CMSC 250 - Homework 3 - Summer 2006

Due Fri June 23rd at the beginning of your discussion section.

You must write the solutions to the problems single-sided on your own lined paper, with all sheets stapled together, and with all answers written in sequential order or you will lose points. Please write your solutions in pencil.

1. Given the following arguments in english, if it is valid, convert it to predicate logic and prove it. If it is invalid, draw an Euler diagram to support your finding.

   (a) No human beings are quadrupeds. All men are human beings. Therefore, no men are quadrupeds. (H(x), M(x), Q(x))

   (b) Every member of the committee is wealthy and a Republican. Some committee members are old. Therefore, there are some old Republicans. (C(x), W(x), R(x), O(x))

   (c) Some rational numbers are not integers. Pi is not a rational number. Therefore, Pi is not an integer. (Q(x), Z(x), Pi)

   (d) No positive integers are irrational numbers. No positive integers are negative integers. Therefore, no irrational numbers are negative integers. (P(x), R(x), N(x))

   (e) All rational numbers are real numbers. Some rationals are integers. Therefore, some real numbers are integers. (Q(x), R(x), Z(x))

   (f) No Republican or Democrat is a Socialist. Noam Chomsky is a Socialist. Therefore, Noam Chomsky is not a Republican and Noam Chomsky is not a Democrat. (R(x), D(x), S(x), Chomsky)

   (g) Some dolphins give kisses. Some dolphins leap high in the air. Therefore, some dolphins can do both tricks. (D(x), K(x), L(x))

   (h) No pusher is an addict. Some addicts are people with records. Therefore, some people with a record are not pushers. (P(x), A(x), R(x))

   (i) All dolphins swim in the ocean. Flipper swims in the ocean. Therefore, Flipper is a dolphin. (D(x), O(x), Flipper)

2. Use the rules of inference you were given to prove the following:

(a)

P1	$\forall x \in D, P(x) \wedge (Q(x) \vee R(x))$
P2	$\forall x \in D, S(x) \rightarrow \sim P(x)$
P3	$\forall x \in D, \sim Q(x) \rightarrow S(x)$
Therefore	$\forall x \in D, Q(x)$

(b)

P1	$\forall x \in D, B(x) \rightarrow L(x)$
P2	$\forall x \in D, C(x) \rightarrow \sim D(x)$
P3	$\forall x \in D, L(x) \rightarrow D(x)$
Therefore	$\forall x \in D, B(x) \rightarrow \sim C(x)$

(c)

P1	$\forall x \in D, B(x) \vee A(x)$
P2	$\forall x \in D, \sim B(x) \vee A(x)$
P3	$\exists x \in D, \sim B(x) \vee \sim C(x)$
Therefore	$\forall x \in D, A(x)$

(d)

P1	$\forall x \in D, F(x) \rightarrow (\sim M(x) \vee N(x))$
P2	$\forall y \in D, \sim (M(y) \vee Y(y)) \rightarrow N(y)$
P3	$\exists z \in D, \sim Y(z) \wedge Z(z)$
P4	$\forall x \in D, \sim Z(x) \vee \sim N(x)$
Therefore	$\exists m \in D, \sim F(m) \wedge Z(m)$

(e)

P1	$\sim (D(a) \vee \sim C(a))$ with $a \in P$
P2	$\forall x \in P, A(x) \vee (B(x) \wedge C(x))$
P3	$\forall x \in P, B(y) \rightarrow \sim C(y)$
P4	$\forall z \in P, A(z) \rightarrow (D(z) \vee E(z))$
Therefore	$\exists w \in P, E(w) \wedge A(w)$

(f)

P1	$P(d)$ with $d \in D$
P2	$\forall x \in D, (\sim Q(x) \vee P(x)) \rightarrow (S(x) \wedge T(x))$
P3	$\exists x \in D, P(x) \rightarrow (T(x) \vee \sim E(x))$
Therefore	$T(d) \vee \sim E(d)$

3. Prove that there is a perfect square that can be written as a sum of two other perfect squares.

4. Prove that the product of four consecutive integers is one less than a perfect square.

5. This problem has to do with expressions of the form $(x - r)(x - s)$ being multiplied out:

   (a) What can be said about the coefficients of the polynomial obtained by multiplying out $(x - r)(x - s)$ when both r and s are odd integers? when both r and s are both even? when one is odd and the other is even?

   (b) It follows from part (a) that $x^2 - 1253 + 255$ cannot be written as a product of two polynomials with integer coefficients. Explain why this is so.

6. Prove that for all real numbers c, if c is a root of a polynomial with rational coefficients, then c is a root of a polynomial with integer coefficients.

7. Prove that for all integers a, b and c, if $a|b$ and $a|c$ then $a|(b + c)$.

8. Suppose that in standard factored form $a = p_1^{e_1} p_2^{e_2} \dots p_k^{e_k}$, where k is a positive integer; $p_1, p_2, \dots, p_k$ are prime numbers; and $e_1, e_2, \dots, e_k$ are positive integers.

   (a) What is the standard factored form of a^3?

   (b) Find the least positive integer k such that $2^4.3^5.7.11^2.k$ is a perfect cube.