Problem Set #5

Quantum Error Correction
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Due Tuesday, Apr. 23, 2024

Problem #1. Correctness with weaker versions of fault tolerance (30 points)

For this problem, we will consider a fault-tolerant protocol for a QECC capable of correcting t errors.

a) (15 points) Suppose we consider weaker versions of the ECRP and the GPP. The weak ECRP has a $2s$-filter on the right-hand-side for the output instead of an s-filter when there are s faults in the EC gadget, with $2s \leq t$. The weak GPP (for a single-block gate) has an $(r + 2s)$-filter instead of an $(r + s)$-filter for the output on the right-hand-side when the input passes an r-filter and there are s faults in the circuit, with $r + 2s \leq t$. The correctness conditions are unchanged. For a single-block gate exRec, how many faults can we allow within the exRec and still guarantee that the exRec is correct?

b) (15 points) Consider the weak version of the ECRP and a very weak version of the GPP, with a $(2r + 2s)$-filter for the output on the right-hand-side instead of a $(r + 2s)$-filter, for $2r + 2s \leq t$. With these properties, how many faults can we allow within the exRec and still guarantee that the exRec is correct?

Problem #2. Malignant sets of faults (30 points)

Recall that a set of faults is *malignant* for an extended rectangle if there exists an assignment of errors to those faults such that causes the correctness property to fail for that extended rectangle. For this problem, we will study malignant sets for an extended rectangle for a transversal Hadamard gate for the 7-qubit code. The error correction gadgets in the extended rectangle use Steane error correction and measure the phase errors first.

For each of the following sets, determine if it is a malignant set or not. If it is malignant, give example errors that cause correctness to fail and if it is not malignant, explain why not.

a) (5 points) CNOT gates for the 1st and 3rd qubits of the code block in the bit flip error correction part of the leading error correction step.

b) (5 points) CNOT gates for the 1st and 3rd qubits of the code block in the bit flip error correction part of the trailing error correction step.

c) (5 points) CNOT for the 1st qubit of the code block in the phase error correction part of the leading error correction step and the 3rd qubit of the transversal Hadamard gate.

d) (5 points) Measurement of the 1st qubit of the ancilla in the phase error correction part of the leading error correction step and the 3rd qubit of the transversal Hadamard gate.

e) (5 points) Measurement of the 3rd qubit of the ancilla in the phase error correction part of the leading error correction step and the 3rd qubit of the transversal Hadamard gate.

f) (5 points) The 2nd qubit of the state preparation (after checking) for the ancilla for phase error correction in the leading error correction step and the 5th qubit of the state preparation (after checking) for the ancilla for phase error correction in the leading error correction step.