Problem 1. Use the integral method to get upper and lower bounds for $\sum_{j=0}^{n} j^2$. The two values should have exactly the same high order term. Show your work.

Problem 2. Draw the Decision Tree for Bubble Sort on three distinct elements a, b, c. Note that Bubble Sort is inefficient, so it does some redundant comparisons and some nodes will have only one child. What is the average number of comparisons?

Problem 3. Assume that your computer has special hardware that finds the minimum of k (or fewer) elements in one comparison step. Your answers to this question should have n and k as parameters.

(a) Design (in English) an efficient algorithm based on Merge sort to sort n elements using this special hardware. (This is an upper bound.)

(b) Analyze your algorithm. Get the higher order term exactly.

(c) Use decision trees (no need to draw) to find a lower bound for sorting when using this special hardware.

(d) Compare your upper and lower bounds.