CMSC 250 - Homework 1 - Summer 2006

Due Fri June 9th at the beginning of your discussion section.

You must write the solutions to the problems single-sided on your own lined paper, with all sheets stapled together, and with all answers written in sequential order or you will lose points. Please write your solutions in pencil.

1. Translate the following into propositional logic. Tell what the atomic propositions represent and how they are connected to make the global statement. Each statement must be made from at least two atomic propositions.

   (a) Either it is raining or someone left the shower on.

   (b) If it is foggy tonight, then either John will stay home or he will take a taxi.

   (c) John will sit, and he or George will wait.

   (d) I will go either by bus or by taxi.

   (e) If a football game between team A and team B ends in a tie, then neither team A won the game nor team B won the game.

   (f) If, and only if, the irrigation ditches are dug will the crops. survive; should the crops not survive, then the farmers will go bankrupt and leave.

   (g) If I am either tired or hungry, then I cannot study.

   (h) If John gets up and goes to school, he will be happy; and if he does not get up, he will not be happy.

   (i) Either to be or not to be.

2. Let c be "today is clear", r be "it is raining today", s be "it is snowing today" and y be "yesterday was cloudy". Translate the following into acceptable english:

   (a) $c \to \sim (r \wedge s)$

   (b) $y \leftrightarrow c$

   (c) $y \wedge (c \vee r)$

   (d) $(y \to r) \vee c$

   (e) $(c \to r) \wedge (\sim s \vee y)$

3. Construct the complete truth table of the following pairs of statements and determine if they are logically equivalent.

   (a) $a \vee b$, $b \vee a$

   (b) $a \wedge b$, $a \vee b$

   (c) $a \leftrightarrow b$, $b \to a$

   (d) $a \vee a$, a

   (e) $a \vee (b \wedge c)$, $(a \vee b) \wedge c$

   (f) $(p \to q) \wedge (p \to r)$, $p \to (q \wedge r)$

   (g) $p \leftrightarrow q$, $(p \to q) \wedge (q \to p)$

   (h) $(p \to q) \vee (p \to r)$, $p \to (q \vee r)$

4. Use truth tables to verify the distributive laws.

5. Use the equivalence rules in Theorem 1.1.1 to prove the following:

   (a) $p \wedge (\sim (\sim p \wedge p)) \equiv p$

   (b) $\sim (p \vee (\sim p \wedge q)) \equiv (\sim p \wedge \sim q)$

6. Use the equivalence rules in Theorem 1.1.1 and the definition of implication and the definition of biconditional to prove the following:

   (a) $(p \to q) \equiv (\sim q \to \sim p)$

   (b) $(p \leftrightarrow q) \equiv (p \wedge q) \vee (\sim p \wedge \sim q)$

   (c) $(\sim p \leftrightarrow q) \equiv (p \leftrightarrow \sim q)$