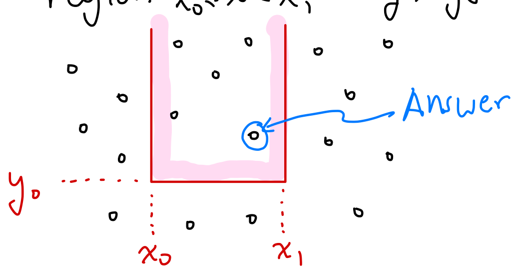
Range Tree Applications:

- Range trees can be applied to a variety of query problems
- Methods:
  - Minimization/Maximization
  - Transform coordinates
  - Adding new coordinates

Minimization/Maximization – 3-Sided Min Query

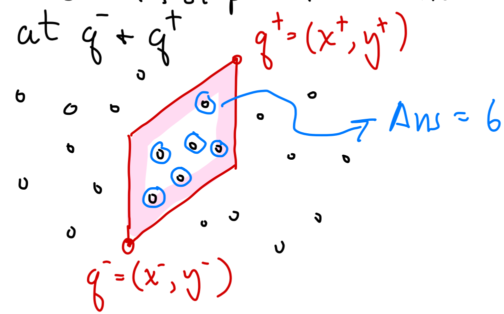
Given a set P of n pts in $\mathbb{R}^2$, a query consists of x-interval $[x_0, x_1]$ and y value y_0. Return the lowest pt in 3-sided region $x_0 \le x \le x_1$ + $y \ge y_0$



Transforming coordinates:

Skewed rectangle query:

Given a set P of n pts in $\mathbb{R}^2$, a skewed rectangle is given by 2 pts $q^- = (x^-, y^-)$ and $q^+ = (x^+, y^+)$ and consists of pts in parallelogram with two vertical sides and two with slope +1 + corners at q^- + q^+

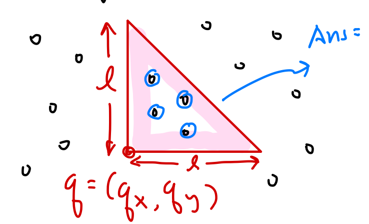


Return a count of the number of pts of P inside the skewed rectangle.

Adding New Coordinates:

NE Right Triangle Query

Given a set P of n pts in $\mathbb{R}^2$ and scalar $\ell > 0$, a NE triangle is a 45-45 right triang with lower left corner at q and side length ℓ.

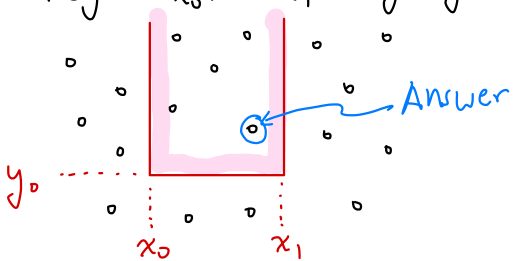


Return a count of the number of pts of P lying within the triangle.

3-Sided Min Query

Return lowest in region

region $x_0 \le x \le x_1$ + $y \ge y_0$



Data structure:

- Build a range tree for x
- Aux. trees are range trees for y that support findLarger

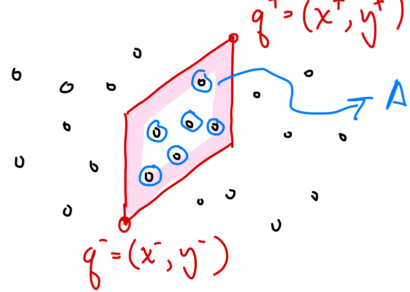
Query Processing:

- Do 1D range search in main tree for interval $[x_0, x_1]$.
- For each maximal subtree in range, do findLarger(y_0)
- Return smallest of these.

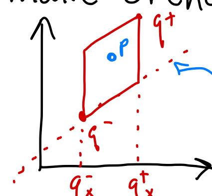
Analysis:

- Same as 2D range tree
- Space: $O(n \log n)$ Time: $O(\log^2 n)$

Skewed rectangle query:



Transform coordinates to make orthog range query



$q_x^- \le p_x \le q_x^+$

Line equation: $y = x + (q_y^- - q_x^-)$

$$p_x + (q_y^- - q_x^-) \le p_y \le p_x + (q_y^+ - q_x^+)$$

$$\Longleftrightarrow \quad q_y^- - q_x^- \le \underbrace{p_y - p_x}_{p_y' = p_y - p_x} \le q_y^+ - q_x^+$$

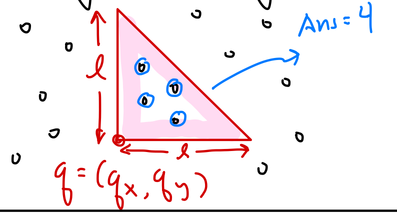
Map each $p = (p_x, p_y) \in P$ to $p' = (p_x', p_y') \triangleq (p_x, p_y - p_x)$

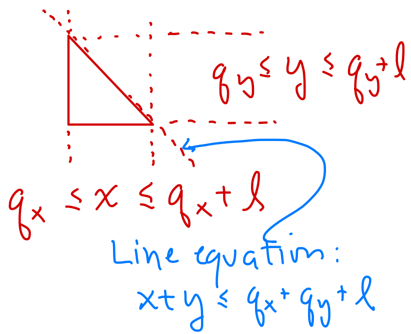
Let P' be resulting set.

Build std. range tree for P'. Return ans. to query

$$\left\{ \begin{array}{l} q_x^- \le x \le q_x^+ \\ q_y^- - q_x^- \le y \le q_y^+ - q_x^+ \end{array} \right.$$

NE Right Triangle Query





- Add new coord:

  $z = x + y$

- Map pts:

  $p = (p_x, p_y) \rightarrow p' = (p_x, p_y, p_x + p_y)$

- Let P' be resulting set

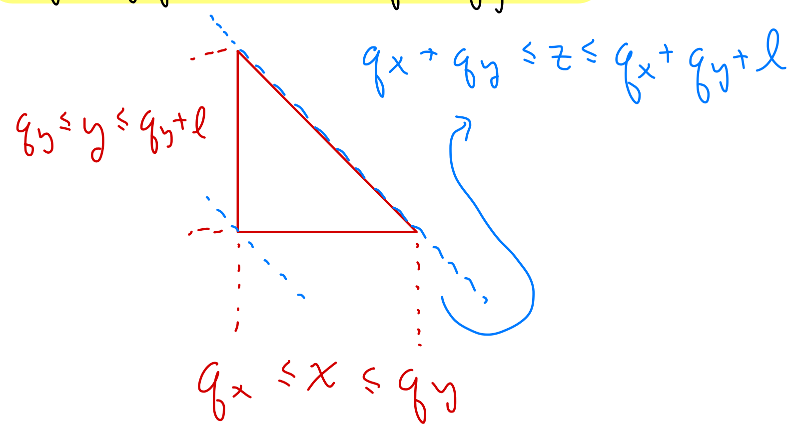
Build a 3D range tree on P'

NE triangle query becomes:

$$q_x \le x \le q_x + l$$

$$q_y \le y \le q_y + l$$

$$q_x + q_y \le z \le q_x + q_y + l$$



Space:

$O(n \log^2 n)$

Query time:

$O(\log^3 n)$