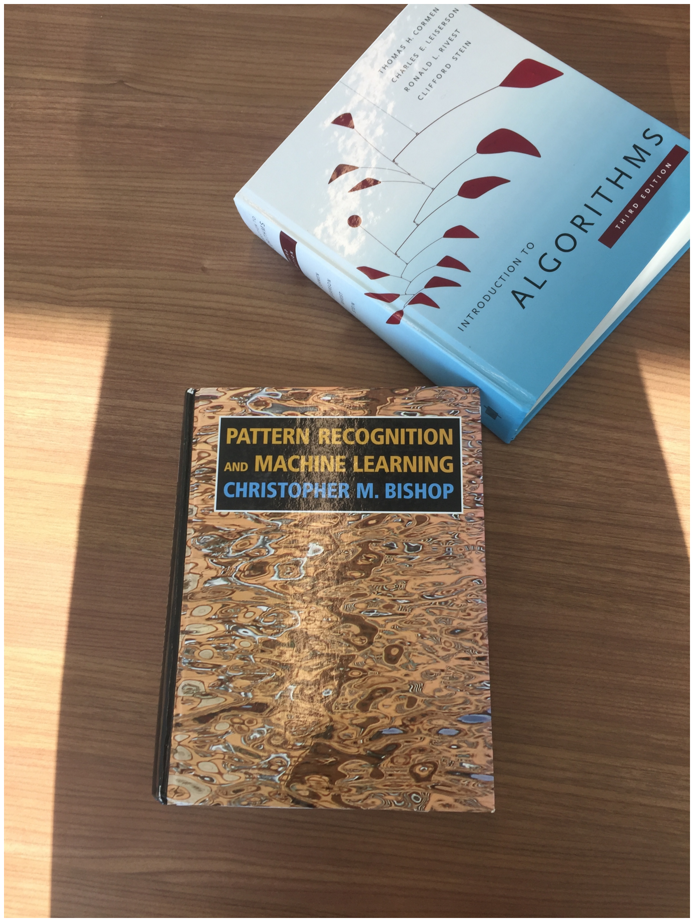
Homography

Homography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PATTERN RECOGNITION
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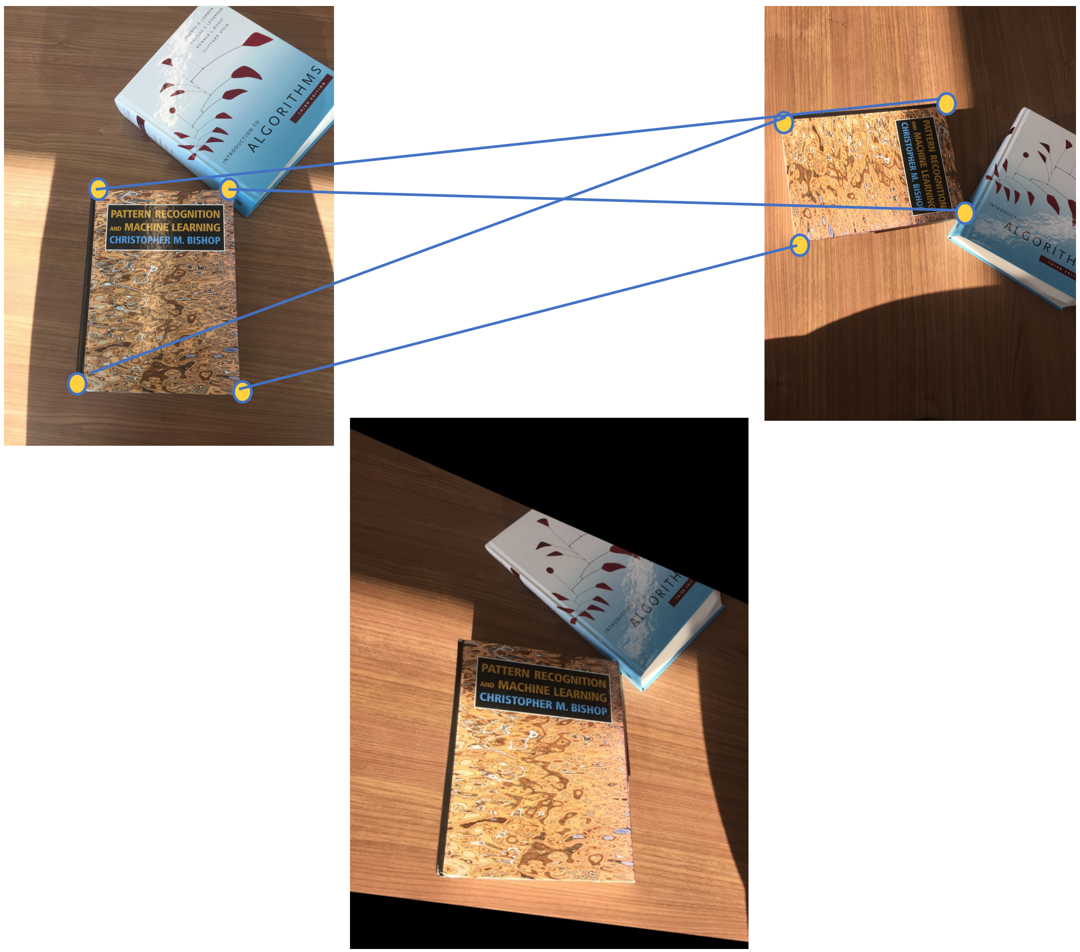
INTRODUCTION TO

Homography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2D homography (projective transformation)

Definition:

A 2D *homography* is an invertible mapping h from P^2 to itself such that three points x_1, x_2, x_3 lie on the same line if and only if $h(x_1), h(x_2), h(x_3)$ do.

Line preserving

Theorem:

A mapping h: $P^2 \rightarrow P^2$ is a homography if and only if there exist a non-singular 3x3 matrix $\mathbf{H}$ such that for any point in P^2 represented by a vector x it is true that $h(x)=\mathbf{H}x$

Definition: Homography

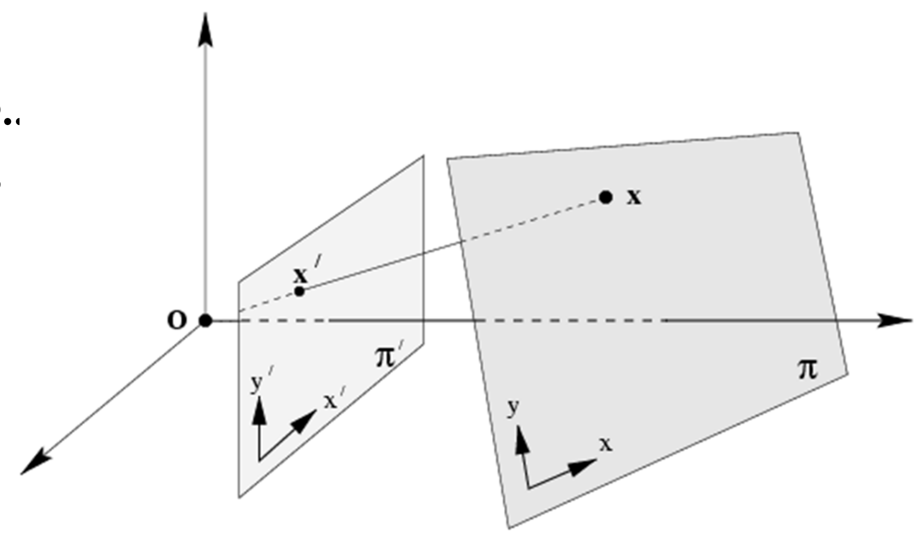
$$\begin{bmatrix} x_2 \\ y_2 \\ z_2 \end{bmatrix} = \begin{bmatrix} h_{00} & h_{01} & h_{02} \\ h_{10} & h_{11} & h_{12} \\ h_{20} & h_{21} & h_{22} \end{bmatrix} \begin{bmatrix} x_1 \\ y_1 \\ z_1 \end{bmatrix}$$

Homography=projective transformation=projectivity=collineation

General homography

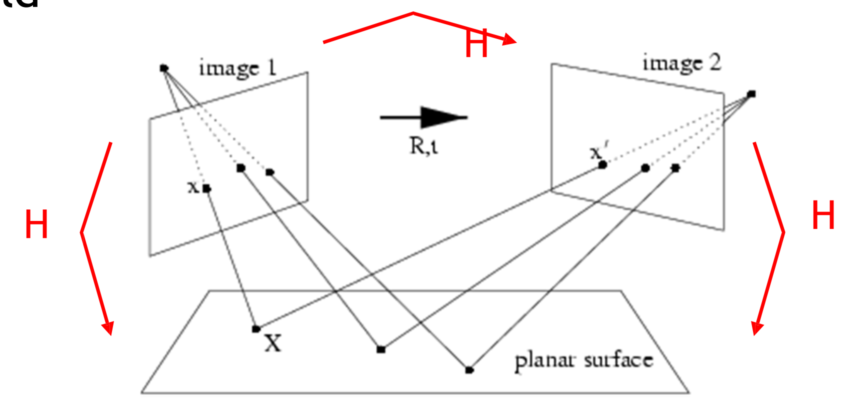
- Note: homographies are not restricted to P^2
- General definition:
  A homography is a non-singular, line preserving, projective mapping h: $P^n \rightarrow P^n$.
  It is represented by a square (n + 1)-dim matrix
  with $(n + 1)^2-1$ DOF

- Now back to the 2D case…
- Mapping between planes



Homographies in Computer vision

Rotating/translating camera, planar world

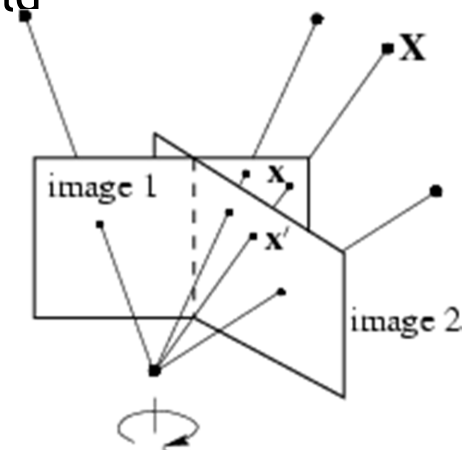


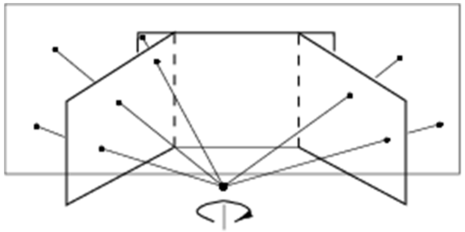
$$(x, y, 1)^T = x \propto PX = \boldsymbol{K}[\boldsymbol{r}_1 \boldsymbol{r}_2 \cancel{\boldsymbol{r}_3} \boldsymbol{t}] \begin{pmatrix} X \\ Y \\ \cancel{0} \\ 1 \end{pmatrix} = H \begin{pmatrix} X \\ Y \\ 1 \end{pmatrix}$$

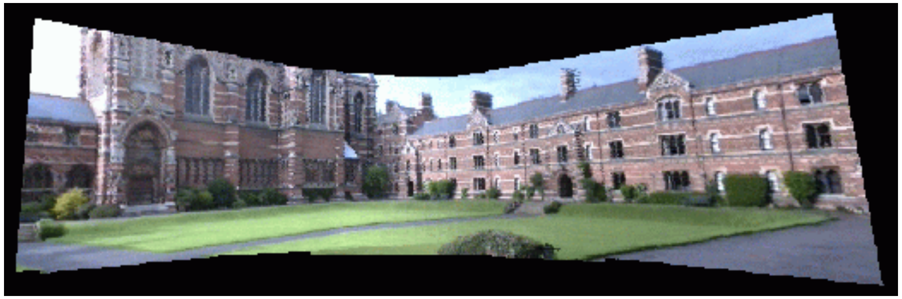
What happens to the P-matrix, if Z is assumed zero?

Homographies in Computer vision

Rotating camera, arbitrary world

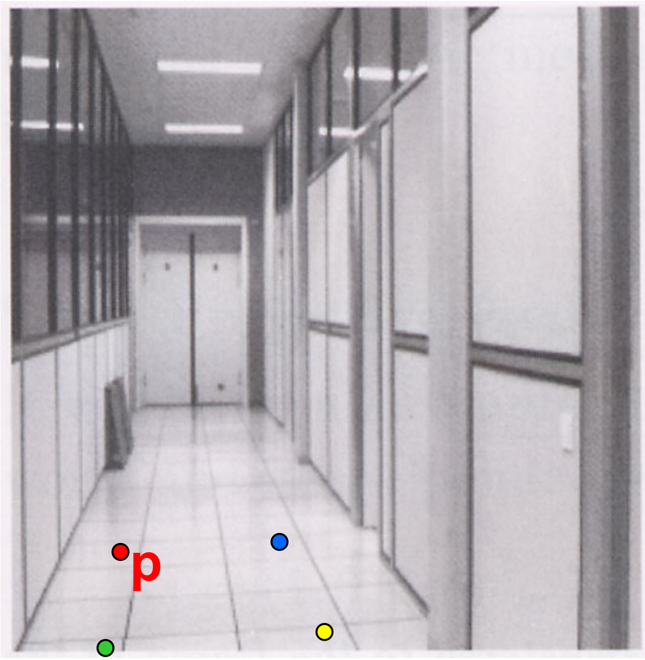


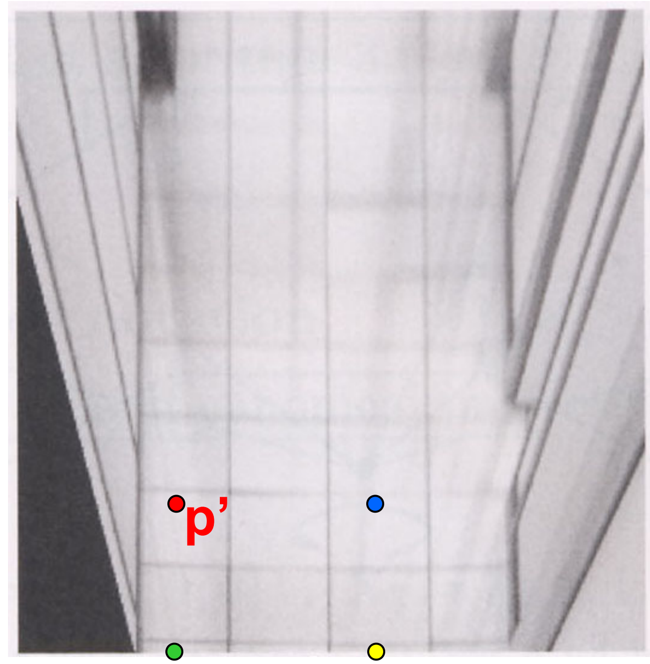




$$(x, y, 1)^T = x \propto PX = K\left(r_1\, r_2\, r_3\, \cancel{t}\right)\begin{pmatrix} X \\ Y \\ Z \\ \cancel{1} \end{pmatrix} \propto KRK^{-1}x' = Hx'$$

What happens to the P-matrix, if t is assumed zero?





To unwarp (rectify) an image

- solve for homography **H** given **p** and **p’**
- solve equations of the form: w**p’** = **Hp**
  - linear in unknowns: w and coefficients of **H**
  - H is defined up to an arbitrary scale factor
  - how many points are necessary to solve for **H**?

Solving for homographies

$$\begin{bmatrix} x_i' \\ y_i' \\ 1 \end{bmatrix} \cong \begin{bmatrix} h_{00} & h_{01} & h_{02} \\ h_{10} & h_{11} & h_{12} \\ h_{20} & h_{21} & h_{22} \end{bmatrix} \begin{bmatrix} x_i \\ y_i \\ 1 \end{bmatrix}$$

$$\begin{aligned} x_i' &= \frac{h_{00}x_i + h_{01}y_i + h_{02}}{h_{20}x_i + h_{21}y_i + h_{22}} \\ y_i' &= \frac{h_{10}x_i + h_{11}y_i + h_{12}}{h_{20}x_i + h_{21}y_i + h_{22}} \end{aligned}$$

$$\begin{aligned} x_i'(h_{20}x_i + h_{21}y_i + h_{22}) &= h_{00}x_i + h_{01}y_i + h_{02} \\ y_i'(h_{20}x_i + h_{21}y_i + h_{22}) &= h_{10}x_i + h_{11}y_i + h_{12} \end{aligned}$$

$$\begin{bmatrix} x_i & y_i & 1 & 0 & 0 & 0 & -x_i'x_i & -x_i'y_i & -x_i' \\ 0 & 0 & 0 & x_i & y_i & 1 & -y_i'x_i & -y_i'y_i & -y_i' \end{bmatrix} \begin{bmatrix} h_{00} \\ h_{01} \\ h_{02} \\ h_{10} \\ h_{11} \\ h_{12} \\ h_{20} \\ h_{21} \\ h_{22} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

Solving for homographies

$$
\underbrace{\begin{bmatrix} x_1 & y_1 & 1 & 0 & 0 & 0 & -x'_1 x_1 & -x'_1 y_1 & -x'_1 \\ 0 & 0 & 0 & x_1 & y_1 & 1 & -y'_1 x_1 & -y'_1 y_1 & -y'_1 \\ & & & & \vdots & & & & \\ x_n & y_n & 1 & 0 & 0 & 0 & -x'_n x_n & -x'_n y_n & -x'_n \\ 0 & 0 & 0 & x_n & y_n & 1 & -y'_n x_n & -y'_n y_n & -y'_n \end{bmatrix}}_{\substack{\mathbf{A} \\ \mathbf{2n \times 9}}}
\underbrace{\begin{bmatrix} h_{00} \\ h_{01} \\ h_{02} \\ h_{10} \\ h_{11} \\ h_{12} \\ h_{20} \\ h_{21} \\ h_{22} \end{bmatrix}}_{\substack{\mathbf{h} \\ \mathbf{9}}}
=
\underbrace{\begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \\ 0 \end{bmatrix}}_{\substack{\mathbf{0} \\ \mathbf{2n}}}
$$

- These equations ate linear in the elements of H
- Four point correspondences lead to eight such linear equations
- These are sufficient to solve for H.
- Condition: No three points should be collinear.

Solving for homographies

$$\begin{bmatrix} x_1 & y_1 & 1 & 0 & 0 & 0 & -x'_1 x_1 & -x'_1 y_1 & -x'_1 \\ 0 & 0 & 0 & x_1 & y_1 & 1 & -y'_1 x_1 & -y'_1 y_1 & -y'_1 \\ & & & & \vdots & & & & \\ x_n & y_n & 1 & 0 & 0 & 0 & -x'_n x_n & -x'_n y_n & -x'_n \\ 0 & 0 & 0 & x_n & y_n & 1 & -y'_n x_n & -y'_n y_n & -y'_n \end{bmatrix} \begin{bmatrix} h_{00} \\ h_{01} \\ h_{02} \\ h_{10} \\ h_{11} \\ h_{12} \\ h_{20} \\ h_{21} \\ h_{22} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \\ 0 \end{bmatrix}$$

A **h** **0**

2n × 9 **9** **2n**

Linear least squares

- Since **h** is only defined up to scale, solve for unit vector **ĥ**
- Minimize $\|\mathbf{A}\hat{\mathbf{h}}\|^2$

$$\|\mathbf{A}\hat{\mathbf{h}}\|^2 = (\mathbf{A}\hat{\mathbf{h}})^T \mathbf{A}\hat{\mathbf{h}} = \hat{\mathbf{h}}^T \mathbf{A}^T \mathbf{A}\hat{\mathbf{h}}$$

- Solution: **ĥ** = eigenvector of $\mathbf{A}^T\mathbf{A}$ with smallest eigenvalue
- Works with 4 or more points

Inhomogeneous solution

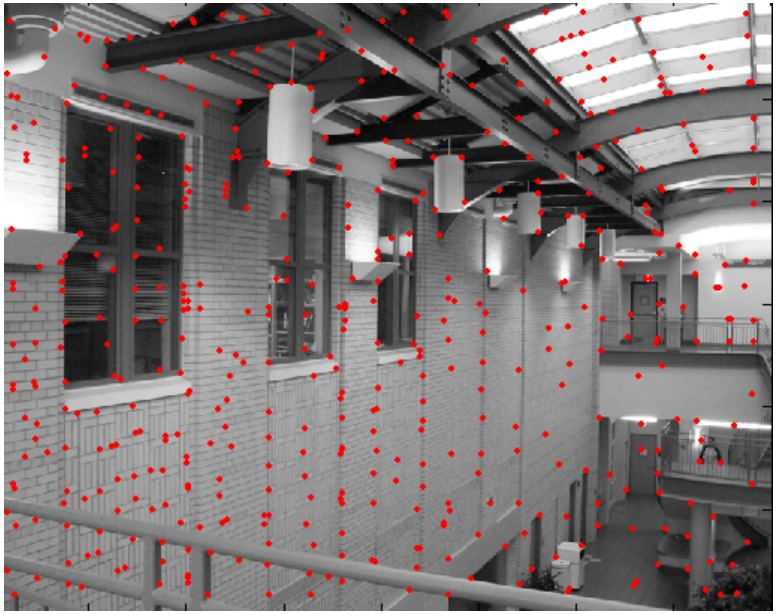
Since h can only be computed up to scale, impose constraint pick h_j=1, e.g. h_9=1, and solve for 8-vector

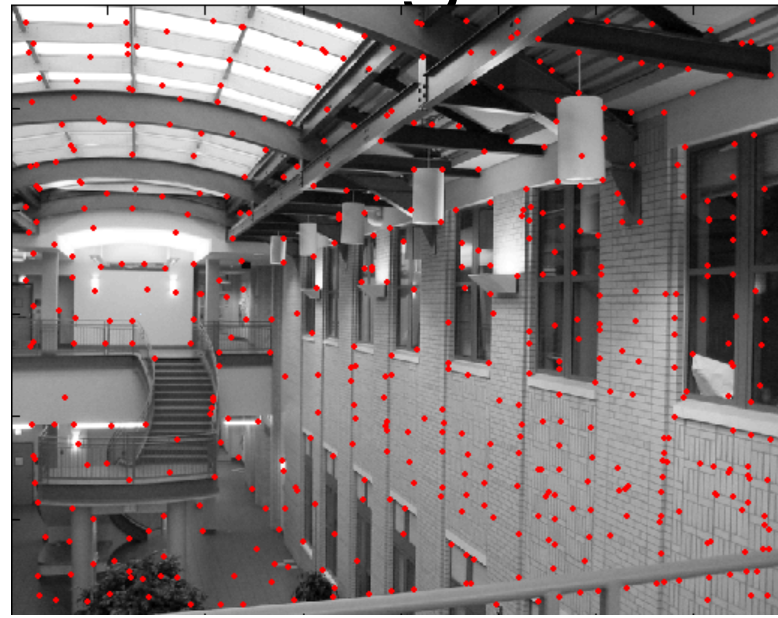
$$\begin{bmatrix} 0 & 0 & 0 & -x_i w_i' & -y_i w_i' & -w_i w_i' & x_i y_i' & y_i y_i' \\ x_i w_i' & y_i w_i' & w_i w_i' & 0 & 0 & 0 & x_i x_i' & y_i x_i' \end{bmatrix} \tilde{h} = \begin{pmatrix} -w_i y_i' \\ w_i x_i' \end{pmatrix}$$

Can be solved using linear least-squares

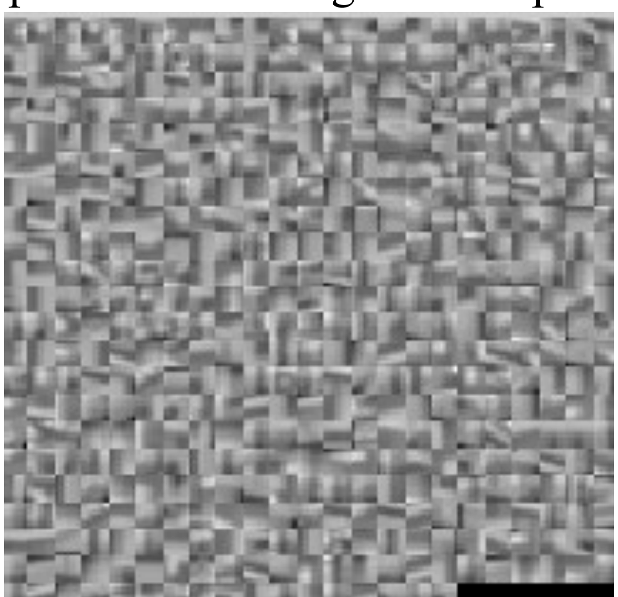
However, if h_9=0 this approach fails Also poor results if h_9 close to zero Therefore, not recommended

Feature matching



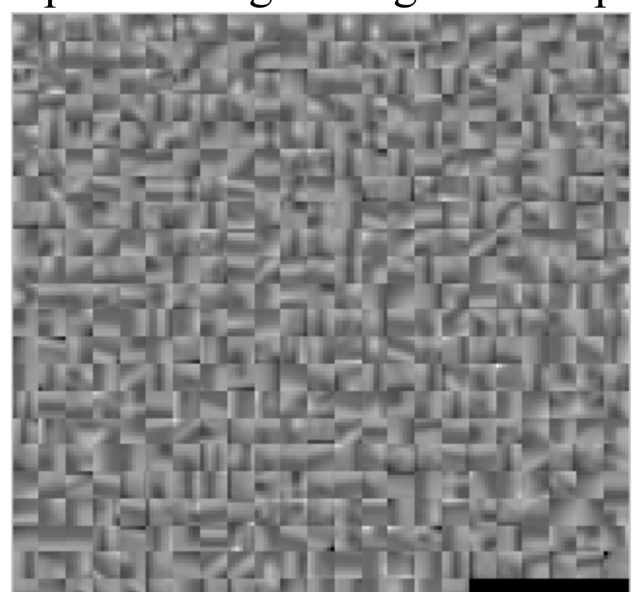


descriptors for left image feature points



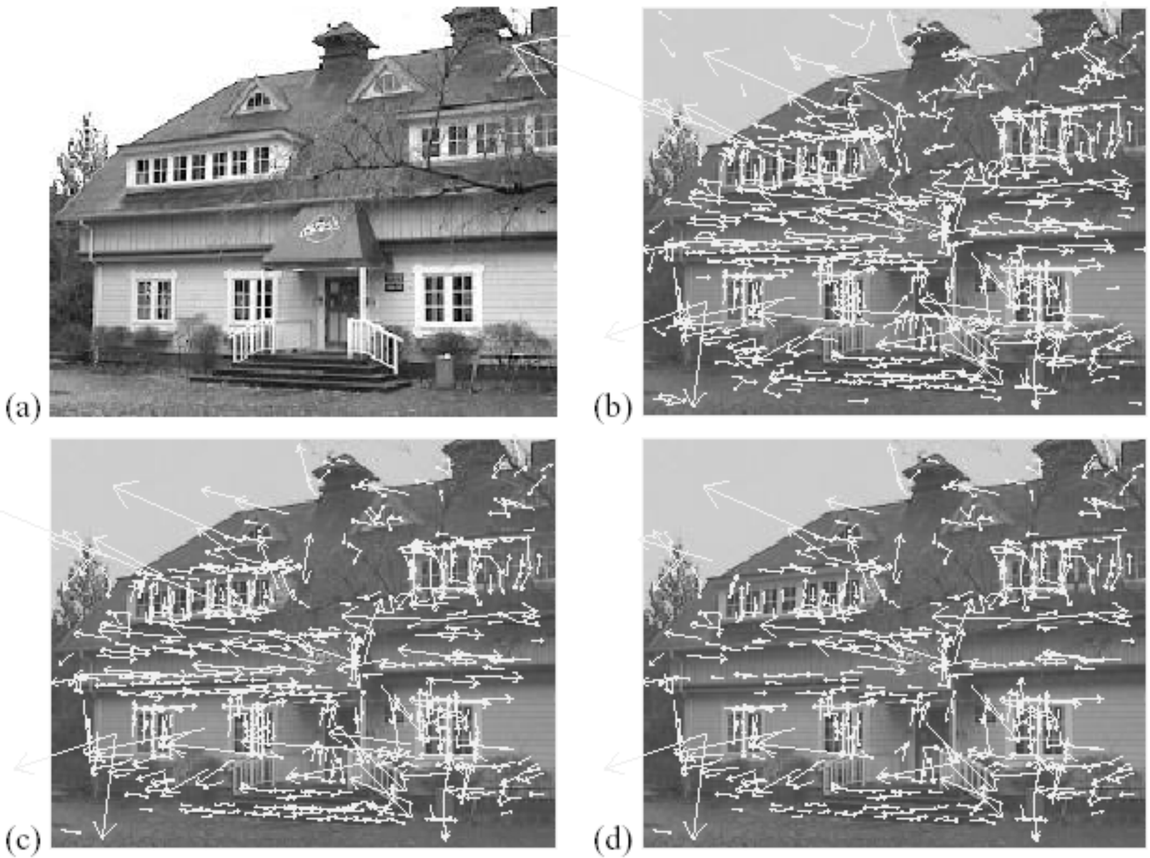
?

descriptors for right image feature points



SIFT features

- Example



(a) 233x189 image
(b) 832 DOG extrema
(c) 729 left after peak value threshold
(d) 536 left after testing ratio of principle curvatures

Strategies to match images robustly

(a)<u>Working with individual features:</u> For each feature point, find most similar point in other image (SIFT distance)

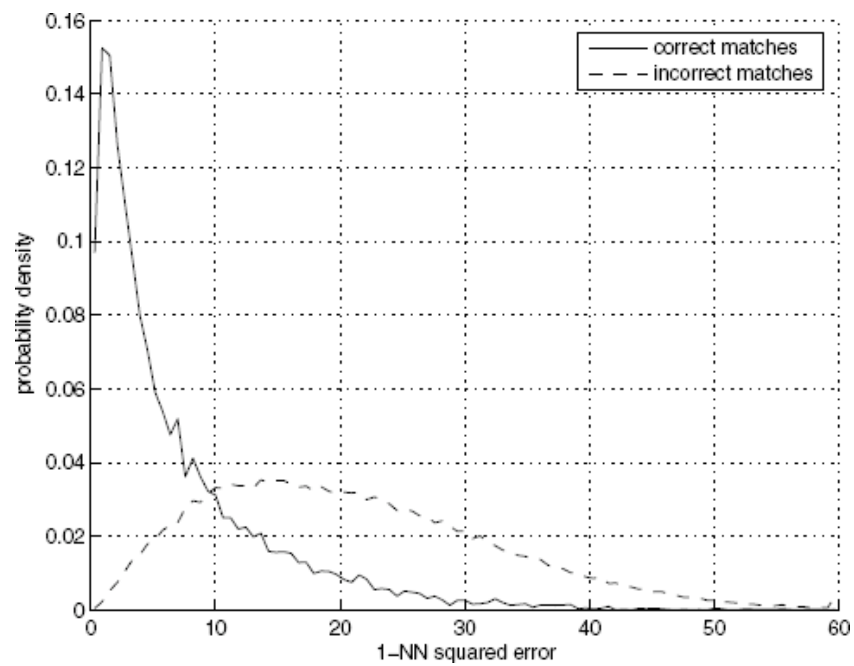
Reject ambiguous matches where there are too many similar points

(b)<u>Working with all the features:</u> Given some good feature matches, look for possible homographies relating the two images

Reject homographies that don't have many feature matches.

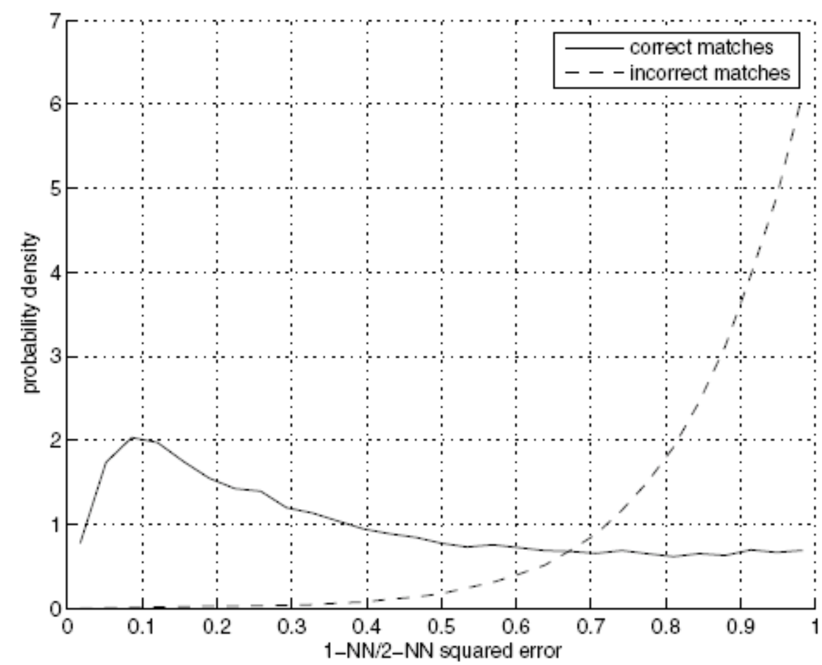
(a) Feature-space outlier rejection

- Let's not match all features, but only these that have "similar enough" matches?
- How can we do it?
  - SSD(patch1,patch2) < threshold
  - How to set threshold? Not so easy.

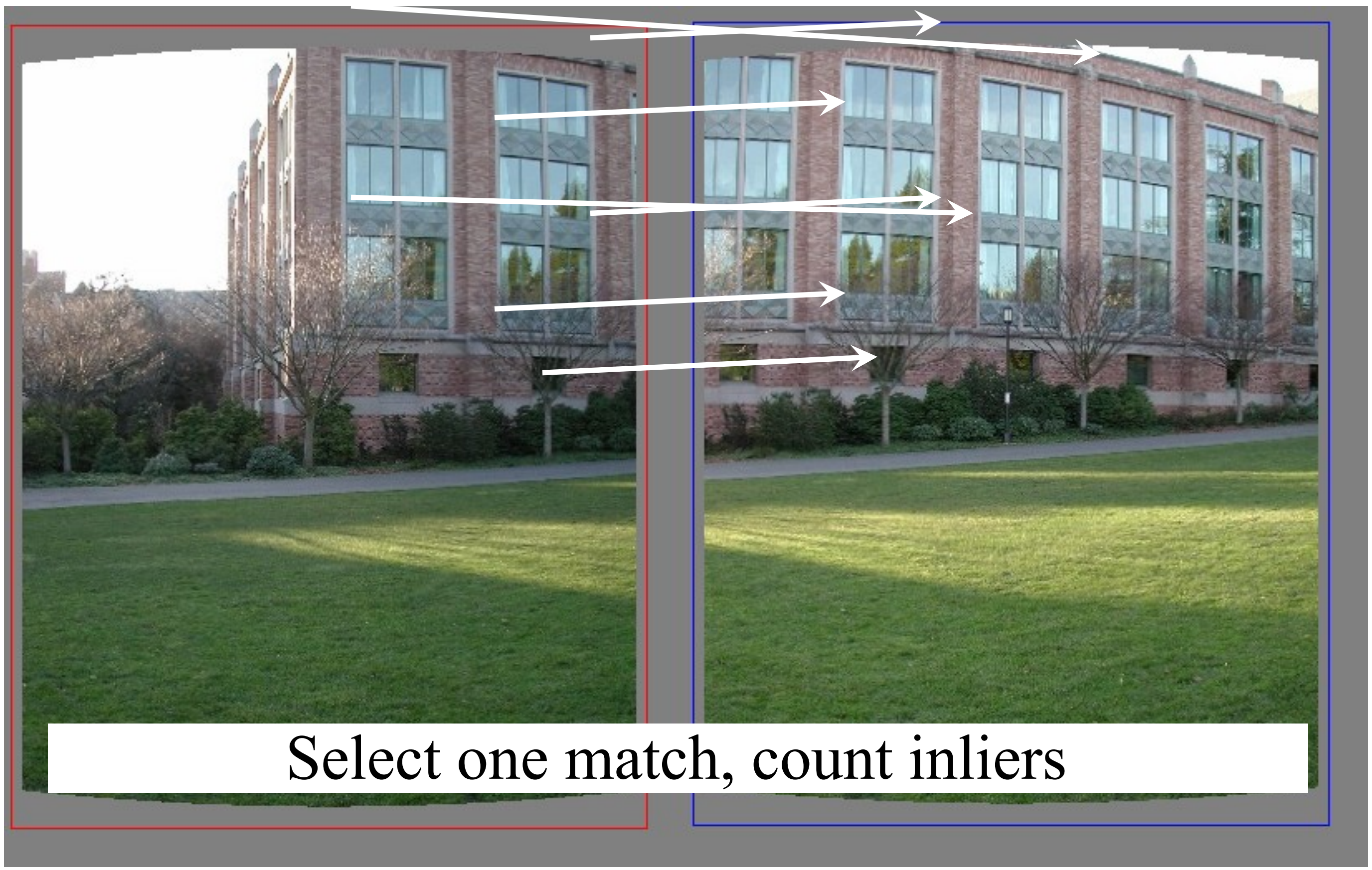


Feature-space outlier rejection

- A better way [Lowe, 1999]:
  - 1-NN: SSD of the closest match
  - 2-NN: SSD of the <u>second-closest</u> match
  - Look at how much better 1-NN is than 2-NN, e.g. 1-NN/2-NN
  - That is, is our best match so much better than the rest?



RAndom SAmple Consensus



RANSAC for estimating homography

RANSAC loop:

Select four feature pairs (at random)

Compute homography H (exact)

Compute inliers where $||p_i', H p_i|| < \varepsilon$

Keep largest set of inliers

Re-compute least-squares H estimate using all of the inliers