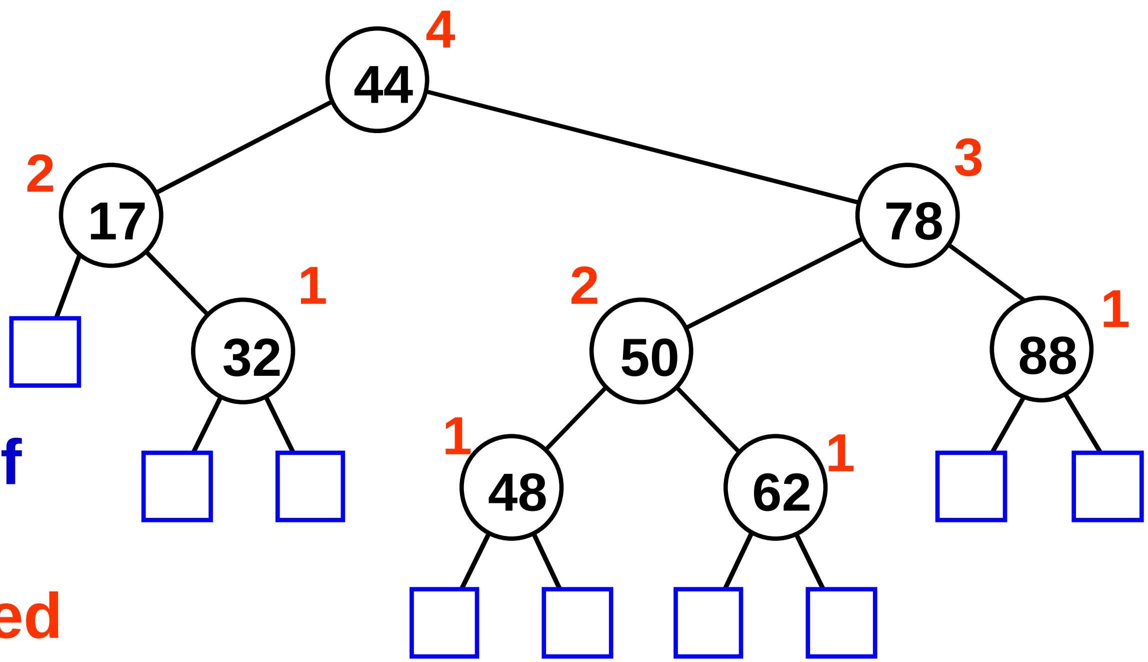
Balanced binary tree

Tree Balance

- **Question**
  - Can we keep tree (mostly) balanced?
- **Self-balancing binary search trees**
  - AVL trees
  - Red-black trees
- **Approach**
  - Select invariant (that keeps tree balanced)
  - Fix tree after each insertion / deletion
    - For example, maintain invariant using rotations
  - Provides operations with O(log(n)) worst case

AVL Trees

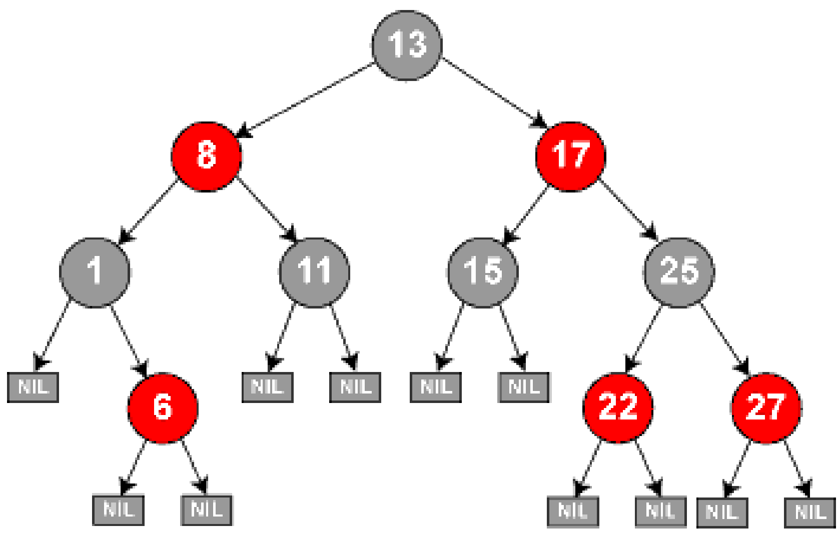
- **Properties**
  - Binary search tree
  - **Heights of children for node differ by at most 1**
- **Example**



Heights of children shown in red

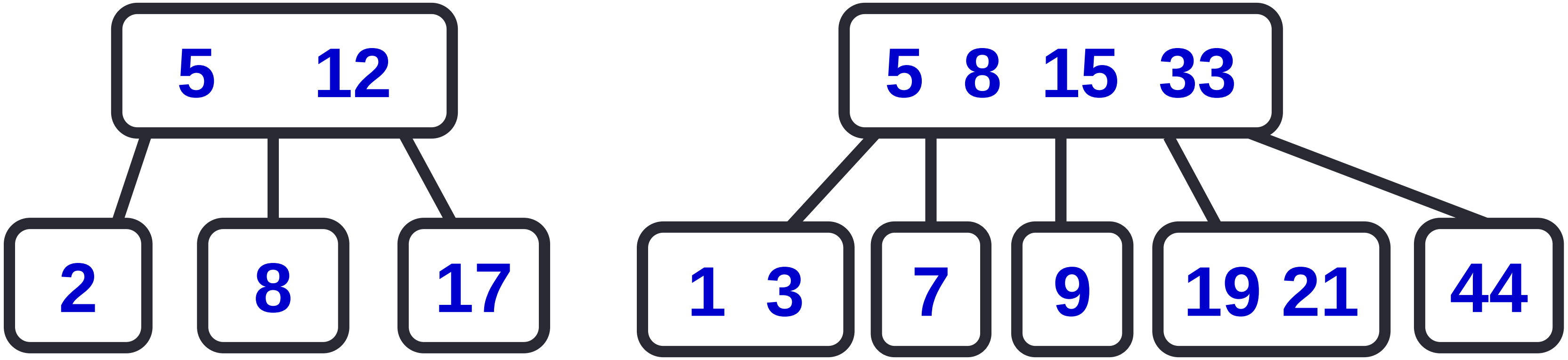
Red-black Trees

- **Java collections**
  - TreeMap and TreeSet use red-black trees
- **Properties**
  - Binary search tree
  - Every node is red or black
- **Characteristics**



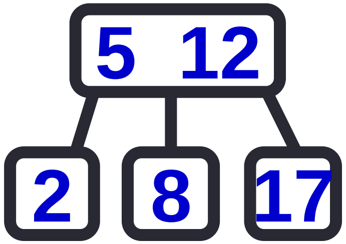
Multi-way Search Trees

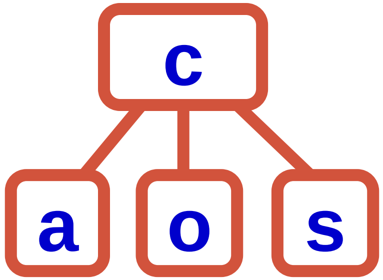
- **Properties**
  - **Generalization of binary search tree**
  - Node contains 1…k keys (in sorted order)
  - Node contains 2…k+1 children
  - Keys in j^{th} child < j^{th} key < keys in $(j+1)^{th}$ child
- **Examples**

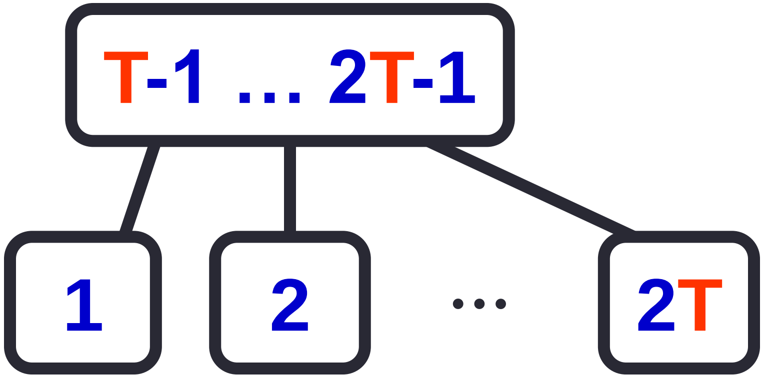


Types of Multi-way Search Trees

- **2-3 Tree**
  - Internal nodes have 2 or 3 children
- **Indexed Search Tree (trie)**
  - Internal nodes have up to 26 children (for strings)
- **B-Tree**
  - **T** = minimum degree
  - Height of tree is O($\log_T(n)$)
  - All leaves have same depth
  - Popular for large databases indices
    - 1 node = 1 disk block

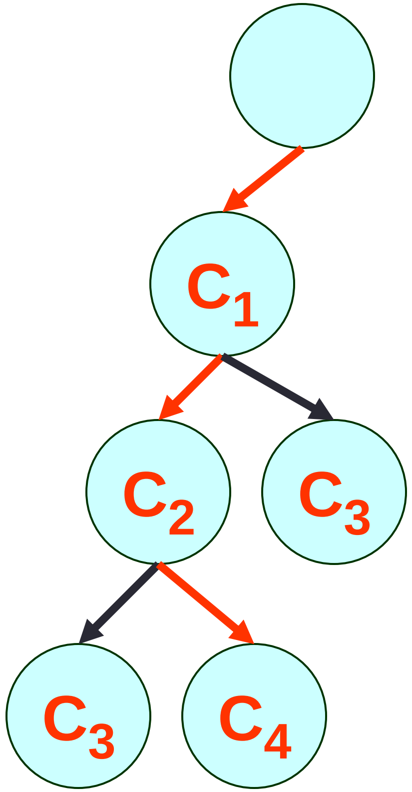






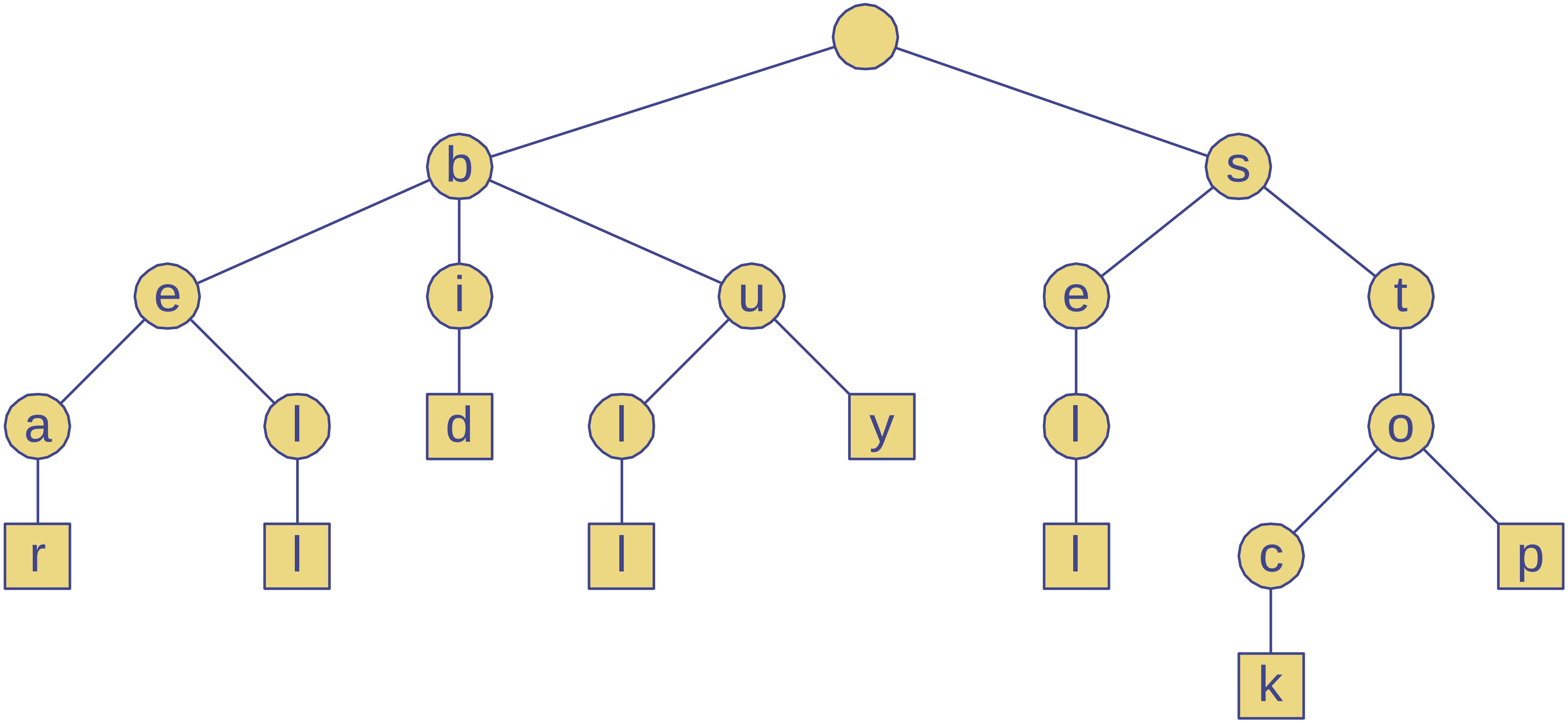
Indexed Search Tree (Trie)

- Special case of tree
- Applicable when
  - Key C can be decomposed into a sequence of subkeys C_1, C_2, … C_n
  - Redundancy exists between subkeys
- Approach
  - Store subkey at each node
  - Path through trie yields full key



Standard Trie Example

- **Example** for strings { bear, bell, bid, bull, buy, sell, stock, stop }



Word Location Trie

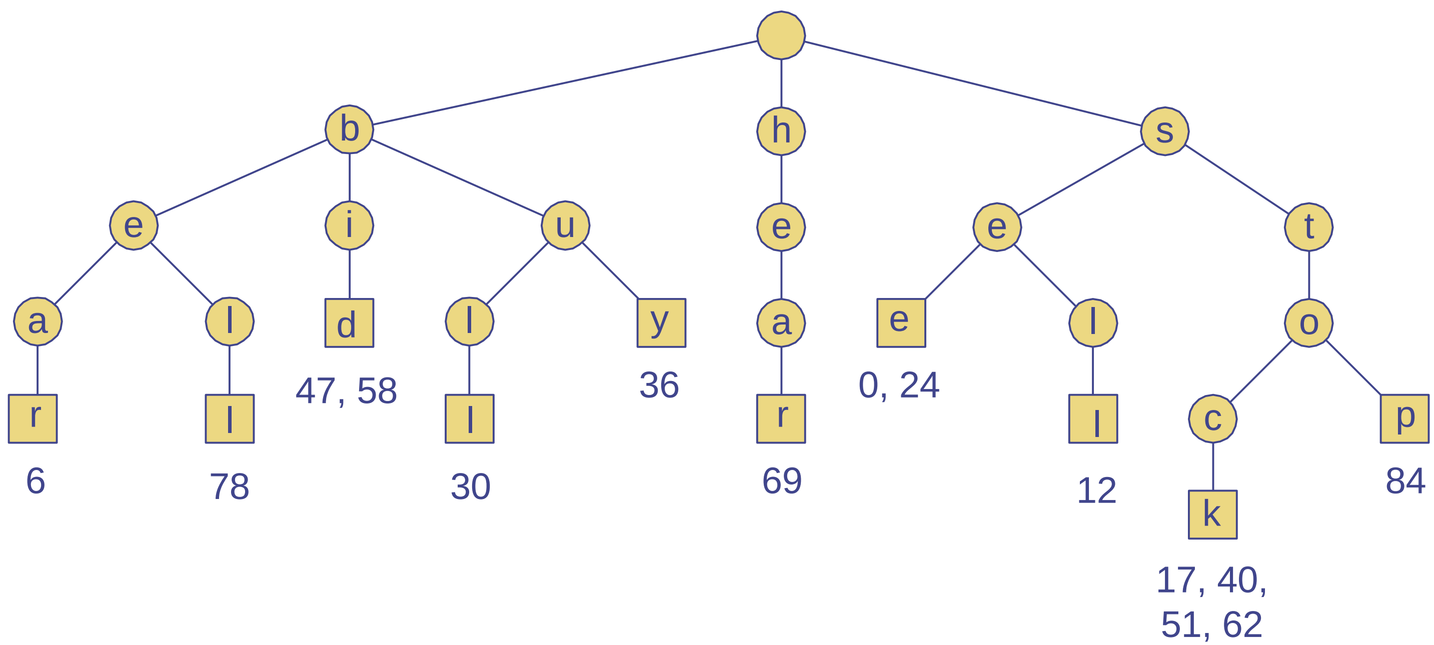
- Insert words into trie
- Each leaf stores locations of word in the text

s	e	e		a		b	e	a	r	?		s	e	l	l		s	t	o	c	k	!	
0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23

s	e	e		a		b	u	l	l	?		b	u	y		s	t	o	c	k	!	
24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40	41	42	43	44	45	46

b	i	d		s	t	o	c	k	!		b	i	d		s	t	o	c	k	!	
47	48	49	50	51	52	53	54	55	56	57	58	59	60	61	62	63	64	65	66	67	68

h	e	a	r		t	h	e		b	e	l	l	?		s	t	o	p	!
69	70	71	72	73	74	75	76	77	78	79	80	81	82	83	84	85	86	87	88

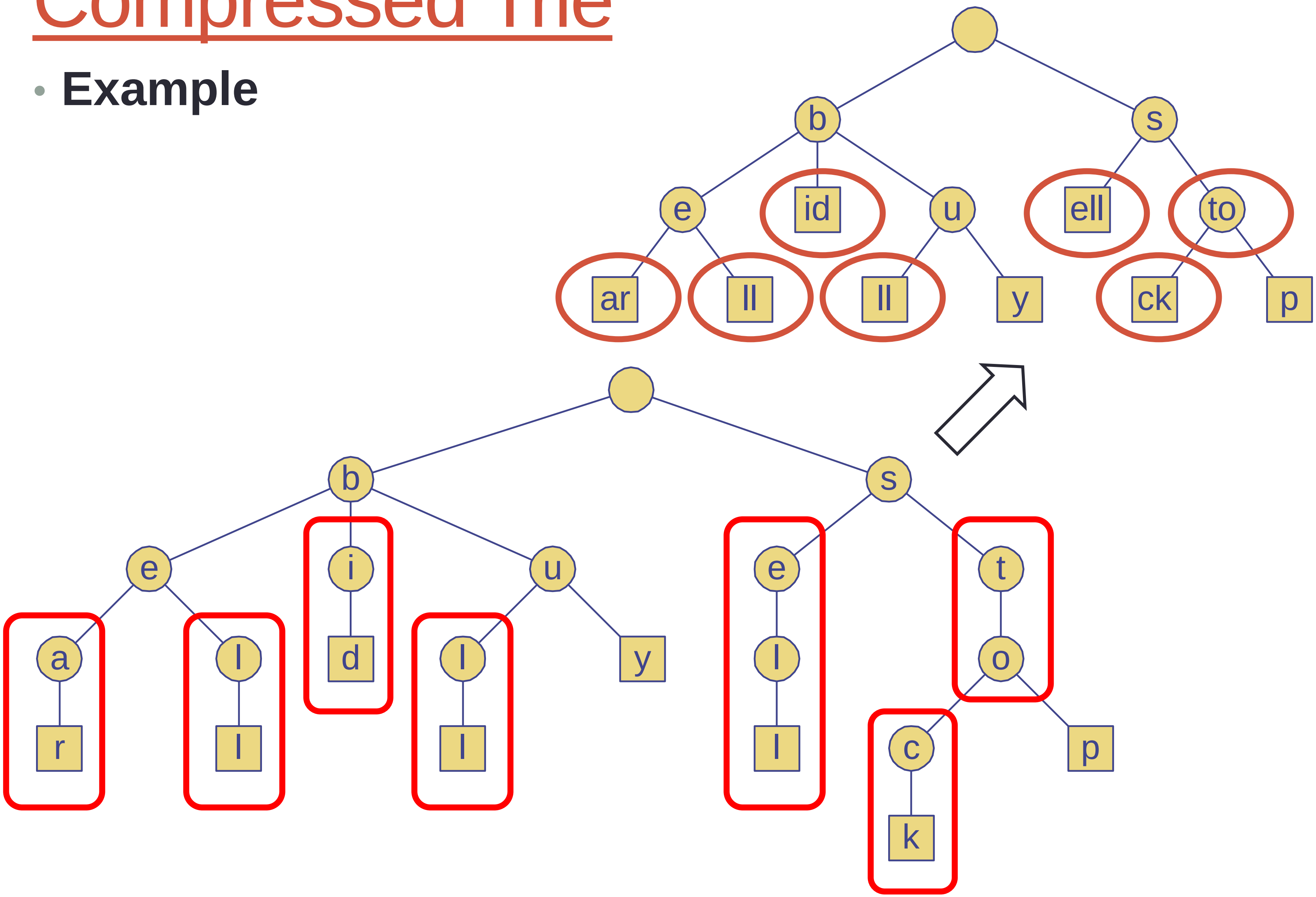


Compressed Trie

- **Observation**
  - Internal node v of T is redundant if v has one child and is not the root
- **Approach**
  - A chain of redundant nodes can be compressed
    - Replace chain with single node
    - Include concatenation of labels from chain
- **Result**
  - Internal nodes have at least 2 children
  - Some nodes have multiple characters

Compressed Trie

- **Example**



Tries and Web Search Engines

- **Search engine index**
  - Collection of all searchable words
  - Stored in compressed trie
- **Each leaf of trie**
  - Associated with a word
  - List of pages (URLs) containing that word
    - Called occurrence list
- **Trie is kept in memory (fast)**
- **Occurrence lists kept in external memory**
  - Ranked by relevance