A Comparison of Sorting Algorithms for the Connection Machine CM-2

presented by Benjamin Black

Optimizing sorting on CM-2

- CM-2: hypercube network connect
- Algorithms to optimize
  - Bitonic sort
  - Sample sort
  - Radix sort

Primitives

- Arithmetic (Map)
- Send across network
- Scan (Cumulative sum)
- Cube swap (send along each dimension of hypercube)

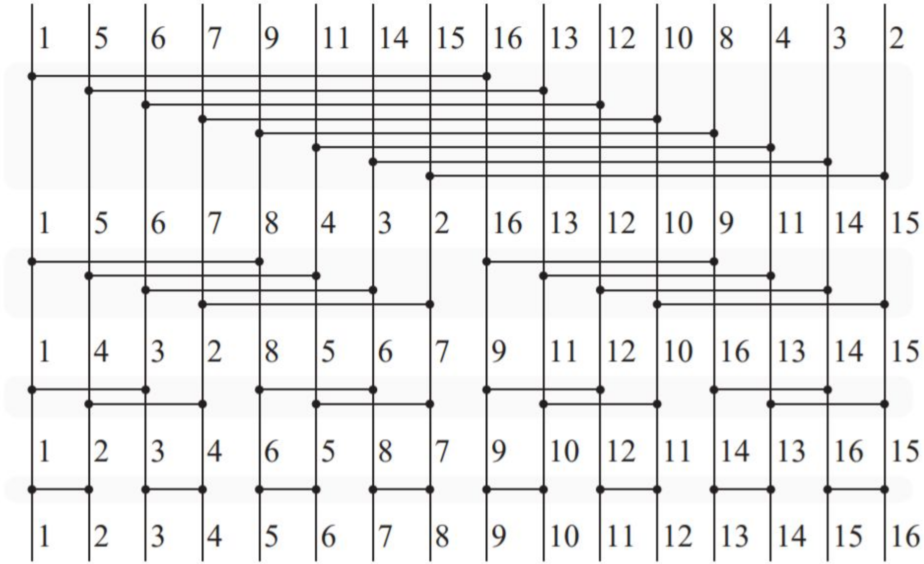
Table 1. The time required for operations on a 32K Connection Machine CM-2.*

Operation	Symbolic time	Actual time
Arithmetic	$A \cdot (n/p)$	$1 \cdot (n/1024)$
Cube Swap	$Q \cdot (n/p)$	$40 \cdot (n/1024)$
Send (routing)	$R \cdot (n/p)$	$130 \cdot (n/1024)$
Scan (parallel prefix)	$3A \cdot (n/p) + S$	$3 \cdot (n/1024) + 50$

*The value p is the number of processors (Sprint nodes), and n is the total number of elements being operated on. All operations are on 64-bit words, except for scans which are on 32-bit words. All times are in microseconds.

Bitonic-sort

- Similar to merge-sort except every other sub-sequence is sorted in reverse order
- Key operation-bitonic merge
  - Naturally organized like a hypercube
  - Most efficient for small numbers of keys



Blelloch et. al. 1991

Bitonic sort optimizations

- Optimization pipelined-bitonic sort
  - Multiple keys per processor
  - exchange all keys before they are needed

Radix sort

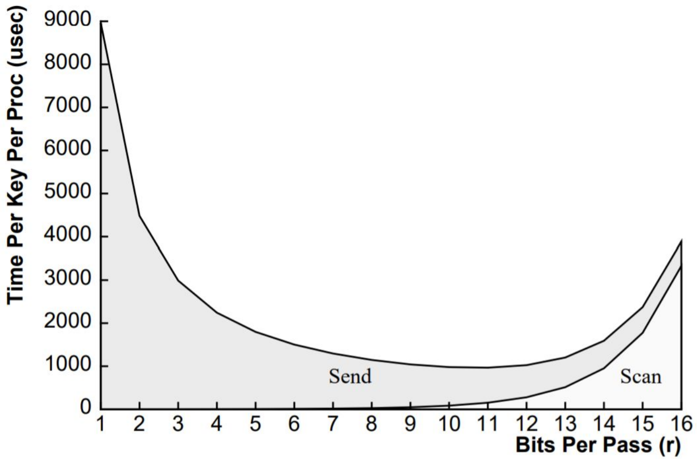
- Iterated bucket sort
- In place sort
- Need to know not only which bucket each key is in, but which rank the key is within each bucket
- Counting rank for each bucket be done with a Scan operation

Optimizing radix sort

- More efficient to calculate all ranks internally for each processor, combine across processors
- Choosing parameter r

$$\begin{aligned} T_{\text{simple-rank}} &= 2^r \cdot (3A \cdot (n/p) + S) + 2^r(2A)(n/p) \\ &= A \cdot ((2+3)2^r(n/p)) + S \cdot 2^r \ , \end{aligned}$$

$$T_{\text{rank}} = A \cdot (2 \cdot 2^r + 10(n/p)) + S \cdot 2^r ,$$

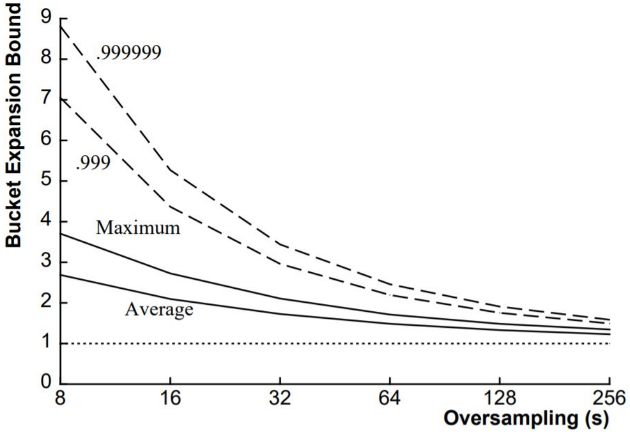


Sample Sort

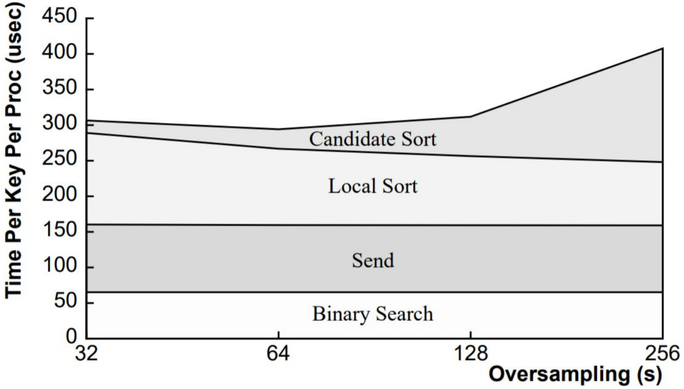
- Divide and conquer algorithm--Similar to quicksort
- Quicksort:
    - find 1 pivot value using a random strategy
    - Divide input below and above pivot point
    - Sort each half independently.
- Sample sort
    - Sample p-1 pivot values using a random strategy (p is the number of processors)
    - Sort pivot values
    - Send all pivot values to all processors
    - Divide input on each processors into the p buckets in parallel
        - binary search
    - Sort each bucket on each processor

Sample sort optimizations

- Oversampling:
  - To make evenly sized buckets, *s(p-1)* samples are selected, where *p* is the number of processors and *s* is the oversampling ratio.
  - Pivots are the samples in the s, 2s, 3s, … (p-1)s ranks in the sample
  - Chose value 64 for s empirically for 16384 keys per processor



Blelloch et. al. 1991



Final results