CMSC 330: Organization of Programming Languages

Closures
(Implementing Higher Order Functions)

Returning Functions as Results

- In OCaml you can pass functions as arguments
  - to **`map`**, **`fold`**, etc.
- and you can return functions as results

```
# let pick_fn n =
    let plus_three x = x + 3 in
    let plus_four x = x + 4 in
    if n > 0 then plus_three else plus_four
val pick_fn : int -> (int->int) = <fun>
```

- Here, **`pick_fn`** takes an **`int`** argument, and returns a function

```
# let g = pick_fn 2;;
val g : int -> int = <fun>
# g 4;;    (* evaluates to 7 *)
```

Multi-argument Functions

- Consider a rewriting of the prior code (above)

```
let pick_fn n =
  if n > 0 then (fun x -> x+3) else (fun x -> x+4)
```

- Here's another version

```
let pick_fn n =
  (fun x -> if n > 0 then x+3 else x+4)
```

- … the shorthand for which is just

```
let pick_fn n x =
  if n > 0 then x+3 else x+4
```

I.e., a multi-argument function!

Currying

- Multi-argument functions not a separate concept
  - Can encode one as a *function that takes a single argument and returns a function that takes the rest*
- This encoding is called currying the function
  - Named after the logician Haskell B. Curry
  - But Schönfinkel and Frege discovered it
    - So maybe it should be called Schönfinkelizing or Fregging

Curried Functions In OCaml

- OCaml syntax defaults to currying. E.g.,

```
let add x y = x + y
```

  - is identical to all of the following:

```
let add = (fun x -> (fun y -> x + y))
let add = (fun x y -> x + y)
let add x = (fun y -> x+y)
```

- Thus:
  - `add` has type `int -> (int -> int)`
  - `add 3` has type `int -> int`
    - `add 3` is a function that adds 3 to its argument
  - `(add 3) 4 = 7`

Syntax Conventions for Currying

- Because currying is so common, OCaml uses the following conventions:
  - `->` associates from the right
    - Thus `int -> int -> int` is the same as
    - `int -> (int -> int)`
  - function application associates from the left
    - Thus `add 3 4` is the same as
    - `(add 3) 4`

Quiz 1: Which f definition is equivalent?

```
let f a b = a / b;;
```

```
A.  let f b = fun a -> a / b;;
B.  let f = fun a -> (fun b -> a / b);;
C.  let f = fun a | b -> a / b;;
D.  let f (a, b) = a / b;;
```

Quiz 1: Which f definition is equivalent?

```
let f a b = a / b;;
```

```
A.  let f b = fun a -> a / b;;
B.  let f = fun a -> (fun b -> a / b);;
C.  let f = fun a | b -> a / b;;
D.  let f (a, b) = a / b;;
```

Multiple Arguments, Partial Application

- Another way you could encode support for multiple arguments is using tuples
  - `let f (a,b) = a / b (* int*int -> int *)`
  - `let f a b = a / b (* int -> int-> int *)`
- Is there a benefit to using currying instead?
  - Supports **partial application** – useful when you want to provide some arguments now, the rest later
  - `let add a b = a + b;;`
  - `let addthree = add 3;;`
  - `addthree 4;; (* evaluates to 7 *)`

Quiz 2: What does this evaluate to?

```
let f a b = a * b in
let g = f 2 in
let a = 3 in
g 4
```

A. 8
B. 6
C. 2
D. 3

Quiz 2: What does this evaluate to?

```
let f a b = a * b in
let g = f 2 in
let a = 3 in
g 4    (* f 2 4 = 8 *)
```

A. 8
B. 6
C. 2
D. 3

Currying is Standard In OCaml

- Pretty much all functions are curried
  - Like the standard library map, fold, etc.
  - See /usr/local/ocaml/lib/ocaml on Grace
    - In particular, look at the file list.ml for standard list functions
    - Access these functions using **`List.<fn name>`**
    - E.g., **`List.hd`**, **`List.length`**, **`List.map`**

- OCaml works hard to make currying efficient
  - Because otherwise it would do a lot of useless allocation and destruction of closures
  - What are those, you ask? Let's see …

Closures

Remember our partial application example

```
let add = fun a -> fun b -> a + b;;
```

```
let addthree = add 3 in
addthree 4
```

- Let's evaluate it the expression (using substitution)

```
let addthree = add 3 in addthree 4
 let addthree = (fun a -> fun b -> a+b) 3 in …
 let addthree = (fun b -> 3+b) in addthree 4
 (fun b -> 3+b) 4
 3+4  7
```

Using Substitution “Remembered” the a is 3

```
let add = fun a -> fun b -> a + b;;
```

```
let addthree = add 3 in
addthree 4
```

- Let’s evaluate it the expression (using substitution)

```
let addthree = add 3 in addthree 4
 let addthree = (fun a -> fun b -> a+b) 3 in …
 let addthree = (fun b -> 3+b) in addthree 4
 (fun b -> 3+b) 4
 3+4  7
```

How to use a stack, not substitution?

- Substitution replaces the occurrence of the variable with the value it is bound to (e.g., at a call)
  - Like changing the code in place!

- In reality, we use a stack to remember variable-to-value mappings

```
let addthree = add 3 in
addthree 4
```

  - But: If calling **`add 3`** pushes **`3`** on the stack, what happens when the call returns? *How does **`addthree`** remember that it was constructed by a call with 3?*

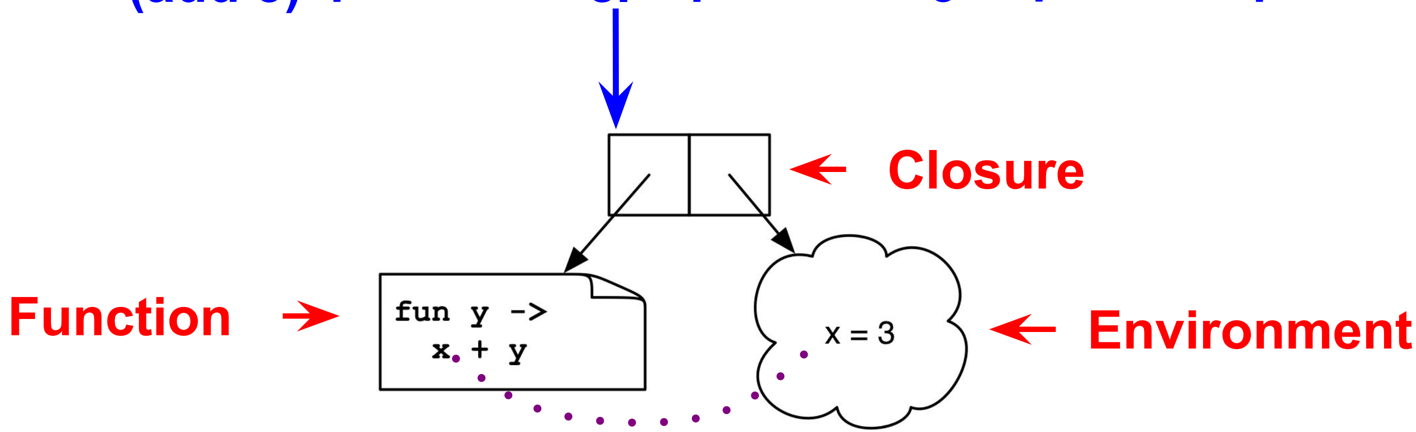
Closures "Remember"

- An environment is a mapping from variables to values
  - Like a stack frame
- A closure is a pair (f, e) consisting of function code f and an environment e
  - Environment "captures" active bindings, when closure is made
  - These include "free variables" – these are mentioned in f's body but are not its formal parameters
- When you invoke a closure, f is evaluated using e

Example 1

```
let add x = (fun y -> x + y)
```

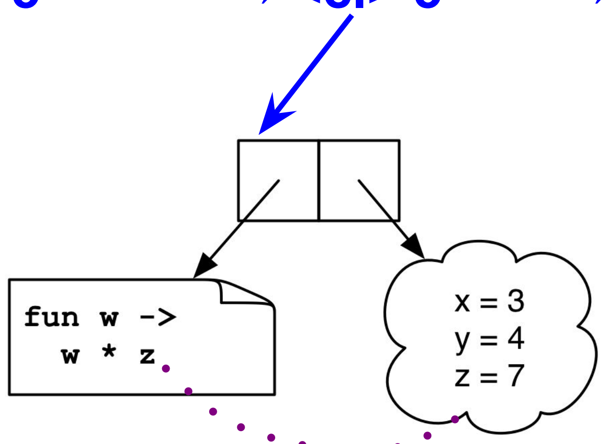
(add 3) 4 → <cl> 4 → 3 + 4 → 7



Example 2

```
let mult_sum (x, y) =
  let z = x + y in
  fun w -> w * z
```

(mult_sum (3, 4)) 5 → <cl> 5 → 5 * 7 → 35



Quiz 3: What is x?

```
let a = 1;;
let a = 0;;
let b = 10;;
let f () = a + b;;
let b = 5;;
let x = f ();;
```

A. **10**

B. **1**

C. **15**

D. Error - variable name conflicts

Quiz 3: What is x?

```
let a = 1;;
let a = 0;;
let b = 10;;
let f = fun () -> a + b;;
let b = 5;;
let x = f ();;
```

A. `10`

B. `1`

C. `15`

D. Error - variable name conflicts

Quiz 4: What is z?

```
let f x = fun y -> x - y in
let g = f 2 in
let x = 3 in
let z = g 4 in
z;;
```

A. `7`

B. `-2`

C. `-1`

D. Type Error – insufficient arguments

Quiz 4: What is z?

```
let f x = fun y -> x - y in
let g = f 2 in
let x = 3 in
let z = g 4 in
z;;
```

A. 7

B. -2

C. -1

D. Type Error – insufficient arguments

Quiz 5: What does this evaluate to?

```
let f x = x+1 in
let g = f in
g (fun i -> i+1) 1
```

A. Type Error

B. 1

C. 2

D. 3

Quiz 5: What does this evaluate to?

```
let f x = x+1 in
let g = f in
(g (fun i -> i+1)) 1
```

A. **Type Error** – Too many arguments passed to g (application is *left associative*)

B. 1

C. 2

D. 3

Scope

- **Dynamic scope**
  - The body of a function is evaluated in the current dynamic environment at the time the function is **called**, not the environment that existed at the time the function was defined
    - Now basically considered a mistake

- **Lexical scope** (aka Static scope)
  - The body of a function is evaluated in the old dynamic environment that existed at the time the function was **defined**, not the current environment when the function is called.
  - **This is implemented by closures**

Dynamic vs. Static Scope

```
let f a b = a * b in
let g = f 2 in
let a = 3 in
g 4
```

A. **8** Answer, if lexical/static scope
B. **12** Answer, if dynamic scope
C. **2**
D. **3**

Higher-Order Functions in C

- C supports function pointers

```
typedef int (*int_func)(int);
void app(int_func f, int *a, int n) {
  for (int i = 0; i < n; i++)
    a[i] = f(a[i]);
}
int add_one(int x) { return x + 1; }
int main() {
  int a[] = {5, 6, 7};
  app(add_one, a, 3);
}
```

Higher-Order Functions in C (cont.)

- C does not support closures
  - Since no nested functions allowed
  - Unbound symbols always in global scope

```
int y = 1;
void app(int(*f)(int), n) {
  return f(n);
}
int add_y(int x) {
  return x + y;
}
int main() {
  app(add_y, 2);
}
```

Higher-Order Functions in C (cont.)

- Cannot access non-local variables in C
- OCaml code

```
let add x y = x + y
```

- Equivalent code in C is illegal

```
int (* add(int x))(int) {
  return add_y;
}
int add_y(int y) {
  return x + y; /* error: x undefined */
}
```

Higher-Order Functions in C (cont.)

- OCaml code

```
let add x y = x + y
```

- Works if C supports nested functions
  - Not in ISO C, but in gcc; but not allowed to return them

```
int (* add(int x))(int) {
  int add_y(int y) {
    return x + y;
  }
  return add_y; }
```

  - Does not allocate closure, so x popped from stack and add_y will get garbage (potentially) when called

Java 8 Supports Lambda Expressions

- Ocaml's

```
fun (a, b) -> a + b
```

- Is like the following in Java 8

```
(a, b) -> a + b
```

- Java 8 supports closures, and variations on this syntax

Java 8 Example

```
public class Calculator {
   interface IntegerMath { int operation(int a, int b);  }
   public int operateBinary(int a, int b, IntegerMath op) {
      return op.operation(a, b);
   }
   public static void main(String... args) {
      Calculator myApp = new Calculator();
      IntegerMath addition = (a, b) -> a + b;
      IntegerMath subtraction = (a, b) -> a - b;
      System.out.println("40 + 2 = " +
         myApp.operateBinary(40, 2, addition));
      System.out.println("20 - 10 = " +
         myApp.operateBinary(20, 10, subtraction));
   }
}
```

Lambda expressions