High Performance Computing Systems (CMSC714)

Lecture 16: Parallel Matrix Multiplication

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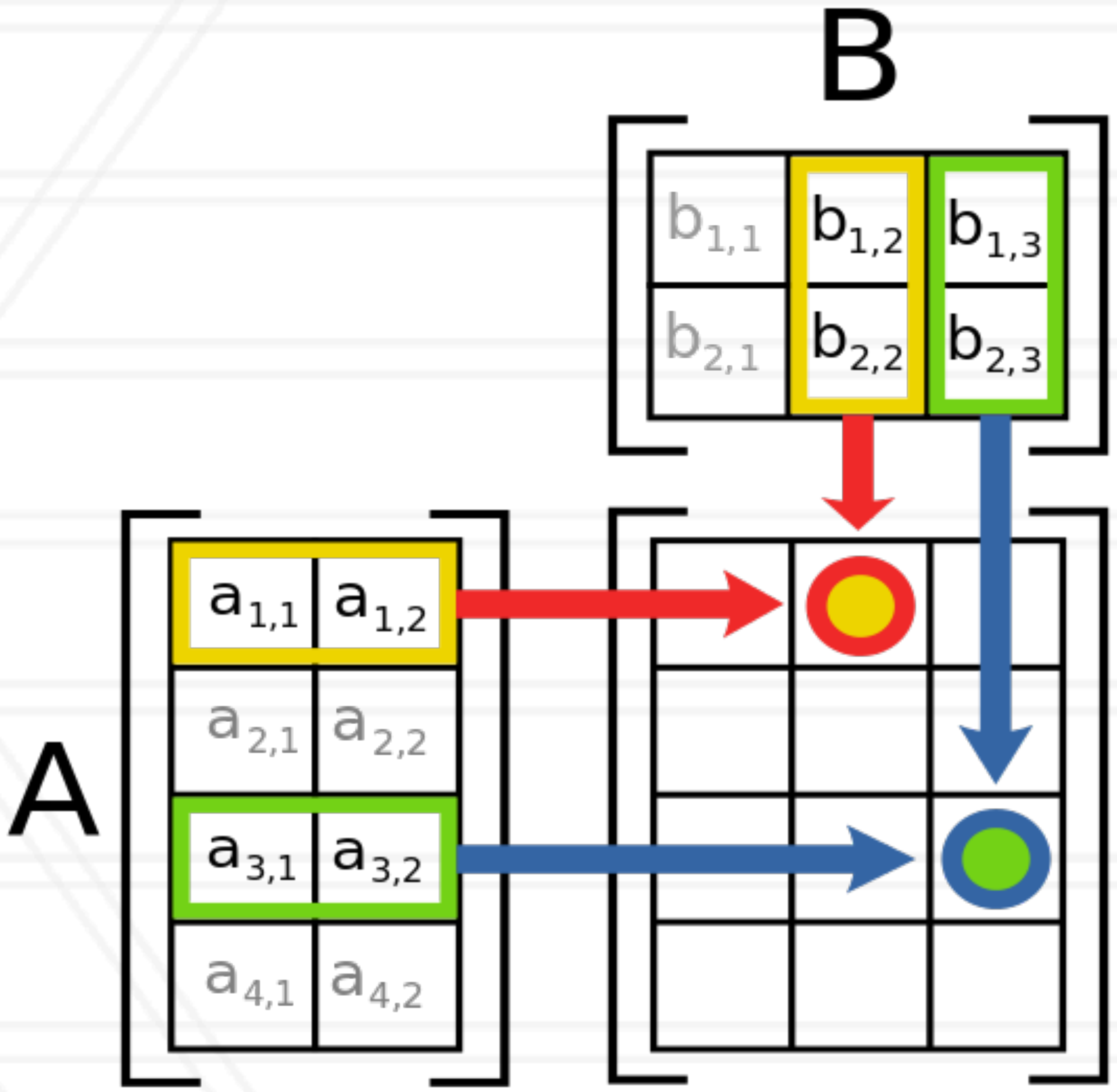
Summary of last lecture

- Parallel sorting is used in many HPC applications
- Two categories of parallel sort algorithms: merge-based and splitter-based
- Sample sort: select p-1 splitters
- Radix sort: look at k bits at a time to place keys in 2^k buckets

Matrix Multiplication

```
for (i=0; i<M; i++)
  for (j=0; j<N; j++)
    for (k=0; k<L; k++)
     C[i][j] += A[i][k]*B[k][j];
```



https://en.wikipedia.org/wiki/Matrix_multiplication

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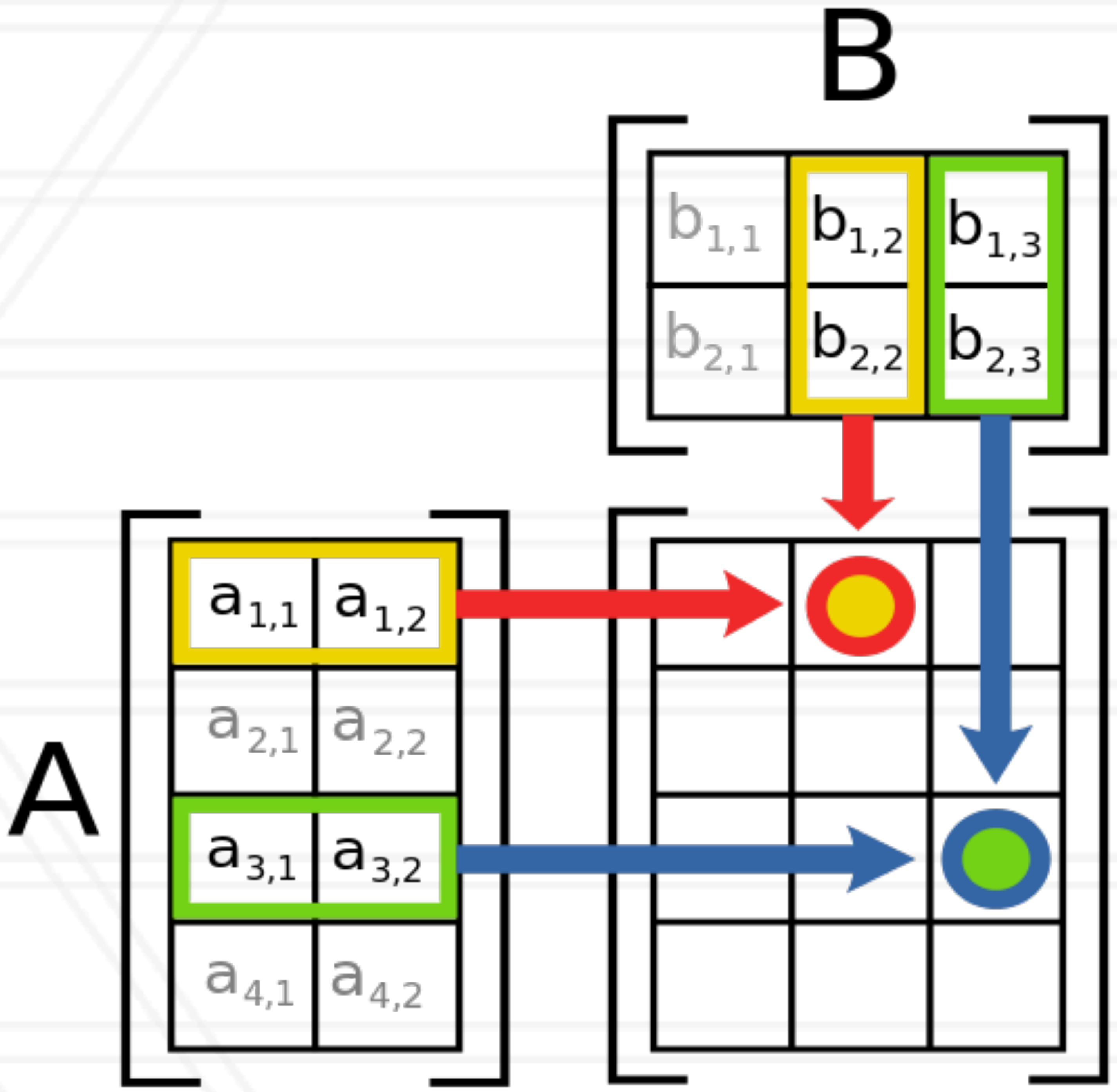

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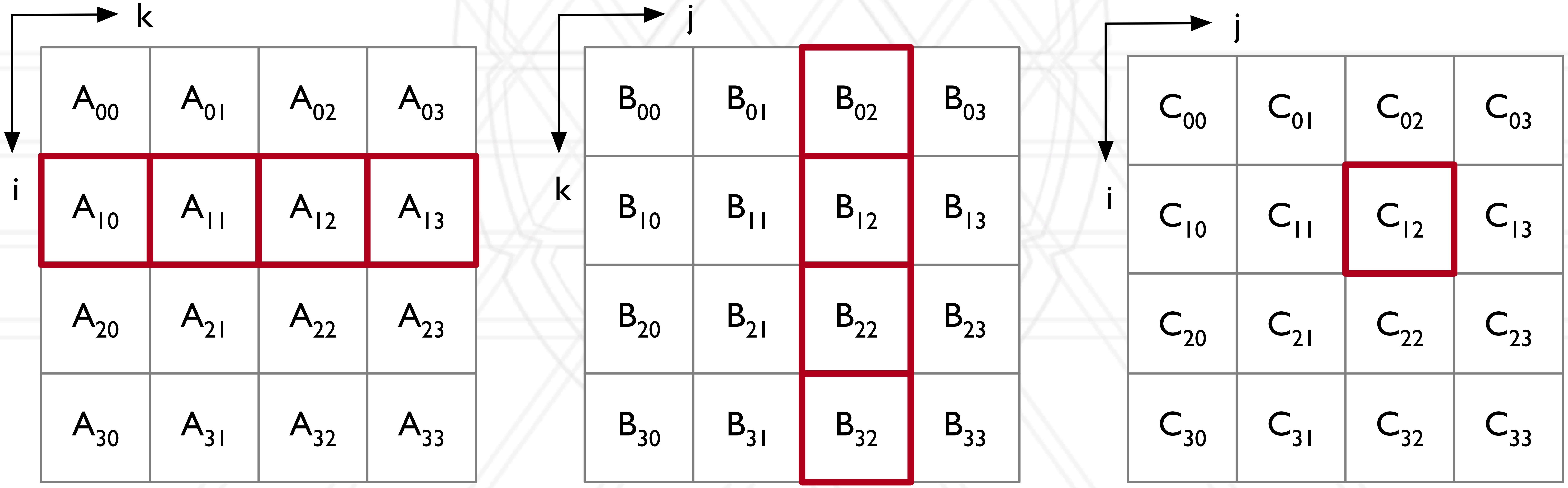
Any performance issues for large arrays?

Blocking to improve cache performance

- Create smaller blocks that fit in cache: leads to cache reuse
- $C_{12} = A_{10} * B_{02} + A_{11} * B_{12} + A_{12} * B_{22} + A_{13} * B_{32}$



Parallel Matrix Multiply

- Store A and B in a distributed manner
- Communication between processes to get the right sub-matrices to each process
- Each process computes a portion of C

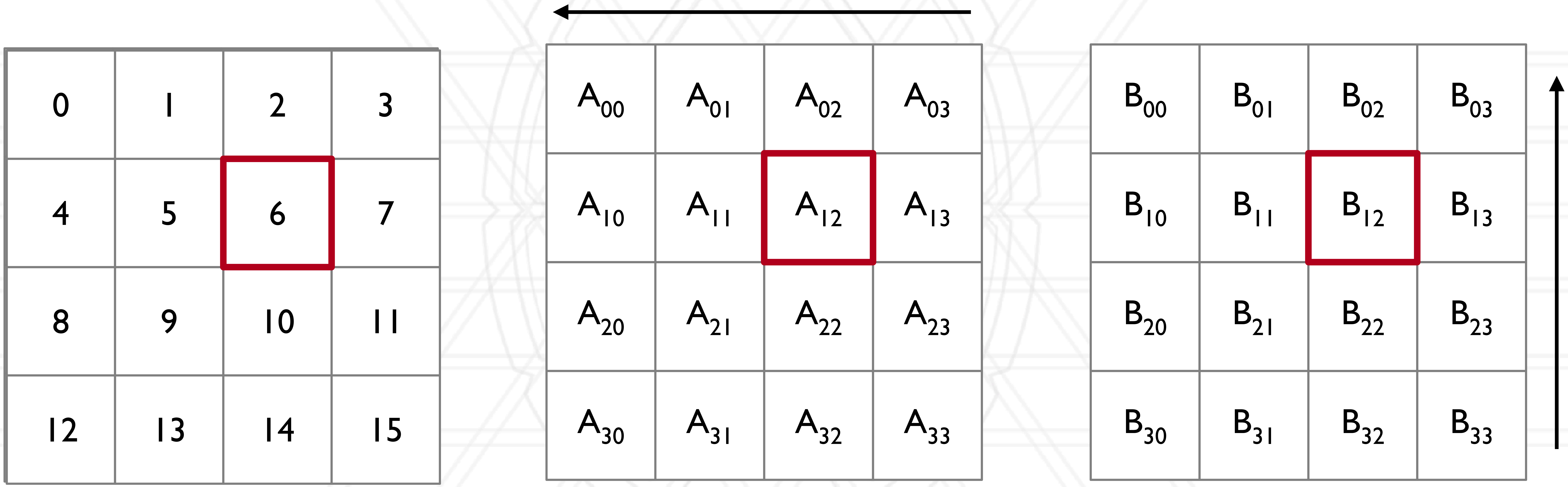
Cannon's 2D matrix multiply

0	1	2	3
4	5	6	7
8	9	10	11
12	13	14	15

A_{00}	A_{01}	A_{02}	A_{03}
A_{10}	A_{11}	A_{12}	A_{13}
A_{20}	A_{21}	A_{22}	A_{23}
A_{30}	A_{31}	A_{32}	A_{33}

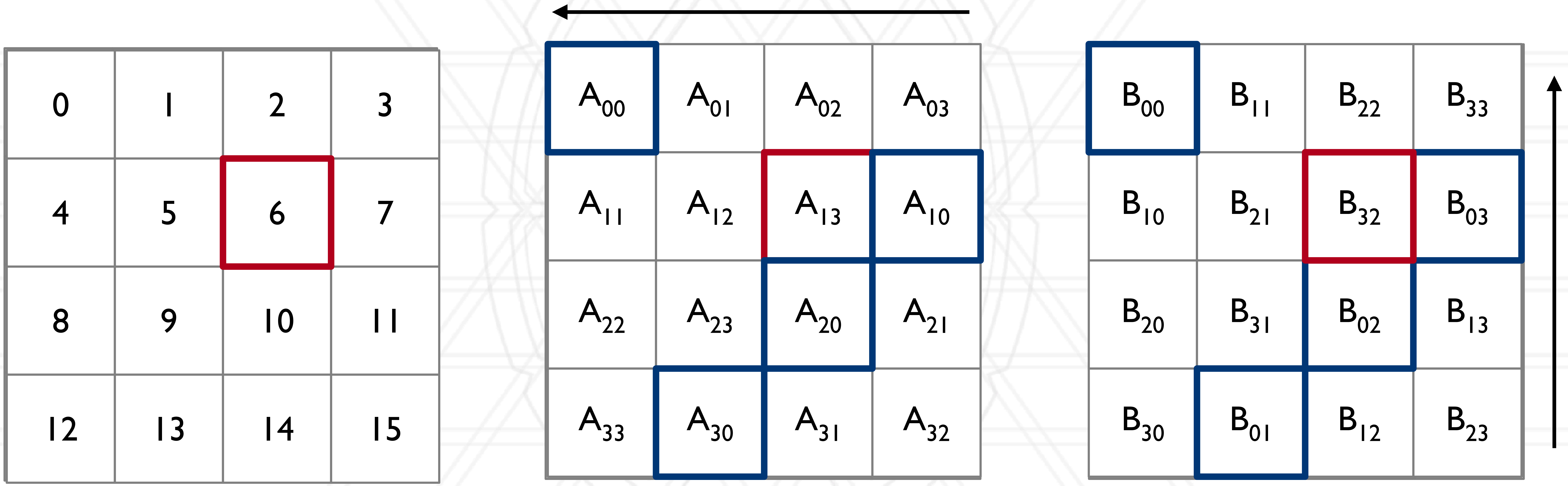
B_{00}	B_{01}	B_{02}	B_{03}
B_{10}	B_{11}	B_{12}	B_{13}
B_{20}	B_{21}	B_{22}	B_{23}
B_{30}	B_{31}	B_{32}	B_{33}

Cannon's 2D matrix multiply



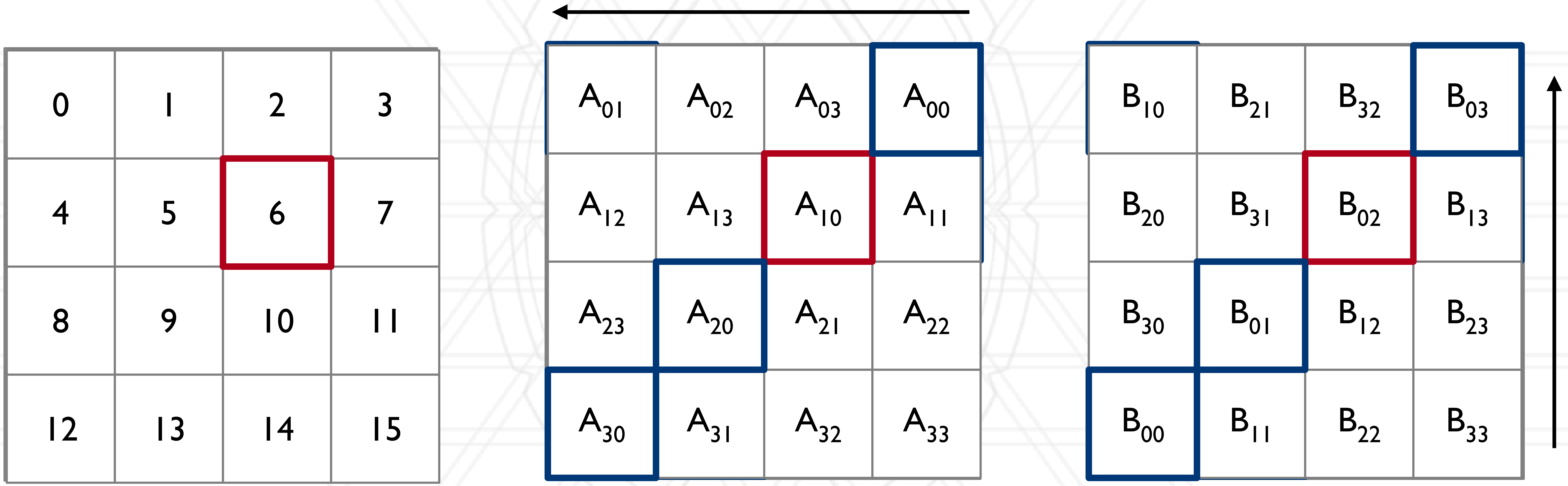
Initial skew

Cannon's 2D matrix multiply



Initial skew

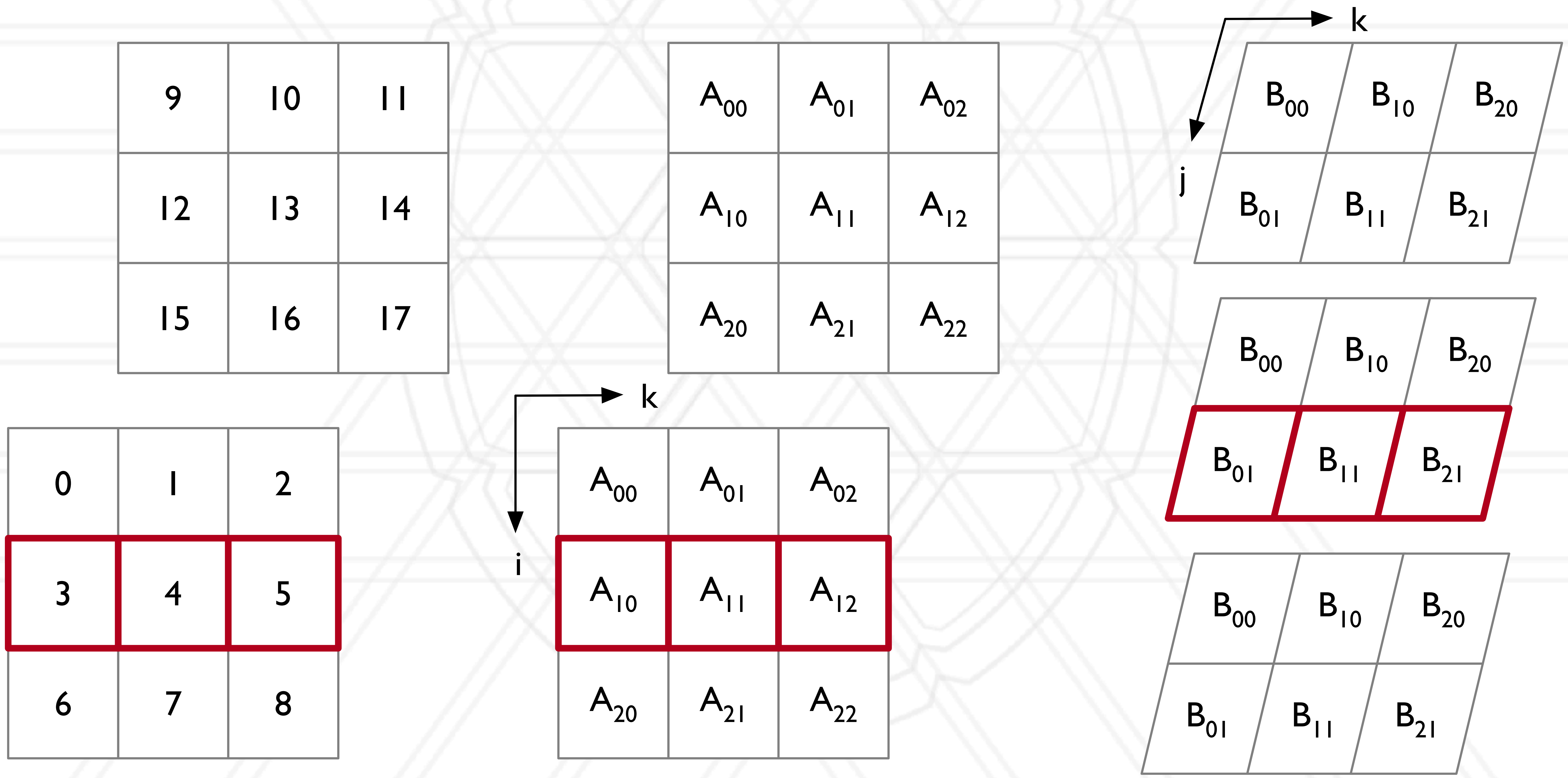
Cannon's 2D matrix multiply



Shift-by-1

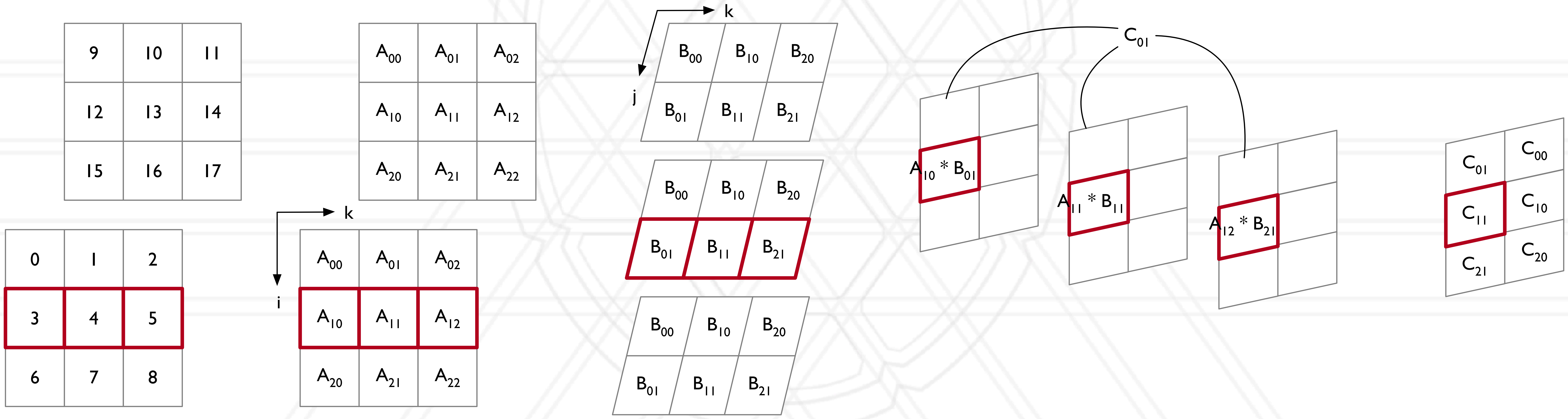
Agarwal's 3D matrix multiply

- Copy A to all i-k planes and B to all j-k planes



Agarwal's 3D matrix multiply

- Perform a single matrix multiply to calculate partial C
- All-to-all along i-j planes to calculate final result



Questions?

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