# CMSC 250 - Problem Set - Summer 2006

**This is a collection of problems of the same type and difficulty of the problems you will find in the midterm. Think of it as a homework you don't have to turn in. You should be able to do each problem in about 10 minutes.**

1. Use induction to prove each of the following equations:
   (a) $1 + 3 + 5 + \cdots + (2n - 1) = n^2$
   (b) $5 + 9 + 11 + \cdots + (2n + 3) = n^2 + 4n$
   (c) $3 + 7 + 11 + \cdots + (4n - 1) = n(2n + 1)$
   (d) $2 + 6 + 10 + \cdots + (4n - 2) = 2n^2$
   (e) $1 + 5 + 9 + \cdots + (4n + 1) = (n + 1)(2n + 1)$
   (f) $2 + 8 + 24 + \cdots + (n2^n) = (n - 1)2^{n+1} + 2$
   (g) $2 + 6 + 12 + \cdots + (n^2 - n) = n(n^2 - 1)/3$
2. Let $\text{sum}(n) = 1 + 2 + \cdots + n$ for all natural numbers $n$. Give and induction proof to show that the following equation is true for all natural numbers $m$ and $n$: $\text{sum}(m + n) = \text{sum}(m) + \text{sum}(n) + mn$.
3. Show by induction that: $\forall n \in Z^{\geq 1}, 7 | 2^{3n} - 1$.
4. Show by induction that: $\forall n \in Z^{\geq 1}, n^2 \geq 2n + 3$.
5. Show by induction that if $|A| = n$ then $|P(A)| = 2^n$, for all $n \geq 0$.
6. Let $e_0 = 1$, $e_1 = 2$, $e_2 = 3$ and $e_k = 2e_{k-1} + 4e_{k-1} + e_{k-3}$ for $k \geq 3$. Show that $e_n \leq 7^n$ for all integers $n \geq 0$.
7. Prove each statement is true or find a specific counterexample. Assume all sets are subsets of a universal set $U$ and that $A$,$B$, and $C$ are arbitrary sets.
   - $A - B \neq B - A$
   - $(A \cup (B \cap C)) \subseteq (A \cup B \cup C)$
   - $(A - B) - C \subseteq (A - C)$
   - $(A \times B) \times C = A \times (B \times C)$
   - $A - (B \cup (B \cap C)) = (A - (B \cup C)) \cup (A \cap B \cap C)$
8. Assume that $R$, $S$ and $T$ are arbitrary relations. Show that:
   (a) $R \circ (S \circ T) = (R \circ S) \circ T$
   (b) $R \circ (S \cup T) = (R \circ S) \cup (R \circ T)$
9. Show that:
   (a) If $R$ is symmetric, then $R^n$ is symmetric.
   (b) If $R$ is transitive, then $R^n$ is transitive.
   (c) If $R$ is reflexive, then $s(R)$ and $t(R)$ are reflexive.
   (d) If $R$ is symmetric, then $r(R)$ and $t(R)$ are symmetric.
   (e) If $R$ is transitive, then $r(R)$ is transitive.

10. Show the double closure properties:

    (a) $rt(R) = tr(R)$

    (b) $rs(R) = sr(R)$

    (c) $st(R) \subseteq ts(R)$

11. Show that if $R$ is transitive and irreflexive then $R$ is asymmetric.