

CMSC 250 - Homework 4 - Summer 2006

Due Fri June 30th at the beginning of your discussion section.

You must write the solutions to the problems single-sided on your own lined paper, with all sheets stapled together, and with all answers written in sequential order or you will lose points. Please write your solutions in pencil.

1. For any integer n , $n^2 - 2$ is not divisible by 4.
2. For all prime numbers a , b , and c , $a^2 + b^2 \neq c^2$.
3. If a , b and c are integers and $a^2 + b^2 = c^2$, then at least one of a or b is even.
4. Prove that for any integer a , $9 \nmid (a^2 - 3)$.
5. There is no greatest integer.
6. Any two consecutive integers have opposite parity.
7. The product of any 4 consecutive integers is divisible by 8.
8. Every prime number except 2 and 3 has the form $6q + 1$ or $6q + 5$ for some integer q .
9. Prove that if $p_1, p_2, \dots, p_n$, are distinct prime numbers with $p_1 = 2$ and $n > 1$, then $p_1 p_2 \dots p_n + 1$ can be written in the form $4k+3$ for some integer k .
10. $a \not\equiv 5 \pmod{5} \iff 5 \mid (a^4 - 1)$
11. $\sqrt{5}$ is irrational.
12. For all primes p , $\sqrt{p}$ is irrational.
13. A positive integer is divisible by 3 if and only if the sum of its digits is divisible by 3.
14. For any positive integer n , $1 + 2 + \dots + n = n(n + 1)/2$.
15. For all positive integers n , $1^2 + 2^2 + \dots + n^2 = (n)(n + 1)(2n + 1)/6$.
16. For all positive integers n , $2 + 4 + \dots + 2n = (n^2 + n)$.
17. For all integers n , $1 + 2 + 2^2 + \dots + 2^n = (2^{n+1} - 1)$.