

Due at the start of class, Friday, June 23.

Let $E(X)$ be the expected value of X , $\text{Var}(X)$ be the variance of X , and $\sigma(X)$ be the standard deviation of X .

Problem 1. Consider a coin that has $2/3$ chance of getting heads. Professor Rosencrantz flips one such coin, and Professor Guildenstern flips two such coins.

- (a) What is the probability that Professor Rosencrantz obtains more heads than Professor Guildenstern obtains tails?
- (b) What is the probability that Professor Rosencrantz obtains at least as many heads as Professor Guildenstern obtains tails?

Problem 2. A carnival game consists of three dice in a cage. A player can bet a dollar on any of the numbers 1 through 6. The cage is shaken, and the payoff is as follows. If the player's number doesn't appear on any of the dice, he loses his dollar. Otherwise if his number appears on exactly k of the three dice, for $k = 1, 2, 3$, he keeps his dollar and wins k more dollars.

- (a) What is his expected gain from playing the carnival game once?
- (b) What is the variance from playing the carnival game once?
- (c) What is the standard deviation from playing the carnival game once?
- (d) What are his expected gain, variance, and standard deviation from playing the carnival game n times?

Problem 3. Consider the bubble-sort algorithm, for an input $(a \ b \ c)$ of the three integer values 1, 2, and 3 in any order, where all input orders are equally likely (uniform distribution).

- (a) Let M be the number of exchanges. What are $E(M)$, $\text{Var}(M)$, and $\sigma(M)$?
- (b) Let x be the number of comparisons. What are $E(M)$, $\text{Var}(M)$, and $\sigma(M)$?
- (c) What percentage of the sample space is within a standard deviation of average.

Problem 4. Again consider the bubble-sort algorithm, for an input $(a \ b \ c)$ of the three integer values 1, 2, and 3 in any order. But now suppose the input orders are *not* equally likely; instead, there is a $1/5$ chance of $a = 1$ being the first item in the input (the $1/5$ distributed equally among those inputs) and $4/5$ of a not being 1 (again the $4/5$ shared equally among all these inputs).

- (a) Let M be the number of exchanges. What are $E(M)$, $\text{Var}(M)$, and $\sigma(M)$?
- (b) Let x be the number of data comparisons made. What are $E(M)$, $\text{Var}(M)$, and $\sigma(M)$?
- (c) What percentage of the sample space is within a standard deviation of average.