

You must work alone on your homework, and homework must be **written legibly**, single-sided on your own lined paper, **or typed**, with the answers clearly labeled and in the sequential order as assigned. You must write your name and university ID number in the upper right-hand corner of your homework. Staple all pages together and be sure that your name appears on every sheet.

1. (3 points) Write your name clearly on each page. Write the time and place of the first midterm.
2. (12 points) Use the definitions of even, odd, prime and composite to justify each of your answers. Assume that r and s are particular integers.
 - (a) Is $4rs$ even?
 - (b) Is $6r + 4s^2 + 3$ odd?
 - (c) If r and s are both positive, is $r^2 + 2rs + s^2$ composite?
3. (10 points) Write a quantified expression for and then prove the following statement: There are distinct integers m and n such that $\frac{1}{m} + \frac{1}{n}$ is an integer.
4. (25 points) Determine whether the property is true for all integers, true for no integers, or true for some integers and false for other integers. Justify your answers.
 - (a) (10 points) $3n^2 - 4n + 1$ is prime
 - (b) (10 points) The average of any two odd integers is odd.
 - (c) (5 points) Write a quantified expression for part (b).
5. (20 points) Follow the directions given for writing proofs of universal statements and prove the following:
 - (a) For all integers n , if n is odd then n^2 is odd.
 - (b) If n is any odd integer, then $(-1)^n = -1$.
6. (30 points) Determine whether the statement is true or false. Justify your answer with a proof or a counterexample, as appropriate.
 - (a) The difference of the squares of any two consecutive integers is odd.
 - (b) If m and n are perfect squares, then $m + n + 2\sqrt{mn}$ is also a perfect square. (Definition: An integer n is called a perfect square if, and only if, $n = k^2$ for some integer k .)
7. **(No points will be awarded for this assignment unless this is done)** Sign your name to the following honor code statement: "I pledge on my honor that I have not given or received any unauthorized assistance on this assignment".