

Problem 1. What is the exact number of atomic multiplications carried out using Karatsuba algorithm to find the product 4352×3748 ? Show your work including all the atomic multiplications in your count.

Note: If at any step the two numbers that you're multiplying do not have the same number of digits, please use leading zeros for the number with the fewer digits to make them the same size. For example, if the multiplication is 12×3 , consider it to be 12×03 , but the multiplication with zero is not counted. This is done only so that the split is nice.

Problem 2. Karatsuba algorithm for integer multiplication has a runtime complexity of $O(n^{1.58})$. We want to test a few multiplication algorithms along the lines of Karatsuba. You may use μ for an atomic multiplication and α for an atomic addition, wherever required. Please follow and answer the following questions:

1. Write a simple (without any clever tricks) recurrence equation for a multiplication algorithm that divides each n digit number into 3 parts rather than 2 parts with the size of each part being $\frac{n}{3}$. We will call this TC algorithm.
2. Solve the recurrence equation for TC algorithm using the recursion tree approach as shown in class.
3. For this and the rest of the parts we will only find the Big-O runtime. What is the runtime of TC algorithm?
4. You probably noticed that TC algorithm is slower than Karatsuba's algorithm. We will try to make it faster. While still splitting the integers into three parts, we will reduce the number of multiplications for TC algorithm by 1. What is the runtime of this algorithm with this modification?
5. What should be the number of multiplications (keep the 3 part split), so that TC algorithm becomes faster than Karatsuba algorithm? What is the runtime?
6. Now we will divide each n -digit number into 4 parts, rather than 3, with the size of each part being $\frac{n}{4}$. How many multiplications do we need so that the runtime of this new algorithm is faster than Karatsuba algorithm?

For each part, show your work.

Challenge Problem (Not graded). For Problem 2(5), show that the reduced number of multiplications is possible when each n -digit number is divided into 3 parts with the size of each part being $\frac{n}{3}$.