Basic Data Structures I

- ADTs
- Lists, Stacks, Queues
- Sequential Allocation

Abstract Data Type (ADT)

- Abstracts the functional elements of a data structure (math) from its implementation (algorithm / programming)

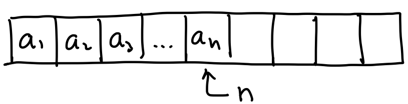
Linear List ADT:

Stores a sequence of elements $\langle a_1, a_2, \ldots, a_n \rangle$. Operations:

- init() - create an empty list
- get(i) - returns a_i
- set(i,x) - sets i^{th} element to x
- insert(i,x) - inserts x prior to i^{th} (moving others back)
- delete(i) - deletes i^{th} item (moving others up)
- length() - returns num. of items

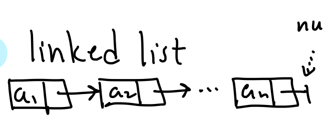
Implementations:

Sequential: Store items in an array

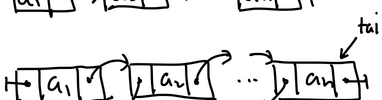


Linked allocation: linked list

Singly: head →

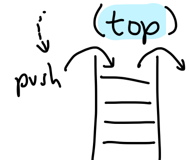


Doubly: head →

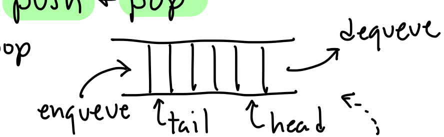


Performance varies with implementation

Stack: All access from one side (top) - push + pop



Queue: FIFO list: enqueue inserts at tail and dequeue deletes from head



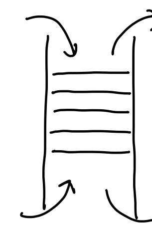
Dynamic Lists + Sequential Allocation: What to do when your array runs out of space?

Doubling Reallocation:

When array of size n overflows

- allocate new array size 2n
- copy old to new
- remove old array

Deque ("deck"): Can insert or delete from either end

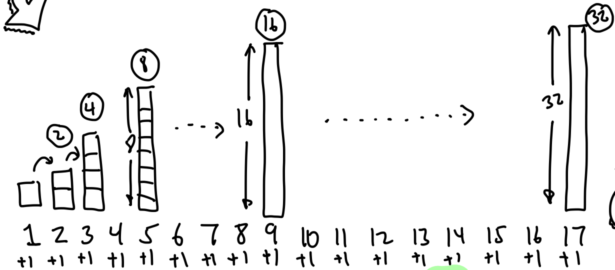


Basic Data Structures II
– Amortized analysis of dynamic stack

Cost model (Actual cost)

Cheap: No reallocation → 1 unit

Expensive: Array of size n is reallocated to size 2n } → 2n+1



Total = 17+(2+4+8+16+32) = 79

Amortized Cost:

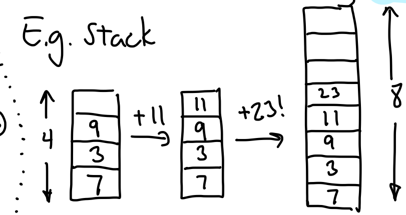
Starting from an empty structure, suppose that any sequence of m ops takes time T(m). The amortized cost is T(m)/m.

Thm: Starting from an empty stack, the amortized cost of our stack operations is at most 5.
[ie. any seq. of m ops has cost ≤ 5·m]

Dynamic (Sequential) Allocation

– When we overflow, double

E.g. Stack



Charging Argument:

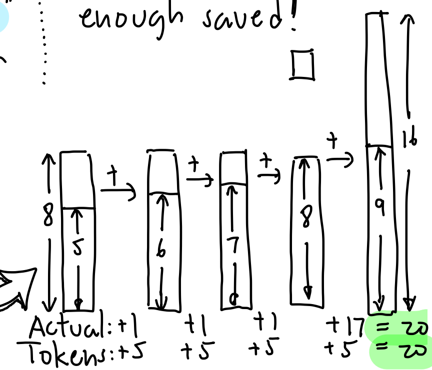
– Each request of push/pop we charge user 5 "work tokens"
– We use 1 token to pay for the operation + put other 4 in bank account.
– Will show there is enough in bank account to pay actual costs.

Proof:

– Break the full sequence after each reallocation → run

1 2 | 3 | 4 5 | 6 7 8 9 | 10 11 … 16 17 |

– At start of a run there are n+1 items in stack and array size is 2n
– There are at least n ops before the end of run
– During this time we collect at least 5n tokens
  → 1 for each op
  → 4 for deposit
– Next reallocation costs 4n, but we have enough saved! □



Fixed Increment: Increase by a fixed constant
$n \longrightarrow n + 100$

Fixed factor: Increase by a fixed constant factor (not nec. 2)
$n \rightarrow 5 \cdot n$

Squaring: Square the size (or some other power)
$n \rightarrow n^2$ or $n \rightarrow \lceil n^{1.5} \rceil$

Which of these provide $O(1)$ amortized cost per operation?

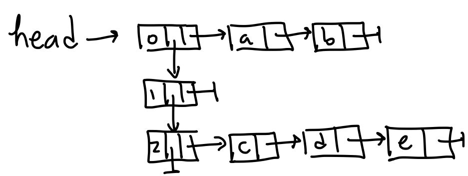
Leave as exercise ☹

(Spoiler alert!)
- Fixed increment → no
- Fixed factor → yes
- Squaring → yes

Basic Data Structures III
- Dynamic Stack - Wrap-up
- Multilists + Sparse Matrices

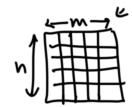
Dynamic Stack:
- Showed doubling ⇒ Amortized $O(1)$
- Other strategies?

Multilists: Lists of lists



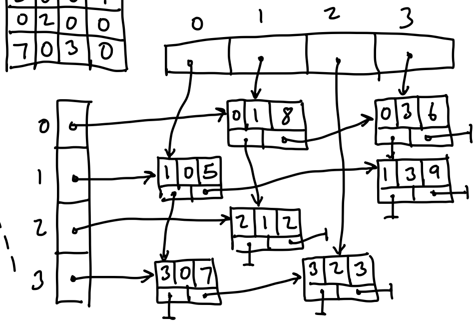
Sparse Matrices:

An $n \times m$ matrix has $n \cdot m$ entries and takes (naively) $O(n \cdot m)$ space

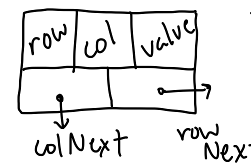


Sparse matrix: Most entries are zero

0	8	0	6
5	0	0	9
0	2	0	0
7	0	3	0



Node:



Idea: Store only nonzero entries linked by row and column