

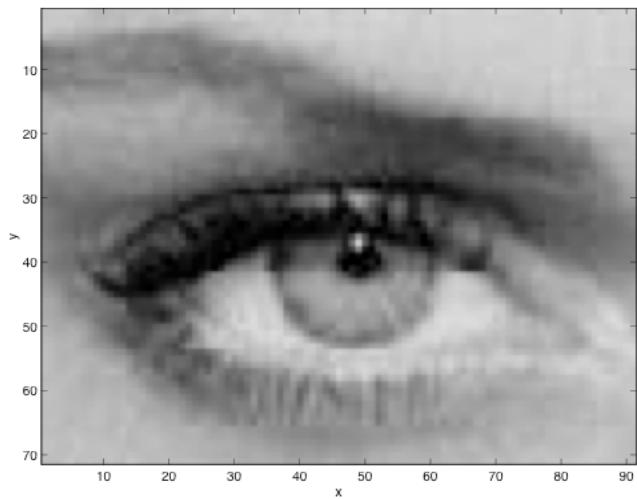
Image Processing

What is an image?

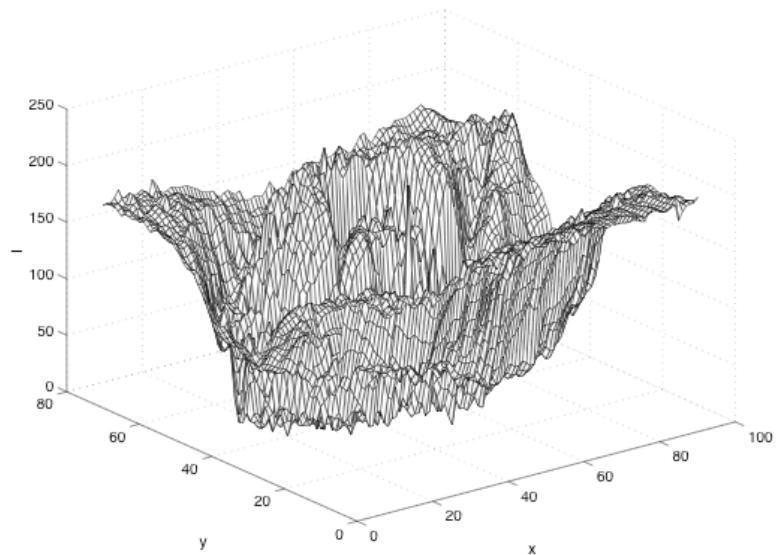
- We can think of an **image** as a function, f , from \mathbb{R}^2 to \mathbb{R} :
 - $f(x, y)$ gives the **intensity** at position (x, y)
 - Realistically, we expect the image only to be defined over a rectangle, with a finite range:
 - $f: [a,b] \times [c,d] \rightarrow [0,1]$
- A color image is just three functions pasted together. We can write this as a “vector-valued” function:

$$f(x, y) = \begin{bmatrix} r(x, y) \\ g(x, y) \\ b(x, y) \end{bmatrix}$$

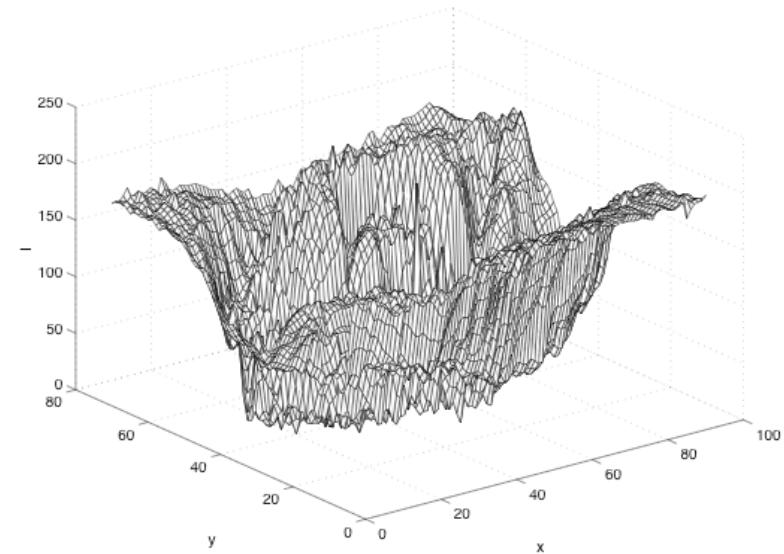
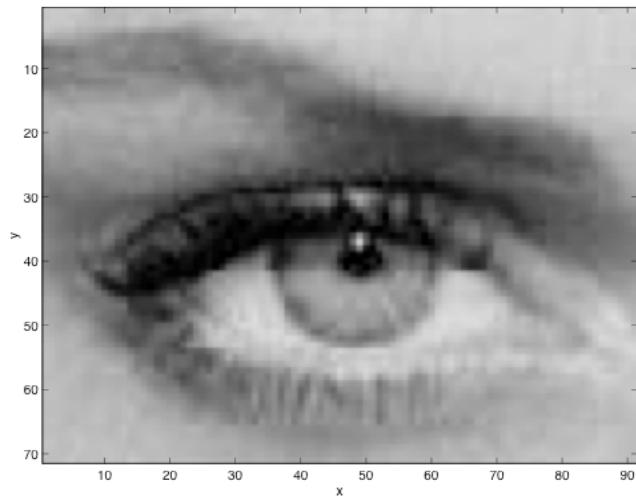
Image



Brightness values



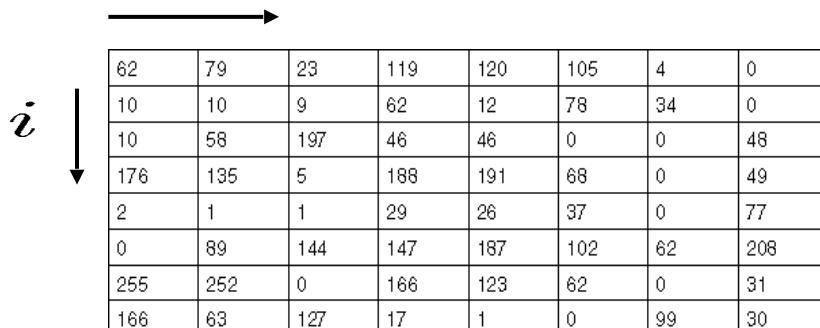
$$I(x,y)$$



62	79	23	119	120	105	4	0
10	10	9	62	12	78	34	0
10	58	197	46	46	0	0	48
176	135	5	188	191	68	0	49
2	1	1	29	26	37	0	77
0	89	144	147	187	102	62	208
255	252	0	166	123	62	0	31
166	63	127	17	1	0	99	30

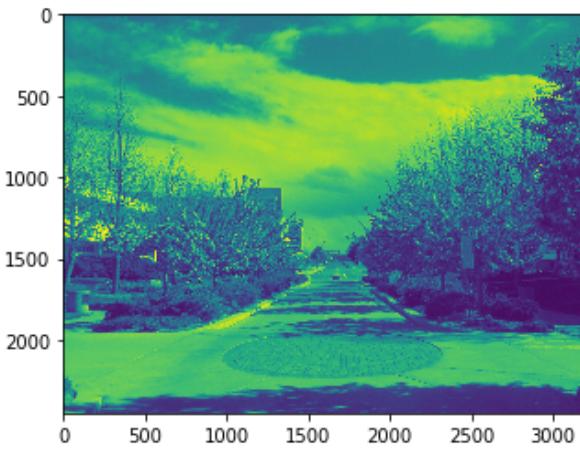
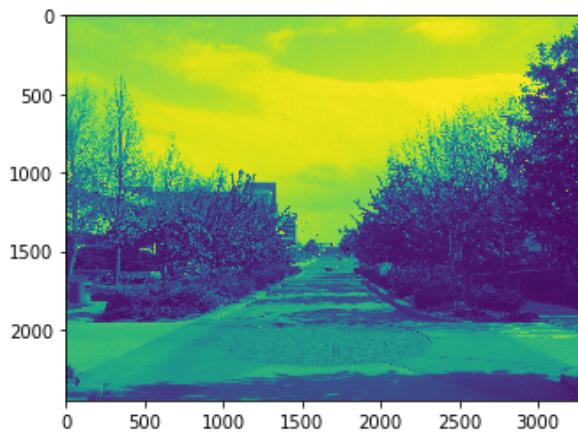
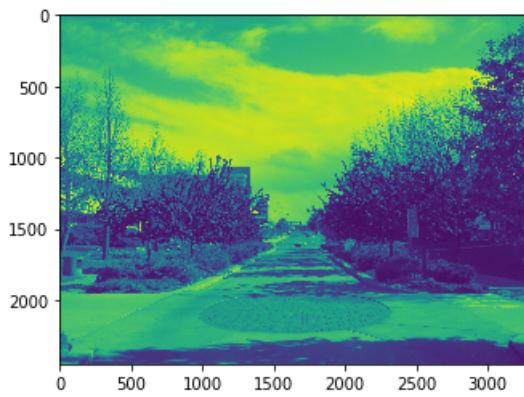
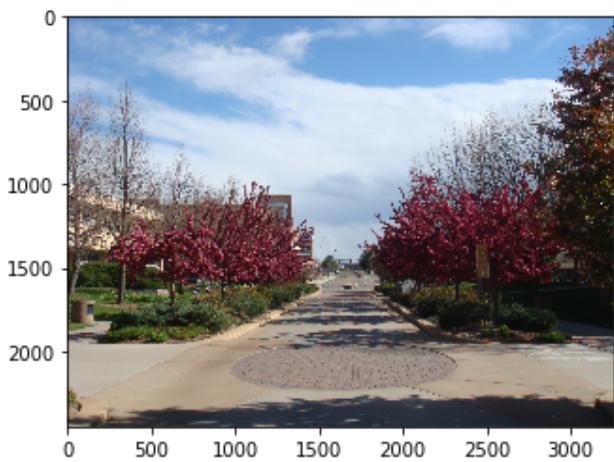
What is a digital image?

- In computer vision we usually operate on **digital (discrete)** images:
 - Sample the 2D space on a regular grid
 - Quantize each sample (round to nearest integer)
- If our samples are Δ apart, we can write this as:
- $f[i, j] = \text{Quantize}\{ f(i\Delta, j\Delta) \}$
- The image can now be represented as a matrix of integer values



62	79	23	119	120	105	4	0
10	10	9	62	12	78	34	0
10	58	197	46	46	0	0	48
176	135	5	188	191	68	0	49
2	1	1	29	26	37	0	77
0	89	144	147	187	102	62	208
255	252	0	166	123	62	0	31
166	63	127	17	1	0	99	30

2448,3264,3



im[1:10,1:10,0]

```
array([[88, 89, 90, 90, 91, 91, 92, 92, 91],  
       [89, 90, 91, 91, 91, 92, 92, 92, 92],  
       [90, 92, 92, 92, 91, 92, 92, 93, 93],  
       [91, 93, 93, 92, 91, 92, 92, 93, 93],  
       [91, 92, 92, 92, 92, 92, 93, 93, 94],  
       [91, 91, 92, 92, 93, 93, 94, 95, 94],  
       [92, 92, 92, 93, 94, 94, 96, 96, 96],  
       [93, 93, 93, 93, 92, 92, 91, 94, 94],  
       [94, 94, 94, 94, 93, 92, 92, 94, 94]], dtype=uint8)
```

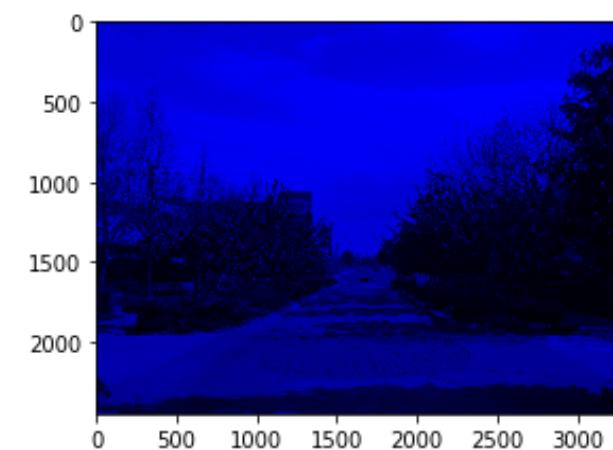
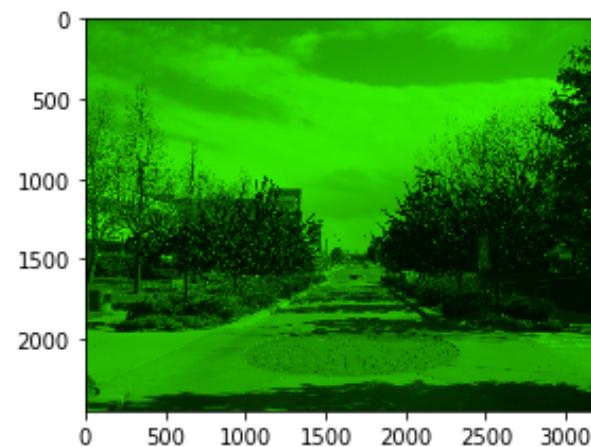
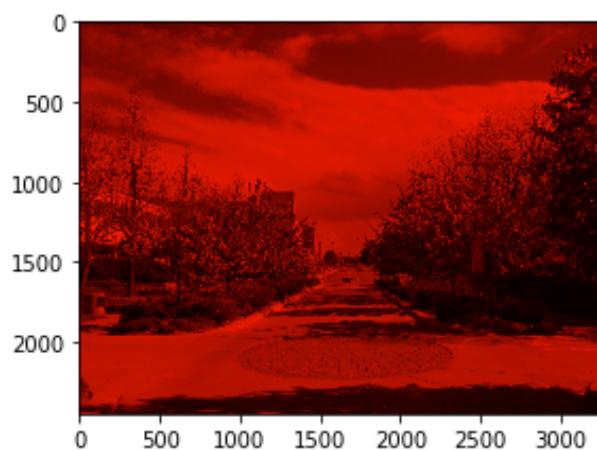
im[1:10,1:10,1]

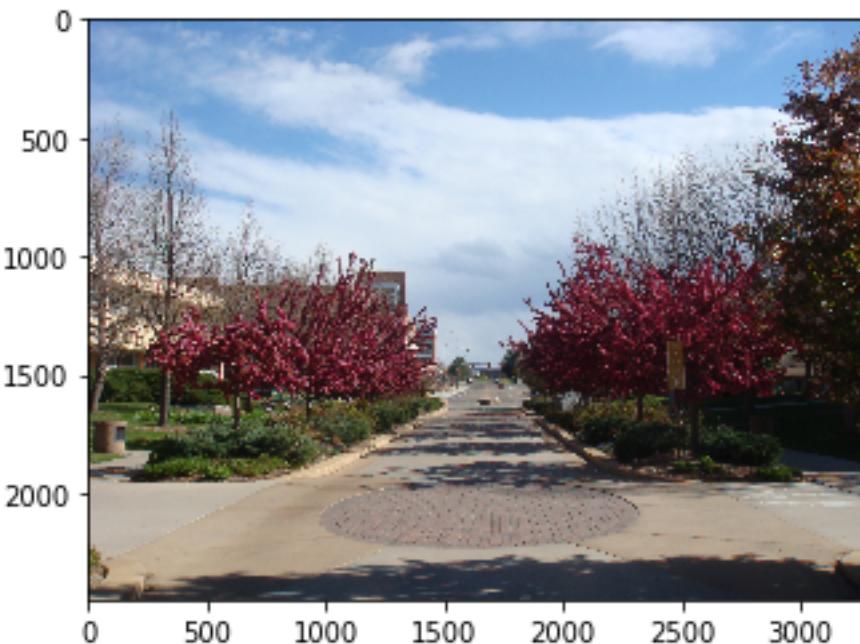
```
array([[134, 135, 136, 136, 137, 137, 138, 138, 137],  
       [135, 136, 137, 137, 138, 138, 138, 138, 138],  
       [136, 138, 138, 138, 137, 138, 138, 139, 139],  
       [137, 139, 139, 138, 137, 138, 138, 139, 139],  
       [137, 138, 138, 138, 138, 139, 139, 139, 139],  
       [137, 137, 138, 138, 139, 140, 140, 140, 139],  
       [138, 138, 138, 139, 140, 140, 141, 141, 141],  
       [137, 137, 137, 137, 138, 138, 137, 140, 140],  
       [138, 138, 138, 138, 139, 138, 138, 140, 140]], dtype=uint8)
```

```
array([[194, 195, 196, 196, 197, 197, 198, 198, 196],  
       [195, 196, 197, 197, 197, 197, 197, 197, 197],  
       [195, 197, 197, 197, 196, 197, 197, 198, 198],  
       [196, 198, 198, 197, 196, 197, 197, 198, 198],  
       [196, 197, 197, 197, 197, 197, 198, 198, 198],  
       [196, 196, 197, 197, 198, 198, 199, 199, 198],  
       [197, 197, 197, 198, 199, 199, 200, 200, 200],  
       [198, 198, 198, 198, 197, 197, 196, 199, 199],  
       [199, 199, 199, 199, 198, 197, 197, 199, 199]], dtype=uint8)
```

im[1:10,1:10,2]

```
array([[194, 195, 196, 196, 197, 197, 198, 198, 196],  
       [195, 196, 197, 197, 197, 197, 197, 197, 197],  
       [195, 197, 197, 197, 196, 197, 197, 198, 198],  
       [196, 198, 198, 197, 196, 197, 197, 198, 198],  
       [196, 197, 197, 197, 197, 197, 198, 198, 198],  
       [196, 196, 197, 197, 198, 198, 199, 199, 198],  
       [197, 197, 197, 198, 199, 199, 200, 200, 200],  
       [198, 198, 198, 198, 197, 197, 196, 199, 199],  
       [199, 199, 199, 199, 198, 197, 197, 199, 199]], dtype=uint8)
```





```
array([[ 127.086,   128.086,   129.086,   129.086,   130.086,   130.086,
       131.086,   131.086,   129.972],
       [ 128.086,   129.086,   130.086,   130.086,   130.086,   130.972,
       130.972,   130.972,   130.972],
       [ 128.972,   130.972,   130.972,   130.972,   129.972,   130.972,
       130.972,   131.972,   131.972],
       [ 129.972,   131.972,   131.972,   130.972,   129.972,   130.972,
       130.972,   131.972,   131.972],
       [ 129.972,   130.972,   130.972,   130.972,   130.972,   130.972,
       131.972,   131.972,   132.271],
       [ 129.972,   129.972,   130.972,   130.972,   131.972,   131.972,
       132.972,   133.271,   132.271],
       [ 130.972,   130.972,   130.972,   131.972,   132.972,   132.972,
       134.271,   134.271,   134.271],
       [ 130.798,   130.798,   130.798,   130.798,   130.972,   130.972,
       129.972,   132.972,   132.972],
       [ 131.798,   131.798,   131.798,   131.798,   131.972,   130.972,
       130.972,   132.972,   132.972]])
```

RGB to Grayscale

- The relationship between grayscale reflectance of a surface and its RGB color equivalent is given by:

$$Y = 0.299 * R + 0.587 * G + 0.114 * B$$

Image Processing

- An **image processing** operation typically defines a new image g in terms of an existing image f .

$$g(x, y) = t(f(x, y))$$

Image Processing at pixel level - Thresholding

One of the simplest operations we can perform on an image is *thresholding*.

For example, take the swan image and threshold it with a value of T .

Make all pixels $\geq T$ into 1, and all pixels $< T$ into 0.



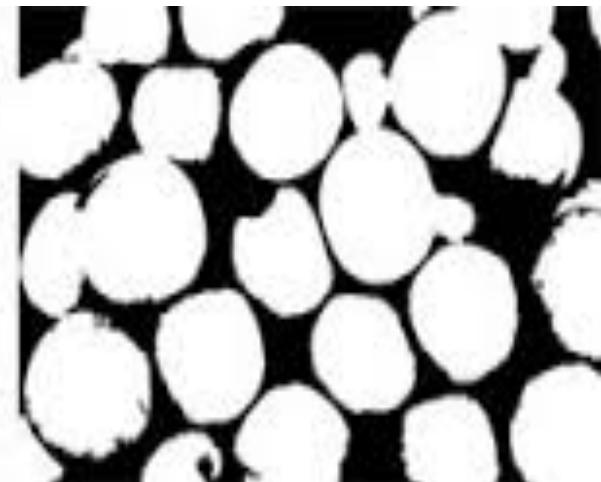
Threshold $T=128$



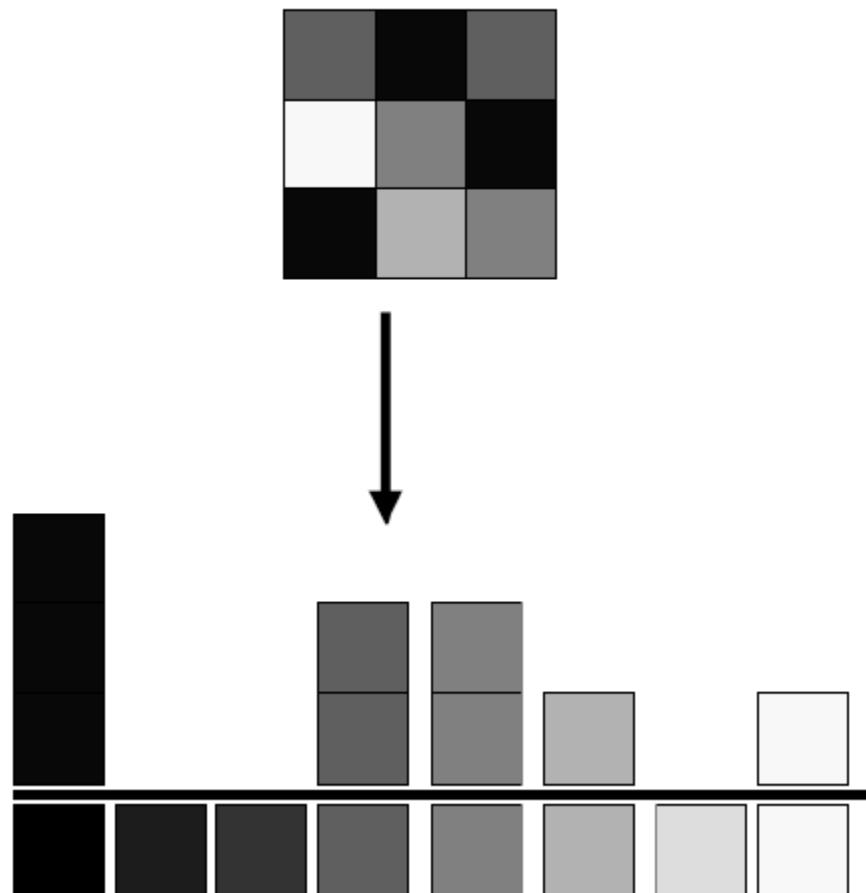
$$im[im > 128] = 255$$

$$im[im \leq 128] = 0$$

Examples



A simple image and its histogram

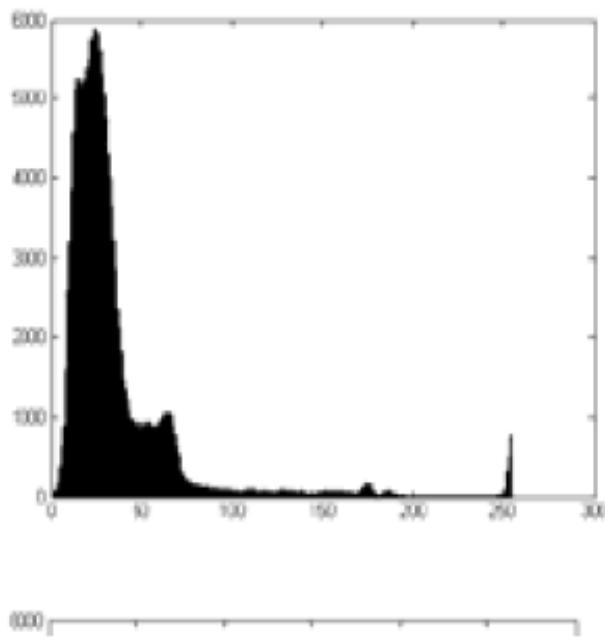


Definition of histogram

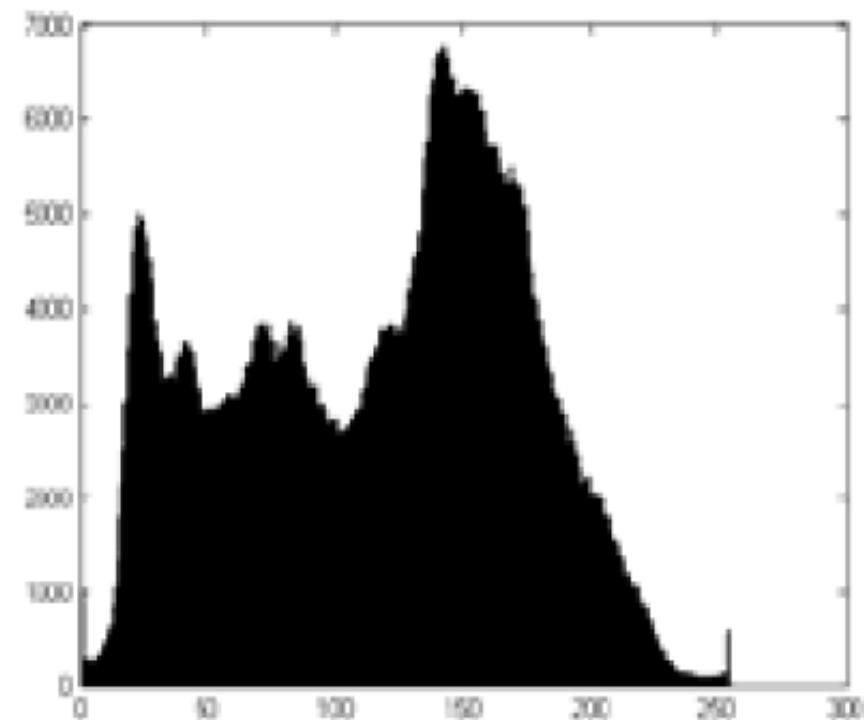
- To write this down, we might say that we have an image, I , in which the intensity at pixel with coordinates (x,y) is $I(x,y)$.
- We would write the histogram h , as $h(i)$ indicating that intensity i , appears $h(i)$ times in the image.
- If we let the expression $(a=b)$ have the value 1 when $a=b$, and 0 otherwise, we can write for histogram $h(i)$:

$$h(i) = \sum_x \sum_y I(x,y)$$

Histogram example



Histogram example



NUMBER OF BINS AND SIZES

Pick a discrete set of bins, then put values into the bins

Equal-length bins:

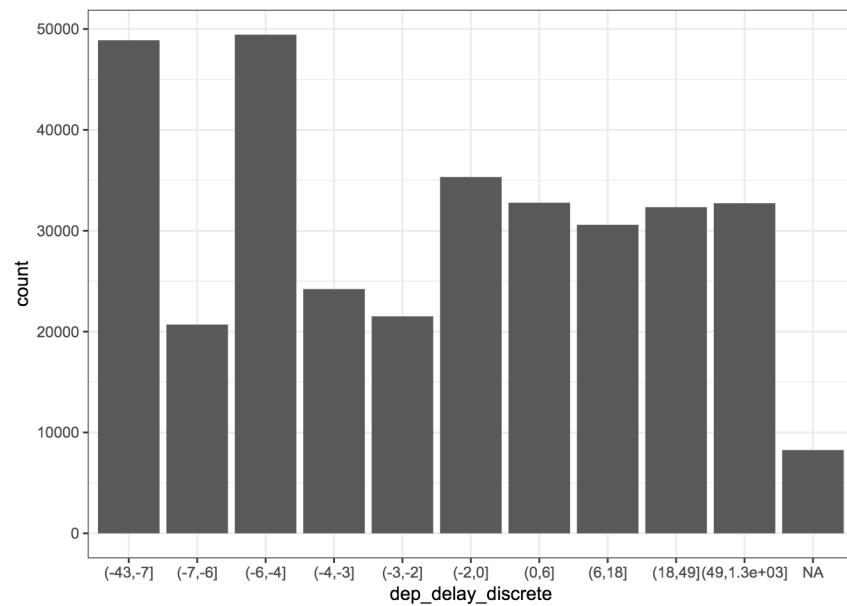
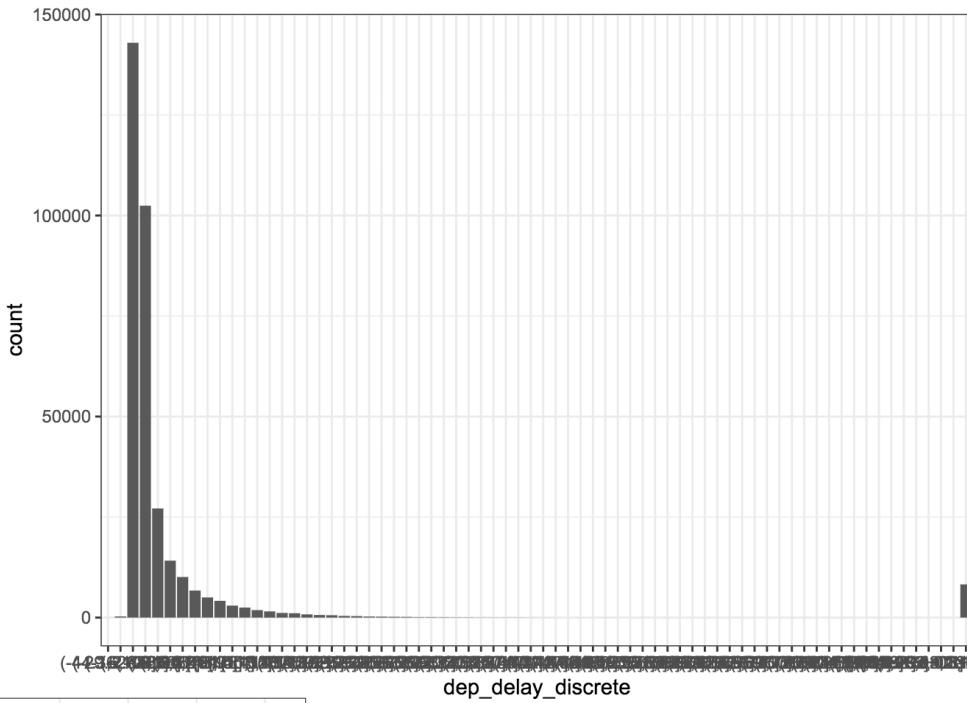
- Bins have an equal-length range and skewed membership
- Good/Bad ????????

Equal-sized bins:

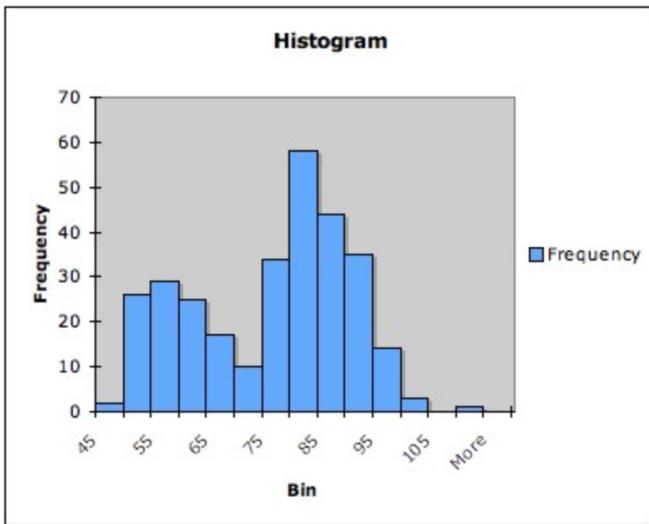
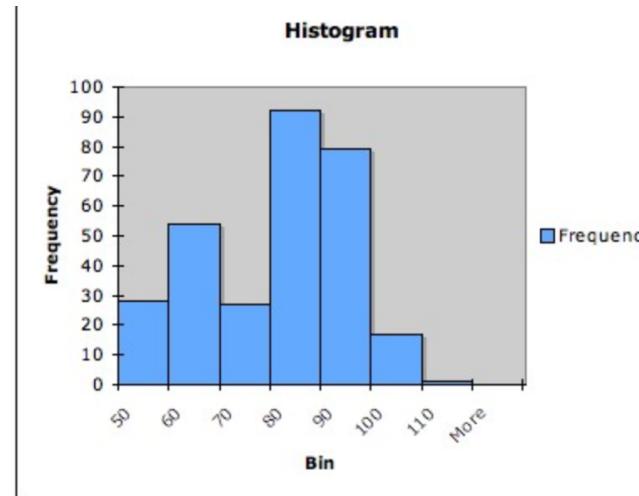
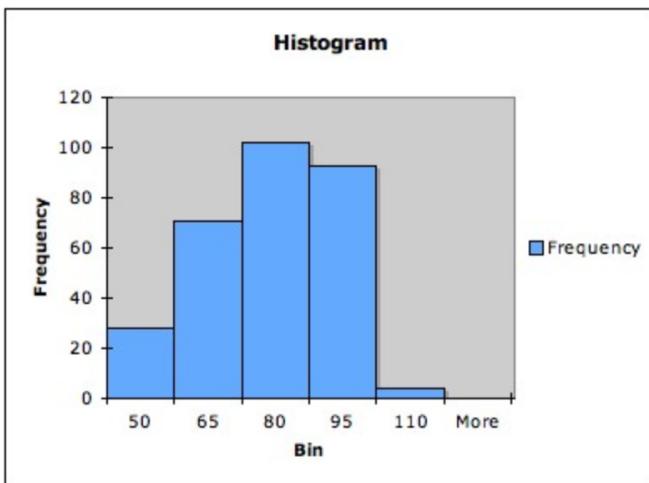
- Bins have variable-length ranges but equal membership
- Good/Bad ????????



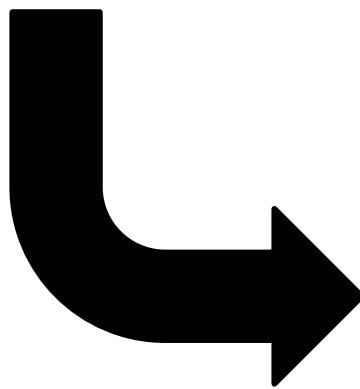
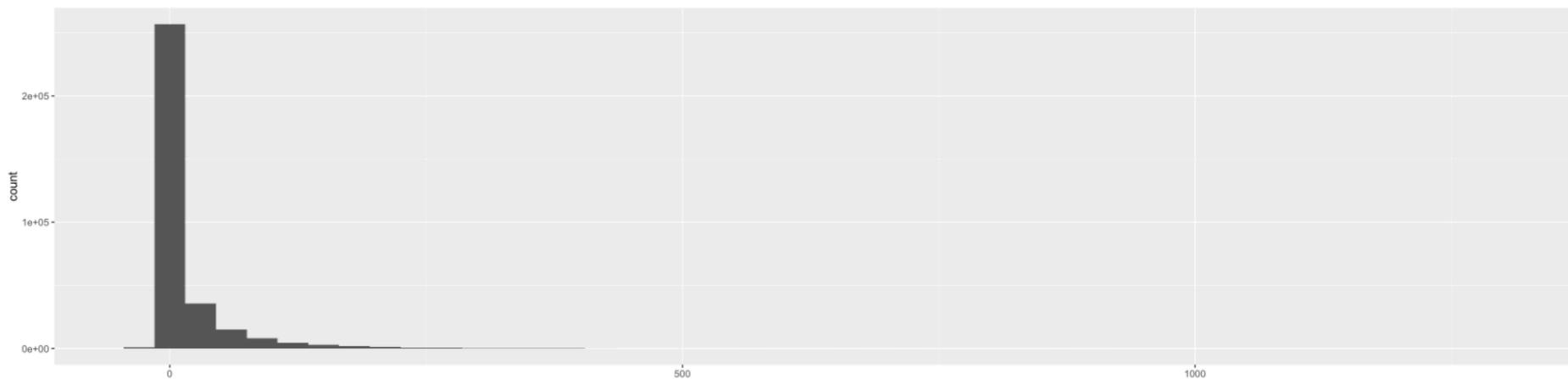
BIN SIZE



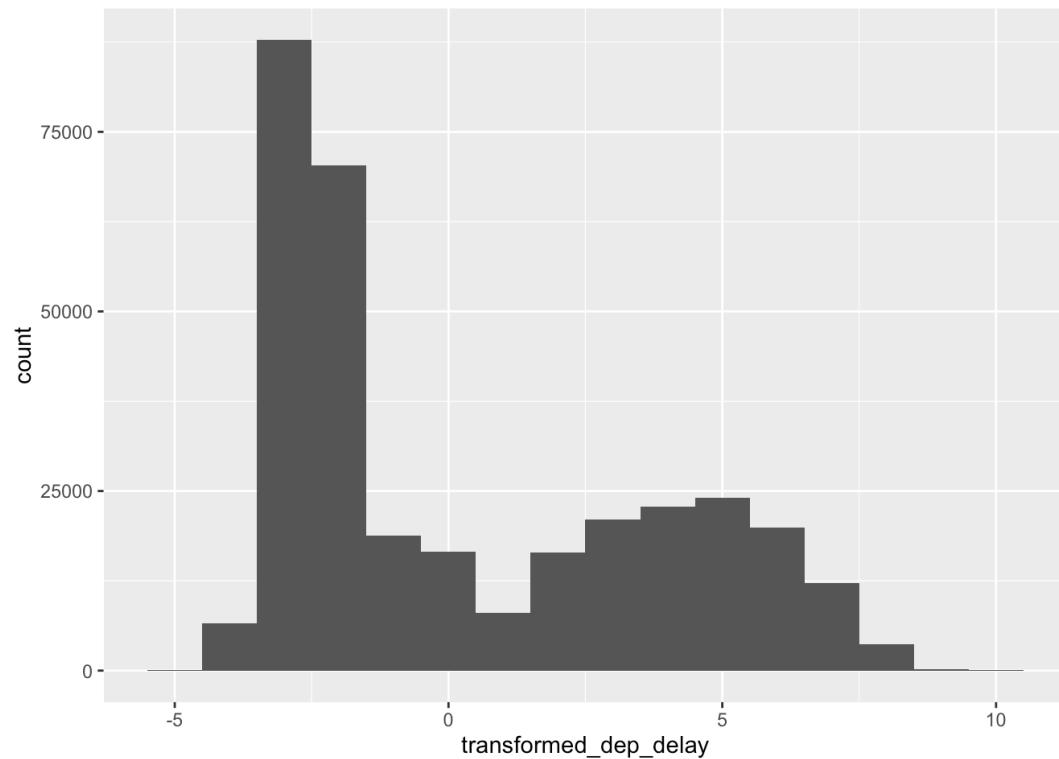
DIFFERENT NUMBER OF BINS



SKEWED DATA



\log_2 transform



HISTOGRAM BINS AND WIDTHS

Square formula

$$bins = \sqrt{n}$$

$$binwidth = \frac{\max(values) - \min(values)}{\sqrt{n}}$$

Sturges formula

$$bins = \lceil \log_2 n \rceil + 1$$

$$binwidth = \frac{\max(values) - \min(values)}{\lceil \log_2 n \rceil + 1}$$

Rice formula

$$bins = 2 \times n^{1/3}$$

$$binwidth = \frac{\max(values) - \min(values)}{bins}$$

Scott formula

$$bins = \frac{\max(values) - \min(values)}{3.5 \times \frac{\text{stdev}(values)}{n^{1/3}}}$$

$$binwidth = 3.5 \times \frac{\text{stdev}(values)}{n^{1/3}}$$

Freedman-Diaconis formula

$$bins = \frac{\max(values) - \min(values)}{2 \times \frac{IQR(values)}{n^{1/3}}}$$

$$binwidth = 2 \times \frac{IQR(values)}{n^{1/3}}$$

Histograms allow image manipulation

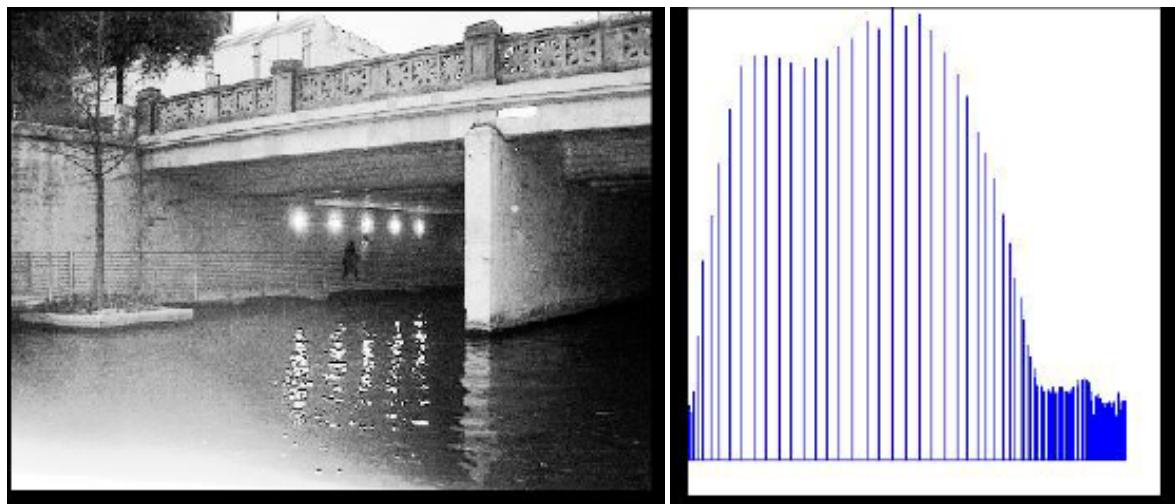
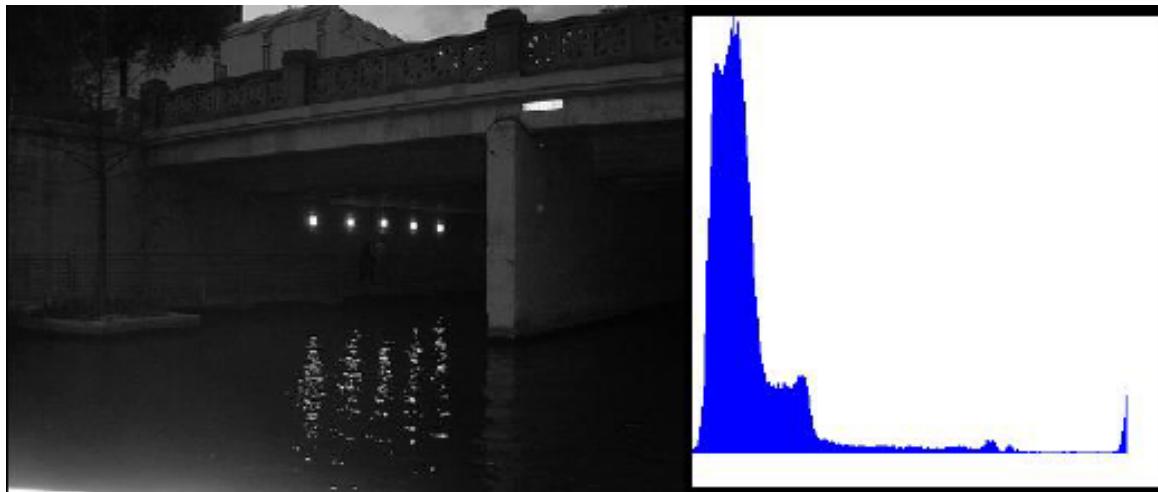
- One reason to compute a histogram is that it allows us to manipulate an image by changing its histogram. We do this by creating a new image, J , in which:
- The trick is to choose an f that will generate a nice or useful image. Typically, we choose f to be monotonic. This means that: if $u < v$ then $f(u) < f(v)$. Non-monotonic functions tend to make an image look truly different, while monotonic changes will be more subtle.

$$J(x, y) = f(I(x, y))$$

Histogram Equalization

- The idea is to spread out the histogram so that it makes full use of the dynamic range of the image.
- For example, if an image is very dark, most of the intensities might lie in the range 0-50. By choosing f to spread out the intensity values, we can make fuller use of the available intensities, and make darker parts of an image easier to understand.
- If we choose f to make the histogram of the new image, J , as uniform as possible, we call this **histogram equalization**.

Example of histogram equalization



8 x 8 Image

52	55	61	59	70	61	76	61
62	59	55	104	94	85	59	71
63	65	66	113	144	104	63	72
64	70	70	126	154	109	71	69
67	73	68	106	122	88	68	68
68	79	60	79	77	66	58	75
69	85	64	58	55	61	65	83
70	87	69	68	65	73	78	90

8 x 8 Image

```
[52 55 61 59 70 61 76 61  
62 59 55 104 94 85 59 71  
63 65 66 113 144 104 63 72  
64 70 70 126 154 109 71 69  
67 73 68 106 122 88 68 68  
68 79 60  
69 85 64  
70 87 69]
```

Value	Count								
52	1	64	2	72	1	85	2	113	1
55	3	65	3	73	2	87	1	122	1
58	2	66	2	75	1	88	1	126	1
59	3	67	1	76	1	90	1	144	1
60	1	68	5	77	1	94	1	154	1
61	4	69	3	78	1	104	2		
62	1	70	4	79	2	106	1		
63	2	71	2	83	1	109	1		

8 x 8 Image

Value	Count	Value	Count	Value	Count
52	1	64	2	72	1
55	3	65	3	73	2
58	2	66	2	75	1
59	3	67	1	76	1
60	1	68	5	77	1
61	4	69	3	78	1
62	1	70	4	79	2
63	2	71	2	83	1

v, Pixel Intensity	cdf(v)	h(v), Equalized v
52	1	0
55	4	12
58	6	20
59	9	32
60	10	36
61	14	53
62	15	57
63	17	65
64	19	73
65	22	85
66	24	93

$$h(v) = \text{round} \left(\frac{\text{cdf}(v) - \text{cdf}_{\min}}{(M \times N) - \text{cdf}_{\min}} \times (L - 1) \right)$$

M – width

M – height

L – Number of gray levels

Comparing histograms

- **SSD** - let $h(i)$ and $g(i)$ be two histograms

$$||h - g|| = \sum_{i=1}^N (h(i) - g(i))^2$$

- **Cosine Distance**

$$\cos(h, g) = \frac{h \cdot g}{||h|| ||g||}$$

- **Chi-square distance** - let $h(i)$ and $g(i)$ be two histograms

$$\chi^2(h(i), g(i)) = \frac{1}{2} \sum_{m=1}^k \frac{(h_i(m) - h_j(m))^2}{h_i(m) + h_j(m)}$$

Continuing operations at a pixel level

Cross - Correlation & Convolution

Correlation & Convolution

- Basic operation to extract information from an image.
- These operations have two key features:
 - shift invariant
 - linear
- Applicable to 1-D and multi dimensional images.

Correlation Example - 1D (Averaging)

Image I

2	3	6	5	5	1	8	9	7
---	---	---	---	---	---	---	---	---

$$G = f(I)$$

$$I[2] = 3$$

$$G[2] = \frac{2 + 3 + 6}{3} = \frac{11}{3}$$

2	$\frac{11}{3}$	6	5	5	1	8	9	7
---	----------------	---	---	---	---	---	---	---

$$I[3] = 6$$

$$G[3] = \frac{3 + 6 + 5}{3} = \frac{14}{3}$$

2	$\frac{11}{3}$	$\frac{14}{3}$	5	5	1	8	9	7
---	----------------	----------------	---	---	---	---	---	---

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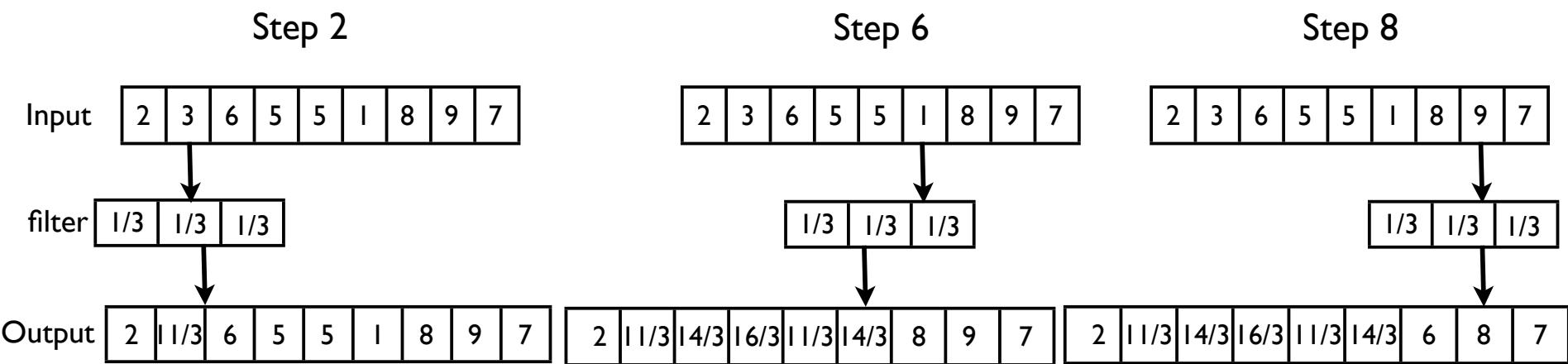
•
•
•

$$I[8] = 9$$

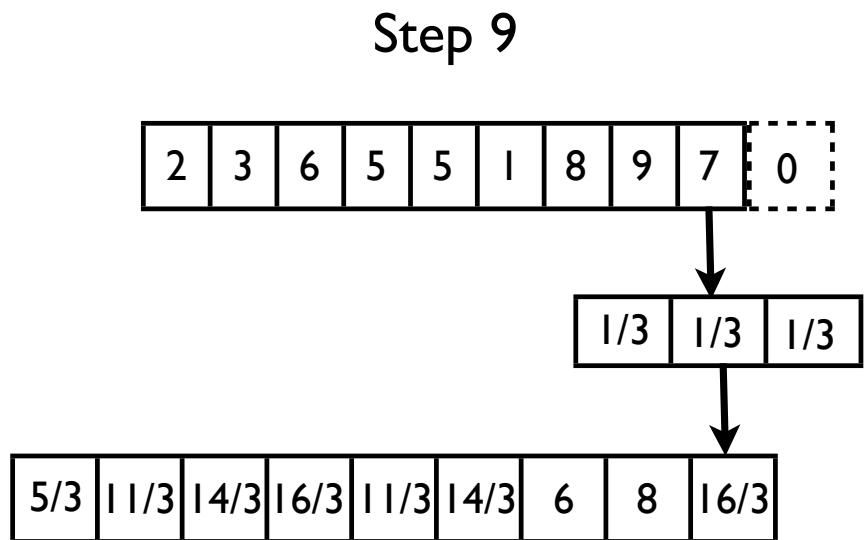
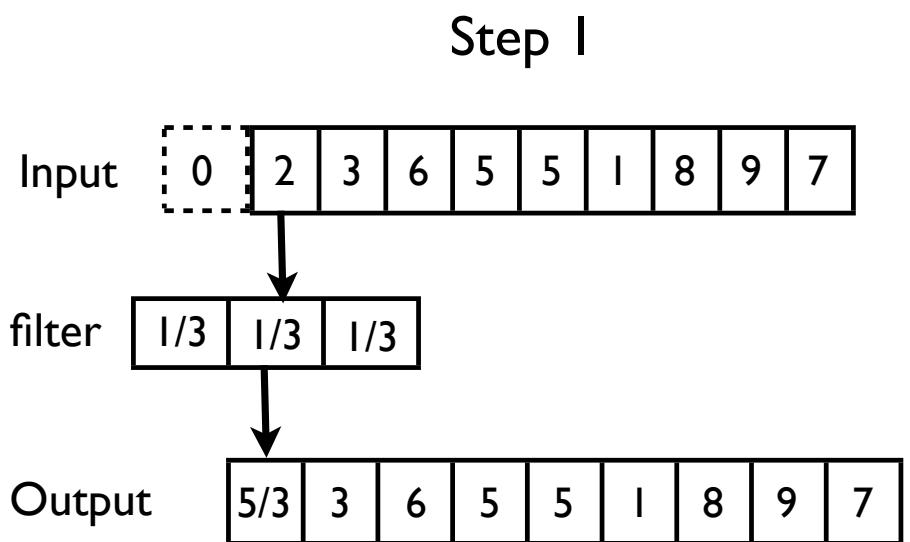
$$G[8] = \frac{8 + 9 + 7}{3} = 8$$

2	$\frac{11}{3}$	$\frac{14}{3}$	$\frac{16}{3}$	$\frac{11}{3}$	$\frac{14}{3}$	6	8	7
---	----------------	----------------	----------------	----------------	----------------	---	---	---

Correlation Example - 1D

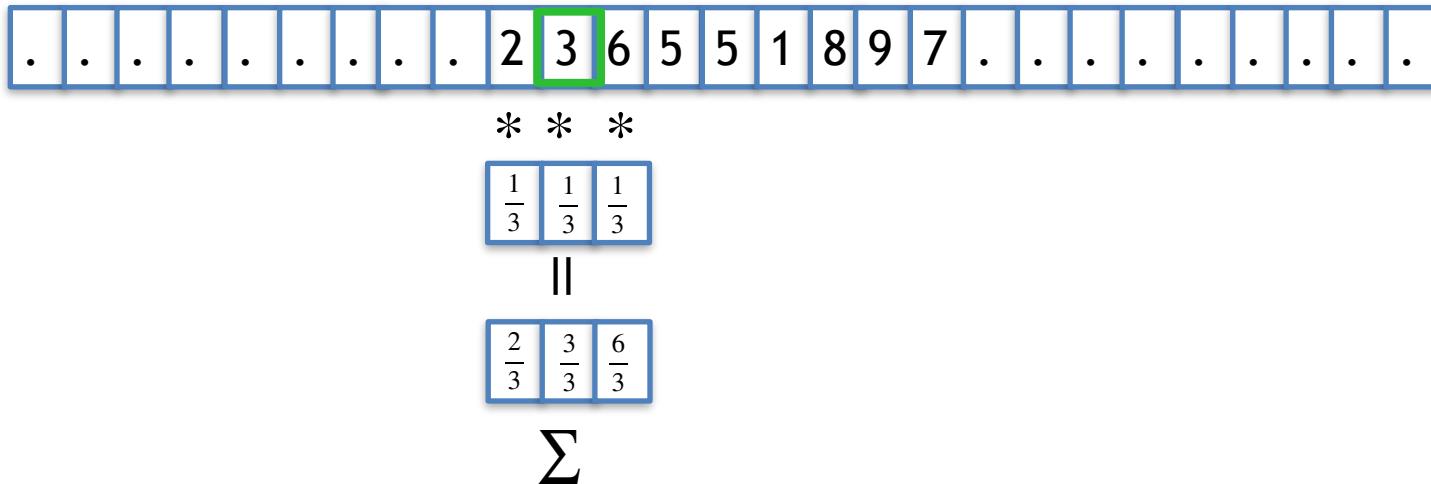


Correlation Example - 1D



Correlation Example - 1D

1



G



Correlation Example - 1D

1



• • •

$$\begin{array}{|c|c|c|} \hline & \frac{1}{3} & \frac{1}{3} & \frac{1}{3} \\ \hline \end{array}$$

11

$$\begin{array}{|c|c|c|} \hline & \frac{3}{3} & \frac{6}{3} & \frac{5}{3} \\ \hline \end{array}$$

Σ

G



Correlation Example - 1D

1



* * *

$$\begin{array}{|c|c|c|} \hline & \frac{1}{3} & \frac{1}{3} \\ \hline & \frac{1}{3} & \end{array}$$

1

$$\begin{array}{|c|c|c|} \hline & \frac{6}{3} & \frac{5}{3} & \frac{5}{3} \\ \hline \end{array}$$

Σ

G



Correlation Example - 1D

I



* * *

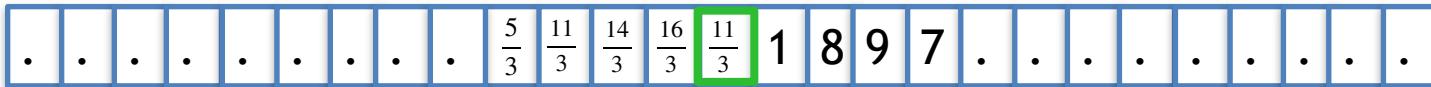
$$\begin{bmatrix} \frac{1}{3} & \frac{1}{3} & \frac{1}{3} \end{bmatrix}$$

||

$$\begin{bmatrix} \frac{5}{3} & \frac{5}{3} & \frac{1}{3} \end{bmatrix}$$

Σ

G



Correlation Example - 1D

I



* * *

$$\begin{array}{|c|c|c|} \hline \frac{1}{3} & \frac{1}{3} & \frac{1}{3} \\ \hline \end{array}$$

||

$$\begin{array}{|c|c|c|} \hline \frac{5}{3} & \frac{1}{3} & \frac{8}{3} \\ \hline \end{array}$$

\sum

G



Correlation Example - 1D

I



* * *

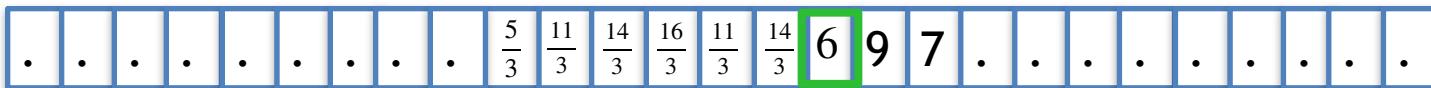
$$\begin{bmatrix} \frac{1}{3} & \frac{1}{3} & \frac{1}{3} \end{bmatrix}$$

||

$$\begin{bmatrix} \frac{1}{3} & \frac{8}{3} & \frac{9}{3} \end{bmatrix}$$

Σ

G



Correlation Example - 1D

I



* * *

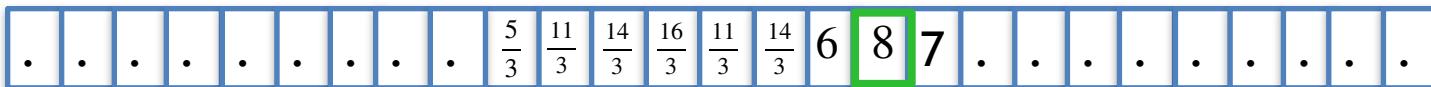
$$\begin{array}{|c|c|c|} \hline \frac{1}{3} & \frac{1}{3} & \frac{1}{3} \\ \hline \end{array}$$

||

$$\begin{array}{|c|c|c|} \hline \frac{8}{3} & \frac{9}{3} & \frac{7}{3} \\ \hline \end{array}$$

Σ

G



Correlation Example - 1D

1



* * *

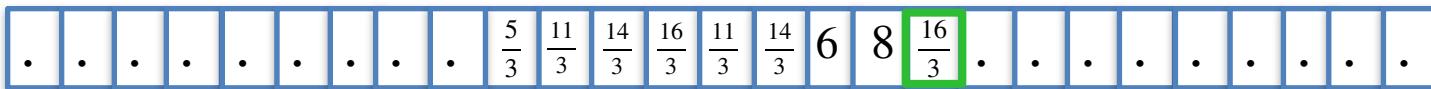
$$\frac{1}{3} \quad \frac{1}{3} \quad \frac{1}{3}$$

11

$$\begin{array}{|c|c|c|} \hline & \frac{9}{3} & \frac{7}{3} & \frac{0}{3} \\ \hline \end{array}$$

Σ

G

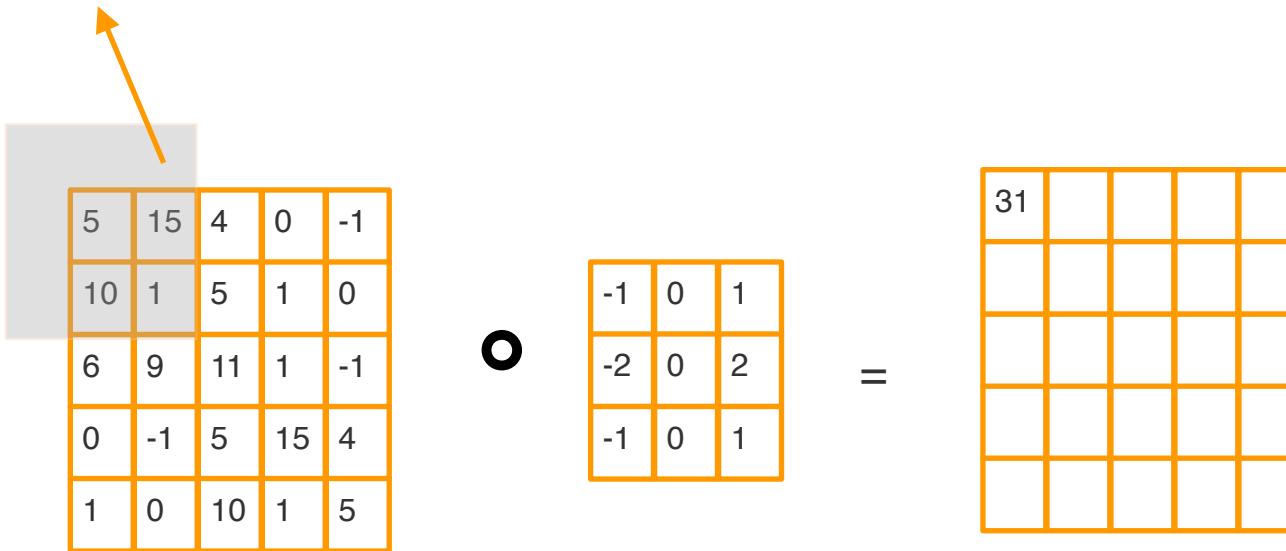


Cross-Correlation and Convolution

$$\begin{matrix} \begin{matrix} 5 & 15 & 4 & 0 & -1 \\ 10 & 1 & 5 & 1 & 0 \\ 6 & 9 & 11 & 1 & -1 \\ 0 & -1 & 5 & 15 & 4 \\ 1 & 0 & 10 & 1 & 5 \end{matrix} \bullet \begin{matrix} -1 & 0 & 1 \\ -2 & 0 & 2 \\ -1 & 0 & 1 \end{matrix} \end{matrix}$$

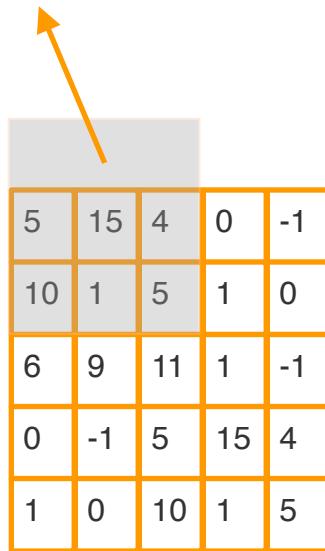
Cross-Correlation and Convolution

$$2 \times 15 + 1 \times 1 = 31$$



Cross-Correlation and Convolution

$$-2 \times 5 + 2 \times 4 - 1 \times 10 + 5 \times 1 = 7$$



5	15	4	0	-1
10	1	5	1	0
6	9	11	1	-1
0	-1	5	15	4
1	0	10	1	5



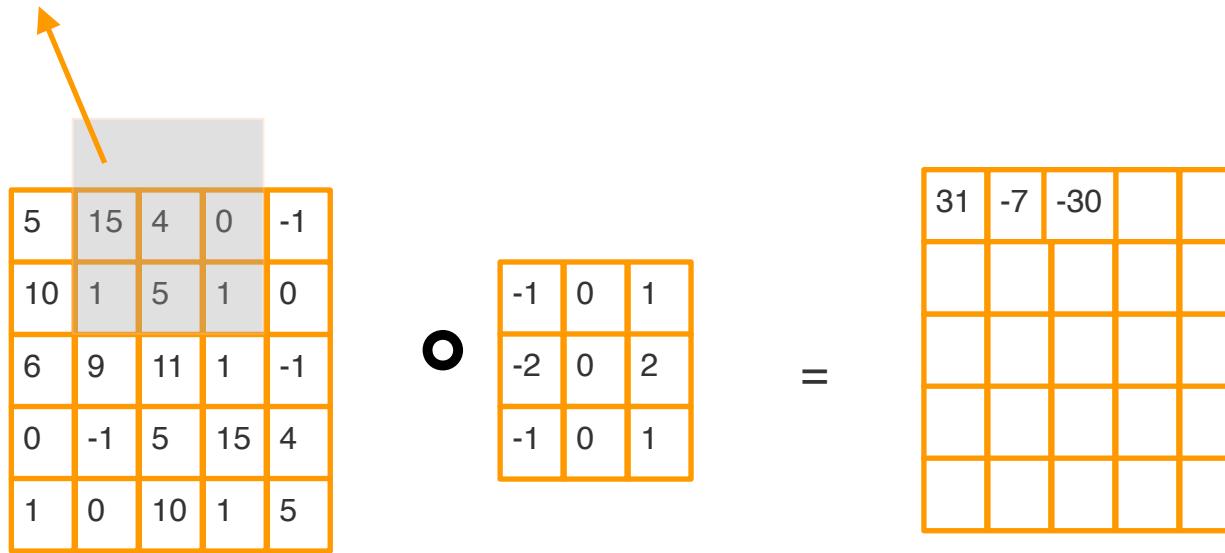
-1	0	1
-2	0	2
-1	0	1

=

31	-7			

Cross-Correlation and Convolution

$$-2 \times 15 - 1 \times 1 + 1 \times 1 = -30$$



Cross-Correlation and Convolution

$$-2 \times 4 - 1 \times 2 - 5 \times 1 = -15$$

5	15	4	0	-1
10	1	5	1	0
6	9	11	1	-1
0	-1	5	15	4
1	0	10	1	5



-1	0	1
-2	0	2
-1	0	1

=

31	-7	-30	-15

Cross-Correlation and Convolution

$$-1 \times 1 = -1$$

5	15	4	0	-1
10	1	5	1	0
6	9	11	1	-1
0	-1	5	15	4
1	0	10	1	5



-1	0	1
-2	0	2
-1	0	1

=

31	-7	-30	-15	-1

Cross-Correlation and Convolution

$$15 \times 1 + 2 \times 1 + 9 \times 1 = 26$$

The diagram illustrates a cross-correlation or convolution operation. On the left, a 5x5 input matrix is shown with its last row highlighted in gray. An orange arrow points from the top-left cell of this row to the result of the computation. The input matrix contains the following values:

5	15	4	0	-1
10	1	5	1	0
6	9	11	1	-1
0	-1	5	15	4
1	0	10	1	5

In the center, a black circle symbol represents the operation. To its right is an equals sign (=). To the right of the equals sign is a 5x5 output matrix, also with its last row highlighted in gray. The output matrix contains the following values:

31	-7	-30	-15	-1
26				

Cross-Correlation and Convolution

$$5 \times (-1) + 10 \times (-2) + 6 \times (-1) + 4 \times 1 + 5 \times 2 + 11 \times 1 = -6$$

The diagram illustrates a cross-correlation or convolution operation between two matrices. On the left is a 5x5 input matrix with orange borders and gray shaded cells. An orange arrow points from the top-left cell (5) to the result matrix. In the center is a 3x3 kernel matrix with an orange border. To its right is an equals sign followed by a 5x5 result matrix with an orange border. The result matrix has values 31, -7, -30, -15, -1 in the first row, and 26, -6 in the second row, with the rest of the matrix being empty.

5	15	4	0	-1
10	1	5	1	0
6	9	11	1	-1
0	-1	5	15	4
1	0	10	1	5

○

-1	0	1
-2	0	2
-1	0	1

=

31	-7	-30	-15	-1
26	-6			

Cross-Correlation and Convolution

$$15 \times (-1) + 1 \times (-2) + 9 \times (-1) + 0 \times 1 + 1 \times 2 + 1 \times 1 = -23$$



5	15	4	0	-1
10	1	5	1	0
6	9	11	1	-1
0	-1	5	15	4
1	0	10	1	5



-1	0	1
-2	0	2
-1	0	1

=

31	-7	-30	-15	-1
26	-6	-23		

Cross-Correlation and Convolution

$$4 \times (-1) + 5 \times (-2) + 11 \times (-1) + (-1) \times 1 + 0 \times 2 + (-1) \times 1 = -27$$



5	15	4	0	-1
10	1	5	1	0
6	9	11	1	-1
0	-1	5	15	4
1	0	10	1	5



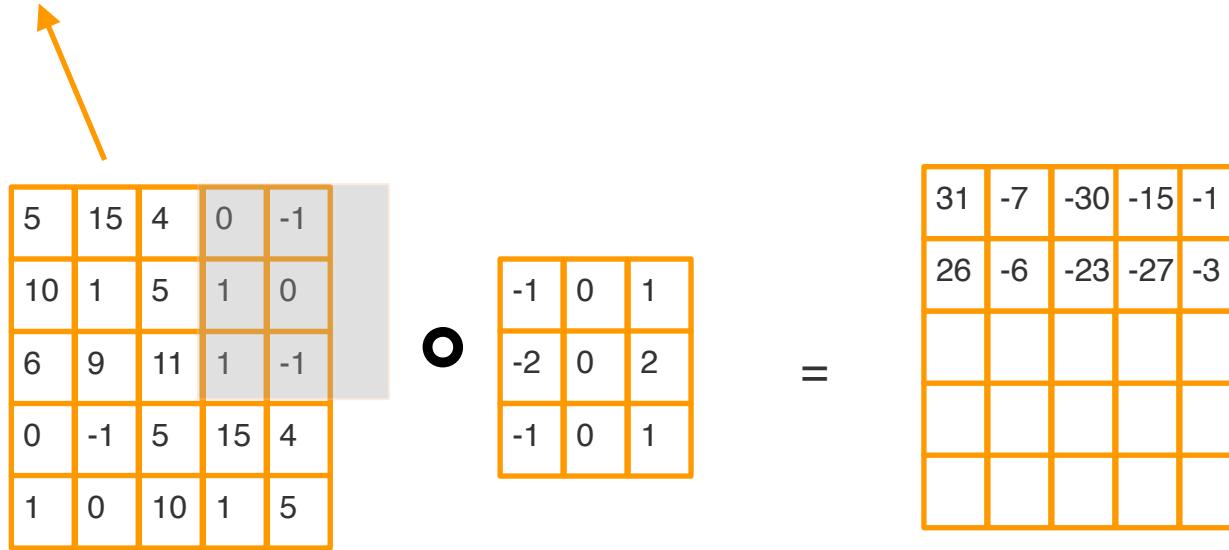
-1	0	1
-2	0	2
-1	0	1

=

31	-7	-30	-15	-1
26	-6	-23	-27	

Cross-Correlation and Convolution

$$0 \times (-1) + 1 \times (-2) + 1 \times (-1) = -3$$



Cross-Correlation and Convolution

$$1 \times 1 + 9 \times 2 + -1 \times 1 = 18$$

The diagram illustrates a cross-correlation or convolution operation between two matrices. On the left, a 5x5 input matrix is shown with a 3x3 receptive field highlighted in gray. An orange arrow points from the top-left cell of the input matrix to the result matrix. In the center, a 3x3 kernel matrix is shown. To the right of the kernel is a circled multiplication symbol (•), followed by an equals sign (=). To the right of the equals sign is the resulting 3x3 output matrix.

5	15	4	0	-1
10	1	5	1	0
6	9	11	1	-1
0	-1	5	15	4
1	0	10	1	5

•

-1	0	1
-2	0	2
-1	0	1

=

31	-7	-30	-15	-1
26	-6	-23	-27	-3
18				

Cross-Correlation and Convolution

$$10 \times (-1) + 6 \times (-2) + 5 \times 1 + 11 \times 2 + 5 \times 1 = 10$$



5	15	4	0	-1
10	1	5	1	0
6	9	11	1	-1
0	-1	5	15	4
1	0	10	1	5



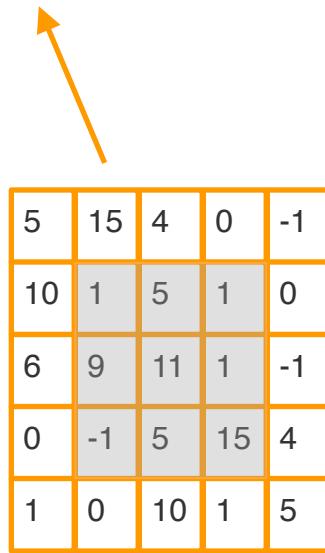
-1	0	1
-2	0	2
-1	0	1

=

31	-7	-30	-15	-1
26	-6	-23	-27	-3
18	10			

Cross-Correlation and Convolution

$$1 \times (-1) + 9 \times (-2) + (-1) \times (-1) + 1 \times 1 + 2 \times 1 + 15 \times 1 = 0$$



5	15	4	0	-1
10	1	5	1	0
6	9	11	1	-1
0	-1	5	15	4
1	0	10	1	5



-1	0	1
-2	0	2
-1	0	1



31	-7	-30	-15	-1
26	-6	-23	-27	-3
18	10	0		

Cross-Correlation and Convolution

$$5 \times (-1) + 11 \times (-2) + 5 \times (-1) + 0 \times 1 + (-1) \times 2 + 4 \times 1 = -30$$



5	15	4	0	-1
10	1	5	1	0
6	9	11	1	-1
0	-1	5	15	4
1	0	10	1	5



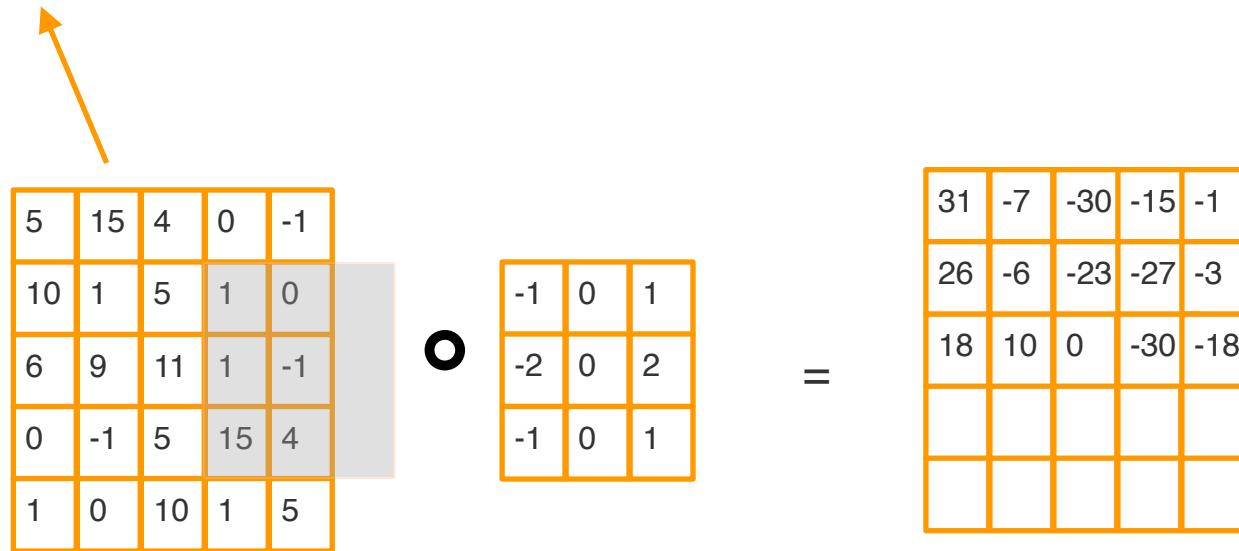
-1	0	1
-2	0	2
-1	0	1

=

31	-7	-30	-15	-1
26	-6	-23	-27	-3
18	10	0	-30	

Cross-Correlation and Convolution

$$1 \times (-1) + 1 \times (-2) + 15 \times (-1) + 0 \times 1 + 0 \times 2 + 0 \times 1 = -18$$



Cross-Correlation and Convolution

$$0 \times (-1) + 0 \times (-2) + 0 \times (-1) + 9 \times 1 + (-1) \times 2 + 0 \times 1 = 7$$

The diagram illustrates a convolution or cross-correlation operation. It shows two input matrices and a kernel matrix. An orange arrow points to the element 9 in the first input matrix. The operation is performed using the formula: $0 \times (-1) + 0 \times (-2) + 0 \times (-1) + 9 \times 1 + (-1) \times 2 + 0 \times 1 = 7$.

The first input matrix has a highlighted 3x3 receptive field in gray. The second input matrix is multiplied by the kernel matrix (indicated by a circled dot). The result is shown in the third matrix.

5	15	4	0	-1
10	1	5	1	0
6	9	11	1	-1
0	-1	5	15	4
1	0	10	1	5

•

-1	0	1
-2	0	2
-1	0	1

=

31	-7	-30	-15	-1
26	-6	-23	-27	-3
18	10	0	-30	-18
7				

Cross-Correlation and Convolution

$$6 \times (-1) + 0 \times (-2) + 1 \times (-1) + 11 \times 1 + 5 \times 2 + 10 \times 1 = 24$$

The diagram illustrates a mathematical operation, likely cross-correlation or convolution, between two matrices. On the left is a 5x5 input matrix with orange borders. A specific element at position (3, 3) is highlighted in gray. An orange arrow points from this element to the result of the calculation shown above. In the center is a 3x3 kernel matrix with orange borders. To the right of the input matrix is a large black circle symbol, indicating the operation. To the right of the kernel is an equals sign (=). To the right of the equals sign is a 5x5 output matrix with orange borders. The element at position (3, 3) of the output matrix is explicitly labeled with the value 24.

5	15	4	0	-1
10	1	5	1	0
6	9	11	1	-1
0	-1	5	15	4
1	0	10	1	5

○

-1	0	1
-2	0	2
-1	0	1

=

31	-7	-30	-15	-1
26	-6	-23	-27	-3
18	10	0	-30	-18
7	24			

Cross-Correlation and Convolution

$$9 \times (-1) + -1 \times (-2) + 0 \times (-1) + 1 \times 1 + 15 \times 2 + 1 \times 1 = 25$$

The diagram illustrates a convolution or cross-correlation operation. It shows two 5x5 input matrices, a 3x3 kernel matrix, and the resulting output matrix.

Input Matrix 1:

5	15	4	0	-1
10	1	5	1	0
6	9	11	1	-1
0	-1	5	15	4
1	0	10	1	5

Input Matrix 2:

31	-7	-30	-15	-1
26	-6	-23	-27	-3
18	10	0	-30	-18
7	24	25		

Kernel:

-1	0	1
-2	0	2
-1	0	1

Operation: \bullet (Cross-Correlation or Convolution)

Result: $=$

31	-7	-30	-15	-1
26	-6	-23	-27	-3
18	10	0	-30	-18
7	24	25		

Cross-Correlation and Convolution

$$11 \times (-1) + 5 \times (-2) + 10 \times (-1) + (-1) \times 1 + 4 \times 2 + 5 \times 1 = -19$$

5	15	4	0	-1
10	1	5	1	0
6	9	11	1	-1
0	-1	5	15	4
1	0	10	1	5



-1	0	1
-2	0	2
-1	0	1

=

31	-7	-30	-15	-1
26	-6	-23	-27	-3
18	10	0	-30	-18
7	24	25	-19	

Cross-Correlation and Convolution

$$1 \times (-1) + 15 \times (-2) + 1 \times (-1) + 0 \times 1 + 0 \times 2 + 0 \times 1 = -32$$

The diagram illustrates a cross-correlation or convolution operation between two matrices. On the left, a larger input matrix (5x5) and a smaller kernel (3x3) are shown. The input matrix has values: 5, 15, 4, 0, -1; 10, 1, 5, 1, 0; 6, 9, 11, 1, -1; 0, -1, 5, 15, 4; 1, 0, 10, 1, 5. A gray vertical bar indicates the stride of 2. The kernel matrix has values: -1, 0, 1; -2, 0, 2; -1, 0, 1. An orange arrow points from the top-left element of the kernel to the value -1 in the input matrix. The result of the operation is shown on the right, enclosed in a box, with values: 31, -7, -30, -15, -1; 26, -6, -23, -27, -3; 18, 10, 0, -30, -18; 7, 24, 25, -19, -32; followed by three empty rows.

31	-7	-30	-15	-1
26	-6	-23	-27	-3
18	10	0	-30	-18
7	24	25	-19	-32

Cross-Correlation and Convolution

$$0 \times (-1) + 0 \times (-2) + 0 \times (-1) + (-1) \times 1 + 0 \times 2 + 0 \times 1 = -1$$

The diagram illustrates a convolution or cross-correlation operation. It shows two 5x5 input matrices and a 3x3 kernel matrix. An orange arrow points from the top-left element of the first input matrix to the result matrix, indicating the calculation of the output value at that position.

The input matrices are:

5	15	4	0	-1
10	1	5	1	0
6	9	11	1	-1
0	-1	5	15	4
1	0	10	1	5

The kernel matrix is:

-1	0	1
-2	0	2
-1	0	1

The result matrix is:

31	-7	-30	-15	-1
26	-6	-23	-27	-3
18	10	0	-30	-18
7	24	25	-19	-32
-1				

Cross-Correlation and Convolution

$$0 \times (-1) + 1 \times (-2) + 0 \times (-1) + 5 \times 1 + 10 \times 2 + 0 \times 1 = 23$$

The diagram illustrates a convolution or cross-correlation operation. It shows two 5x5 input matrices, a 3x3 kernel matrix, and the resulting 5x5 output matrix.

Input Matrix 1:

5	15	4	0	-1
10	1	5	1	0
6	9	11	1	-1
0	-1	5	15	4
1	0	10	1	5

Input Matrix 2:

31	-7	-30	-15	-1
26	-6	-23	-27	-3
18	10	0	-30	-18
7	24	25	-19	-32
-1	23			

Kernel Matrix:

-1	0	1
-2	0	2
-1	0	1

Resulting Output Matrix:

31	-7	-30	-15	-1
26	-6	-23	-27	-3
18	10	0	-30	-18
7	24	25	-19	-32
-1	23			

An orange arrow points from the top-left cell of the first input matrix to the value 23 in the bottom-right cell of the resulting output matrix, indicating the result of the highlighted computation.

Cross-Correlation and Convolution

$$-1 \times (-1) + 0 \times (-2) + 0 \times (-1) + 15 \times 1 + 0 \times 2 + 0 \times 1 = 16$$

The diagram illustrates a cross-correlation or convolution operation between two 5x5 input matrices and a 3x3 kernel matrix.

Input Matrix 1:

5	15	4	0	-1
10	1	5	1	0
6	9	11	1	-1
0	-1	5	15	4
1	0	10	1	5

Input Matrix 2:

31	-7	-30	-15	-1
26	-6	-23	-27	-3
18	10	0	-30	-18
7	24	25	-19	-32
-1	23	16		

Kernel:

-1	0	1
-2	0	2
-1	0	1

Operation: \bullet =

An orange arrow points from the top-left cell of Input Matrix 1 to the value $-1 \times (-1)$, indicating the calculation of the output cell at position (1,1).

Cross-Correlation and Convolution

$$5 \times (-1) + 10 \times (-2) + 0 \times (-1) + 4 \times 1 + 5 \times 2 + 0 \times 1 = -11$$

The diagram illustrates a convolution or cross-correlation operation. On the left, a 5x5 input matrix is shown with orange borders around its rows and columns. A 3x3 kernel matrix is positioned to its right, also with orange borders. The result of the operation is a 3x3 output matrix on the far right, also with orange borders. An orange arrow points from the top-left cell of the input matrix to the result cell in the output matrix, indicating the receptive field of that output unit. The operation is defined by the equation above.

5	15	4	0	-1
10	1	5	1	0
6	9	11	1	-1
0	-1	5	15	4
1	0	10	1	5

○

-1	0	1
-2	0	2
-1	0	1

=

31	-7	-30	-15	-1
26	-6	-23	-27	-3
18	10	0	-30	-18
7	24	25	-19	-32
-1	23	16	-11	

Cross-Correlation and Convolution

$$15 \times (-1) + 1 \times (-2) + 0 \times (-1) + 0 \times 1 + 0 \times 2 + 0 \times 1 = -17$$

The diagram illustrates a convolution or cross-correlation operation. On the left, a 5x5 input matrix is shown with orange borders. A 3x3 kernel matrix is positioned to its right, also with an orange border. The result of the operation is a 3x5 output matrix on the far right, also with an orange border. An orange arrow points from the value -17 in the top-left cell of the output matrix back to the corresponding element in the input matrix. The input matrix has a gray shaded 2x2 block in the bottom-right corner, and the output matrix has a gray shaded 2x2 block in the bottom-right corner.

5	15	4	0	-1
10	1	5	1	0
6	9	11	1	-1
0	-1	5	15	4
1	0	10	1	5

○

-1	0	1
-2	0	2
-1	0	1

=

31	-7	-30	-15	-1
26	-6	-23	-27	-3
18	10	0	-30	-18
7	24	25	-19	-32
-1	23	16	-11	-17

Cross-Correlation and Convolution

5	15	4	0	-1
10	1	5	1	0
6	9	11	1	-1
0	-1	5	15	4
1	0	10	1	5

○

-1	0	1
-2	0	2
-1	0	1

=

31	-7	-30	-15	-1
26	-6	-23	-27	-3
18	10	0	-30	-18
7	24	25	-19	-32
-1	23	16	-11	-17

Image, I

Filter/template

Output image

Cross-Correlation - Mathematically

1D

$$G = F \circ I[i] = \sum_{u=-k}^k F[u]I[i+u] \quad F \text{ has } 2k+1 \text{ elements}$$

Box filter $F[u] = \frac{1}{3}$ for $u = -1, 0, 1$ and 0 otherwise

Cross-correlation filtering - 2D

Let's write this down as an equation. Assume the averaging window is $(2k+1) \times (2k+1)$:

$$G[i, j] = \frac{1}{(2k+1)^2} \sum_{u=-k}^k \sum_{v=-k}^k F[i+u, j+v]$$

We can generalize this idea by allowing different weights for different neighboring pixels:

$$G[i, j] = \sum_{u=-k}^k \sum_{v=-k}^k F[u, v] I[i+u, j+v]$$

This is called a **cross-correlation** operation and written:

$$G = F \circ I$$

F is called the “filter,” “kernel,” or “mask.”

Convolution

Filter is flipped before correlating

1D F has $2k + 1$ elements

$$G = F * I[i] = \sum_{u=-k}^k F[u]I[i-u]$$

Box filter $F[u] = \frac{1}{3}$ for $u = -1, 0, 1$ and 0 otherwise

for example, convolution of 1D image with the filter [3,5,2]

is exactly the same as correlation with the filter [2,5,3]

Convolution filtering - 2D

For 2D the filter is flipped and rotated

$$G[i, j] = \sum_{u=-k}^k \sum_{v=-k}^k F[u, v] I[i - u, j - v]$$

Correlation and convolution are identical for symmetrical filters

Convolution with the filter

1	2	1
0	0	0
-1	-2	-1

-1	0	1
-2	0	2
-1	0	1

is the same as Correlation with the filter