Problem Set #4

Quantum Error Correction
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Due Tuesday, Apr. 9, 2024

Problem #1. Repetition of syndrome measurement in Shor error correction (40 points)

For Shor error correction and a distance 3 code, consider the following method of repeated syndrome measurement: Measure the error syndrome twice. If both syndrome measurements are the same, use that value. If the syndrome measurements differ but the first syndrome measured is 0 (corresponding to no error), deduce the trivial error. If the syndrome measurements differ but the first syndrome is non-zero, use the second syndrome to deduce the error.

a) (15 points) Show that this method of repeating the syndrome and deducing the error satisfies the ECCP for a code correcting 1 error.

b) (15 points) For the 7-qubit code, show that the ECRP is not satisfied by giving a combination of an error on the input state to the EC gadget and a fault during the gadget that cause the ECRP to fail.

c) (10 points) For the 5-qubit code, show that the ECRP is satisfied.

Problem #2. Teleportation of the controlled-phase gate (20 points)

Let U be the gate

$$U = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & i \end{pmatrix}. \tag{1}$$

a) (5 points) Find $UPU^\dagger$ for $P = X \otimes I$, $I \otimes X$, $Z \otimes I$, and $I \otimes Z$.

b) (15 points) Find a *two-qubit* ancilla state and a circuit involving two data qubits and the ancilla consisting of Clifford group gates, Pauli basis measurements, and classical feed-forward, such that the output of the circuit is $U|\psi\rangle$. ($|\psi\rangle$ is the input state of the two data qubits.)