CMSC 754 Quiz 1

This quiz is closed-book and closed-notes, but you may use one sheet of notes, front and back. You may use any algorithms or results given in class. The total point value is 50 points. Good luck!

Problem 1: (20 points)

(a) (4 points) Consider the triangulation of a simple polygon with n sides. How many diagonals and triangles are there?

(b) (6 points) A simple polygon has n *vertices*, of which m are *reflex vertices*, and of these, k are *scan reflex vertices*, where $n \geq m \geq k$. As an exact function of some or all of these quantities, what is the *minimum and maximum number of diagonals* that may be required to be added to decompose such a polygon into x-monotone polygons?

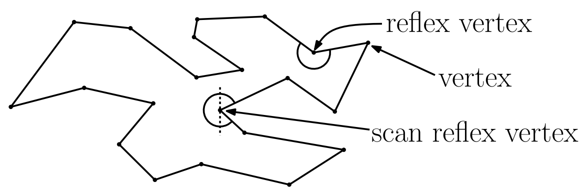


Figure 1: Simple polygon.

(c) (10 points - 5 points each) Recall that Chan's convex-hull algorithm uses multiple iterations, based on an estimate h^* of the actual size of the hull. Suppose that the algorithm were to be run just once, using just a *single choice* of h^*, where $1 \leq h^* \leq n$.

(i) What might go wrong if h^* was *extremely large* compared to the true value h?

(ii) What might go wrong if h^* was *extremely small* compared to the true value h?

Problem 2. (10 points) In each of these problems, give a test that determines whether the given points are in the specified configuration. Throughout, you may assume that the points are in *general position* and the only way to access information about points is by computing *orientations*. Briefly explain.

(a) (5 points) Given four points in the plane a, b, s, and t, test that the line segments $\overline{ab}$ and $\overline{st}$ do *not* intersect, but the infinite linear extension $\overleftrightarrow{ab}$ does intersect the line segment $\overline{st}$ (see Fig. 2(a)).

(b) (5 points) Given four points in the plane a, b, s, and t, test that the line segments $\overline{ab}$ and $\overline{st}$ do *not* intersect, but the ray directed from a through b does intersect the line segment $\overline{st}$ (see Fig. 2(b))

Problem 3. (10 points)

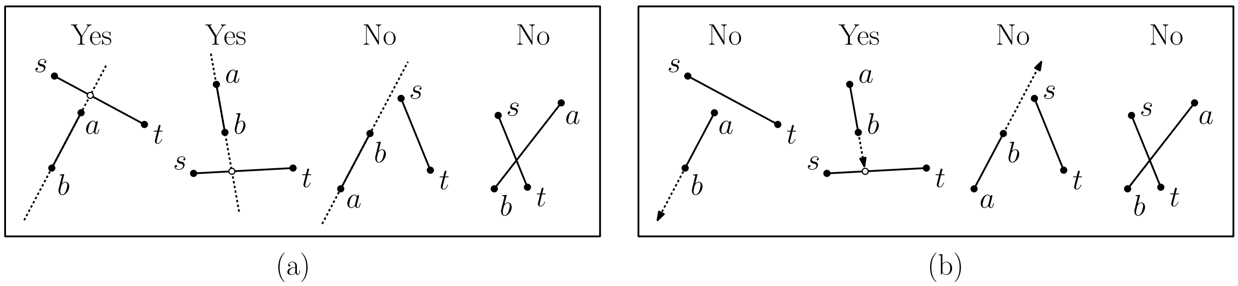


Figure 2: Testing configurations.

(a) (5 points) Given two x-monotone polygonal curves having n and m edges, respectively, prove that the number of intersections between them is at most $n+m-1$ (see Fig. 3(a)). You should assume that the curves are in general position, implying that there are no duplicate x-coordinates or parallel edges.

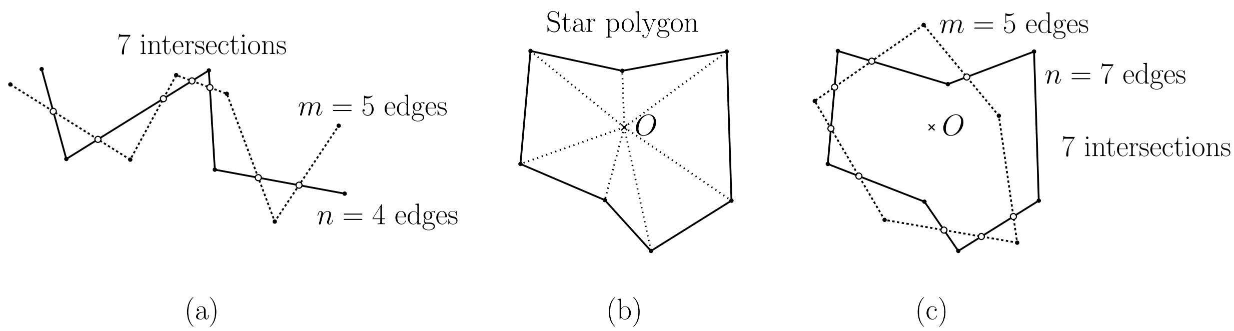
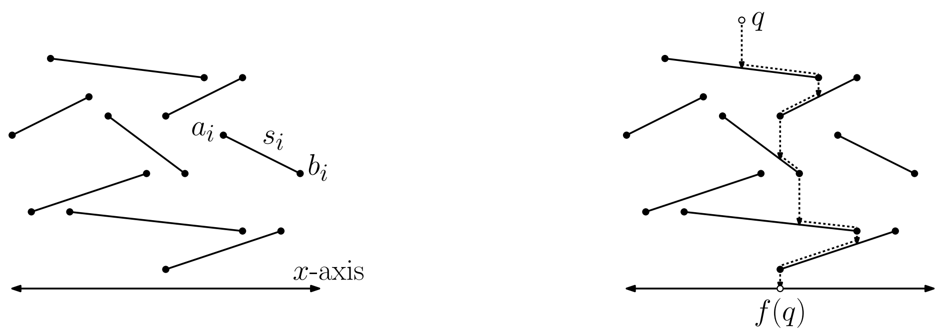


Figure 3: Number of intersections.

(b) (5 points) A simple polygon is a *star* if it contains the origin O in its interior, and for each of its vertices v, the line segment $\overline{Ov}$ lies entirely within the polygon (see Fig. 3(b)). Given two star polygons having n and m edges, respectively, prove that the maximum number of intersections between their boundaries is at most $n+m$ (see Fig. 3(c)). You may assume that $n+m$ is even.

Problem 4. (10 points) You are given a set $S = \{s_1, \ldots, s_n\}$ of n non-intersecting line segments in the plane all lying above the x-axis, where $s_i = \overline{a_i b_i}$. No line segment is horizontal or vertical (see Fig. 4(a)). Given any point $q \in \mathbb{R}^2$ that lies above all the segments, suppose that a drop of water falls at q and drips from one segment to the next until reaching the x-axis. Define $f(q)$ to be the x-coordinate of this point on the x-axis (see Fig. 4(b)).

Present an efficient algorithm, which, given S and q, computes $f(q)$. Explain your algorithm and derive its running time. (Hint: Use a top-down plane sweep. A high-level description suffices. Explain what the sweep-line status stores. Explain the key events and how they are processed. Aim for a running time of $O(n \log n)$.)



(a) (b)

Figure 4: Water drop problem.