# CMSC 330: Organization of Programming Languages

Closures
(Implementing Higher Order Functions)

#### Returning Functions as Results

- In OCaml you can pass functions as arguments
  - to map, fold, etc.
- and you can return functions as results

```
# let pick_fn n =
    let plus_three x = x + 3 in
    let plus_four x = x + 4 in
    if n > 0 then plus_three else plus_four
val pick fn : int -> (int->int) = <fun>
```

Here, pick\_fn takes an int argument, and returns a
function # let g = pick fn 2;;

```
val g : int -> int = <fun>
# g 4;; (* evaluates to 7 *)
```

## Multi-argument Functions

Consider a rewriting of the prior code (above)

```
let pick_fn n =
  if n > 0 then (fun x -> x+3) else (fun x -> x+4)
```

Here's another version

```
let pick_fn n =
  (fun x -> if n > 0 then x+3 else x+4)
```

... the shorthand for which is just

```
let pick_fn n x =
  if n > 0 then x+3 else x+4
```

*I.e., a multi-argument function!* 

## Currying

- Multi-argument functions not a separate concept
  - Can encode one as a function that takes a single argument and returns a function that takes the rest
- This encoding is called currying the function
  - Named after the logician Haskell B. Curry
  - But Schönfinkel and Frege discovered it
    - > So maybe it should be called Schönfinkelizing or Fregging

#### **Curried Functions In OCaml**

OCaml syntax defaults to currying. E.g.,

```
let add x y = x + y
```

is identical to all of the following:

```
let add = (fun x -> (fun y -> x + y))
let add = (fun x y -> x + y)
let add x = (fun y -> x+y)
```

- ▶ Thus:
  - add has type int -> (int -> int)
  - add 3 has type int -> int
     add 3 is a function that adds 3 to its argument
  - (add 3) 4 = 7

## Syntax Conventions for Currying

- Because currying is so common, OCaml uses the following conventions:
  - -> associates from the right
    - > Thus int -> int -> int is the same as
    - > int -> (int -> int)
  - function application associates from the left
    - > Thus add 3 4 is the same as
    - > (add 3) 4

#### Quiz 1: Which f definition is equivalent?

```
let f a b = a / b;;

A. let f b = fun a -> a / b;;

B. let f = fun a -> (fun b -> a / b);;

C. let f = fun a | b -> a / b;;

D. let f (a, b) = a / b;;
```

#### Quiz 1: Which f definition is equivalent?

```
let f a b = a / b;;

A. let f b = fun a -> a / b;;

B. let f = fun a -> (fun b -> a / b);;

C. let f = fun a | b -> a / b;;

D. let f (a, b) = a / b;;
```

## Multiple Arguments, Partial Application

 Another way you could encode support for multiple arguments is using tuples

```
• let f (a,b) = a / b (* int*int -> int *)
• let f a b = a / b (* int -> int-> int *)
```

- Is there a benefit to using currying instead?
  - Supports partial application useful when you want to provide some arguments now, the rest later

```
let add a b = a + b;;
let addthree = add 3;;
addthree 4;; (* evaluates to 7 *)
```

#### Quiz 2: What does this evaluate to?

```
let f a b = a * b in
let g = f 2 in
let a = 3 in
g 4
```

- A. 8
- B. 6
- C. 2
- D. 3

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#### Quiz 2: What does this evaluate to?

```
let f a b = a * b in
let g = f 2 in
let a = 3 in
g 4 (* f 2 4 = 8 *)
A. 8
B. 6
C. 2
D. 3
```

## Currying is Standard In OCaml

- Pretty much all functions are curried
  - Like the standard library map, fold, etc.
  - See /usr/local/ocaml/lib/ocaml on Grace
    - > In particular, look at the file list.ml for standard list functions
    - > Access these functions using List.<fn name>
    - > E.g., List.hd, List.length, List.map
- OCaml works hard to make currying efficient
  - Because otherwise it would do a lot of useless allocation and destruction of closures
  - What are those, you ask? Let's see ...

## Closures

#### Remember our partial application example

```
let add = fun a -> fun b -> a + b;;

let addthree = add 3 in
addthree 4
```

Let's evaluate it the expression (using substitution)

```
let addthree = add 3 in addthree 4

> let addthree = (fun a -> fun b -> a+b) 3 in ...

> let addthree = (fun b -> 3+b) in addthree 4

> (fun b -> 3+b) 4

> 3+4 → 7
```

#### Using Substitution "Remembered" the a is 3

```
let add = fun a -> fun b -> a + b;;

let addthree = add 3 in
addthree 4
```

Let's evaluate it the expression (using substitution)

```
let addthree = add 3 in addthree 4

→ let addthree = (fun a -> fun b -> a+b) 3 in ...

→ let addthree = (fun b -> 3+b) in addthree 4

→ (fun b -> 3+b) 4

→ 3+4 → 7
```

#### How to use a stack, not substitution?

- Substitution replaces the occurrence of the variable with the value it is bound to (e.g., at a call)
  - Like changing the code in place!
- In reality, we use a stack to remember variable-to-value mappings

  let addthree = add 3 in addthree 4
  - But: If calling add 3 pushes 3 on the stack, what happens when the call returns? How does addthree remember that it was constructed by a call with 3?

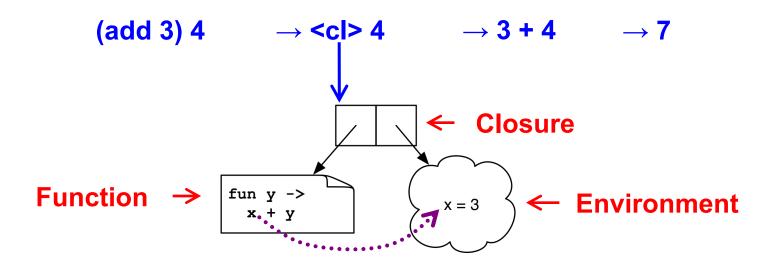
#### Closures "Remember"

- An environment is a mapping from variables to values
  - Like a stack frame
- A closure is a pair (f, e) consisting of function code f and an environment e
  - Environment "captures" active bindings, when closure is made
  - These include "free variables" these are mentioned in f's body but are not its formal parameters

When you invoke a closure, f is evaluated using e

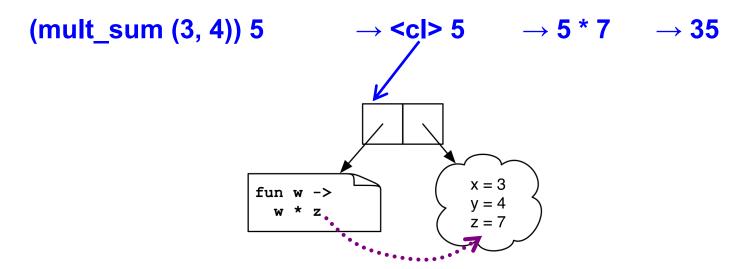
## Example 1

let add 
$$x = (fun y \rightarrow x + y)$$



## Example 2

```
let mult_sum (x, y) =
  let z = x + y in
  fun w -> w * z
```



#### Quiz 3: What is x?

```
let a = 1;;
let a = 0;;
let b = 10;;
let f () = a + b;;
let b = 5;;
let x = f ();;
```

- A. 10
- B. 1
- C. 15
- D. Error variable name conflicts

#### Quiz 3: What is x?

```
let a = 1;;
let a = 0;;
let b = 10;;
let f = fun () -> a + b;;
let b = 5;;
let x = f ();;
```

- A. 10
- B. 1
- C. 15
- D. Error variable name conflicts

#### Quiz 4: What is z?

```
let f x = fun y -> x - y in
let g = f 2 in
let x = 3 in
let z = g 4 in
z;;
```

- A. 7
- B. -2
- C.-1
- D. Type Error insufficient arguments

#### Quiz 4: What is z?

```
let f x = fun y -> x - y in
let g = f 2 in
let x = 3 in
let z = g 4 in
z;;
```

- A. 7
- B. -2
- C.-1
- D. Type Error insufficient arguments

#### Quiz 5: What does this evaluate to?

```
let f x = x+1 in
let g = f in
g (fun i -> i+1) 1
```

- A. Type Error
- B. 1
- C. 2
- D. 3

#### Quiz 5: What does this evaluate to?

```
let f x = x+1 in
let g = f in
(g (fun i -> i+1)) 1
```

- **A.** Type Error Too many arguments passed to g (application is *left associative*)
- B. 1
- C. 2
- D. 3

#### Scope

#### Dynamic scope

- The body of a function is evaluated in the current dynamic environment at the time the function is called, not the environment that existed at the time the function was defined
  - > Now basically considered a mistake
- Lexical scope (aka Static scope)
  - The body of a function is evaluated in the old dynamic environment that existed at the time the function was defined, not the current environment when the function is called.
  - This is implemented by closures

## Dynamic vs. Static Scope

```
let f a b = a * b in
let g = f 2 in
let a = 3 in
g 4
```

- Answer, if lexical/static scope
- B. 12 Answer, if dynamic scope
- c. **2**
- D. 3

## Higher-Order Functions in C

C supports function pointers

```
typedef int (*int func)(int);
void app(int func f, int *a, int n) {
  for (int i = 0; i < n; i++)
    a[i] = f(a[i]);
int add one(int x) { return x + 1; }
int main() {
  int a[] = \{5, 6, 7\};
  app(add one, a, 3);
```

# Higher-Order Functions in C (cont.)

- C does not support closures
  - Since no nested functions allowed
  - Unbound symbols always in global scope

```
int y = 1;
void app(int(*f)(int), n) {
  return f(n);
int add_y(int x) {
  return x + y;
int main()
  app(add_y, 2);
```

# Higher-Order Functions in C (cont.)

- Cannot access non-local variables in C
- OCaml code

```
let add x y = x + y
```

Equivalent code in C is illegal

```
int (* add(int x))(int) {
  return add_y;
}
int add_y(int y) {
  return x + y; /* error: x undefined */
}
```

# Higher-Order Functions in C (cont.)

OCaml code

```
let add x y = x + y
```

- Works if C supports nested functions
  - Not in ISO C, but in gcc; but not allowed to return them

```
int (* add(int x))(int) {
  int add_y(int y) {
    return x + y;
  }
  return add_y; }
```

 Does not allocate closure, so x popped from stack and add\_y will get garbage (potentially) when called

## Java 8 Supports Lambda Expressions

Ocaml's

fun 
$$(a, b) \rightarrow a + b$$

Is like the following in Java 8

$$(a, b) -> a + b$$

Java 8 supports closures, and variations on this syntax

# Java 8 Example

```
public class Calculator {
  interface IntegerMath { int operation(int a, int b); }
  public int operateBinary(int a, int b, IntegerMath op) {
     return op.operation(a, b);
  public static void main(String... args) {
     Calculator myApp = new Calculator();
                                                                   Lambda
     IntegerMath addition = (a, b) \rightarrow a + b;
     IntegerMath subtraction = (a, b) \rightarrow a - b;
                                                                   expressions
     System.out.println("40 + 2 = " +
       myApp.operateBinary(40, 2, addition));
     System.out.println("20 - 10 = " +
       myApp.operateBinary(20, 10, subtraction));
```