

CMSC 250 - Homework 5 - Summer 2006

Due Fri July 7th at the beginning of your discussion section.

You must write the solutions to the problems single-sided on your own lined paper, with all sheets stapled together, and with all answers written in sequential order or you will lose points. Please write your solutions in pencil.

1. Show that if $a, b > 0$, then $(a + b)^n \geq a^n + b^n$ for any positive integer n .
2. Suppose that d_n is defined as follows:

$$\begin{aligned}d_1 &= \frac{9}{10}, \\d_2 &= \frac{10}{11} \\d_k &= d_{k-1} \cdot d_{k-2}, \text{ for all integers } k \geq 3.\end{aligned}$$

Prove that $0 < d_n \leq 1$ for all integers $n \geq 0$.

3. Define a sequence $a_1, a_2, a_3, \dots$ as follows: $a_1 = 1$, $a_2 = 3$, $a_k = a_{k-1} + a_{k-2}$ for all integers $k \geq 3$. (This sequence is known as the Lucas sequence.) Use strong mathematical induction to prove that $a_n \leq (\frac{7}{4})^n$ for all integers $n \geq 1$.
4. Use strong mathematical induction to prove that for any integer $n \geq 2$, if n is even, then any sum of n odd integers is even, and if n is odd, then any sum of n odd integers is odd.
5. Show that the following statements about sets A and B are equivalent to one another:
 - (a) $A \subseteq B$
 - (b) $A \cap B = A$
 - (c) $A \cup B = B$
6. Show that $(A \cap B \cap C \cap X^C) \cup (A^C \cap C) \cup (B^C \cap C) \cup (C \cap X) = C$.
7. Show the following properties of the cartesian product:
 - (a) $A \times \emptyset = \emptyset$
 - (b) $(A \times B) \cap (C \times D) = (A \cap C) \times (B \cap D)$
 - (c) $(A \times B)^C = (A^C \times B^C) \cup (A^C \times B) \cup (A \times B^C)$