
Due on Wednesday 23

Problem 1: Max-flow with forward edges only. From the book: Exercise 7.11.

Problem 2: Escape problem. From the book: Exercise 7.14.

Problem 3: Zero cycle. From the book: Exercise 8.17.

Problem 4: In the *Degree Δ Spanning Tree Problem* we are given a graph $G = (V, E)$. We are asked if there is a spanning tree of G with degree at most Δ . Note that Δ is not part of the input, but rather part of the problem definition. Prove that Degree 2 Spanning Tree is NP-complete. Prove that Degree Δ Spanning Tree is NP-complete for all Δ .

Problem 5: Cycle cover. Exercise 8.41.

Problem (extra credit): Consider the following *Network Design Problem*. Let $G = (V, E)$ be an undirected graph representing a computer network. We would like to set up a virtual private network (VPN) for a subset of the computers $X \subset V$. If the graph induced¹ by X is connected there is no problem setting up the VPN. However, if the graph is not connected additional intermediate routers must be used so that computers in X can communicate with each other. These additional routes can be rented by paying a fixed fee. Since the fee is rather steep we would like to minimize their number of intermediate routers.

Here is a cleaner formulation of the problem we want to solve. Given a graph $G = (V, E)$, a set of terminals X and a number k , we want to know if there is a set $S \subseteq V$ of size at most k such that $G[X \cup S]$ is connected. Prove that this problem is NP-complete.

¹The graph induced by X is defined as $G[X] = (X, E[X])$, where $E[X] = \{(u, v) \in E \mid u, v \in X\}$.