CMSC 250 – Jerry Alan Fails
Due: Friday, July 20, 2007

# HW #12

Student: ____________________
ID #: ____________________

You must work alone on your homework, and homework must be **written legibly**, single-sided on your own lined paper, **or typed**, with the answers clearly labeled and in the sequential order as assigned. You must write your name and university ID number in the upper right-hand corner of your homework. Staple all pages together and be sure that your name appears on every sheet.

1. (-10 points if wrong) Write your name clearly on each page. Write the time and place of the Final Exam.

2. (20 points) Answer each of the following:

   (a) How many integers from 1 through 100 must you pick in order to be sure of getting one that is divisible by 5?
   (b) A certain college class has 40 students. All the students in the class are known to be from 17 through 34 years of age. You want to make a bet that the class contains at least x students of the same age. How large can you make x and be sure to win your bet?
   (c) A group of 15 executives are to share 5 secretaries. Each executive is assigned exactly one secretary, and no secretary is assigned to more than 4 executives. Show that at least 3 secretaries are assigned to 3 or more executives.
   (d) Given a set of 52 distinct integers, show that there must be 2 whose sum or difference is divisible by 100.

3. (9 points) If $f: X \to Y$ and $g: Y \to Z$ are functions and $g \circ f$ is onto, must both $f$ and $g$ be onto? Prove or give a counterexample.

4. (9 points) If $f: X \to Y$ and $g: Y \to Z$ are functions and $g \circ f$ is onto, must $g$ be onto? Prove or give a counterexample.

5. (10 points) Suppose $f: X \to Y$ and $g: Y \to Z$ are both one-to-one and onto. Prove that $(g \circ f)^{-1}$ exists and that $(g \circ f)^{-1} = f^{1} \circ g^{-1}$.

6. (8 points) Let $\boldsymbol{O}$ be the set of all odd integers. Prove that $\boldsymbol{O}$ has the same cardinality as $2\mathbf{Z}$ (the set of all even integers).

7. (12 points) Let $A = \{4,5,6\}$ and $B = \{5,6,7\}$ and define binary relations $R$, $S$, and $T$ from $A$ to $B$ as follows:

   For all $(x,y) \in A \times B$, $(x,y) \in R \Leftrightarrow x \geq y$.
   For all $(x,y) \in A \times B$, $xRy \Leftrightarrow 2 \mid (x - y)$.
   $T = \{(4,7),(6,5),(6,7)\}$.

   (a) Draw arrow diagrams for $R$, $S$, and $T$.
   (b) Indicate whether any of the relations $R$, $S$, and $T$ are functions.

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8. (8 points) Define a relation $R$ from $\mathbf{R}$ to $\mathbf{R}$ as follows:

   For all $(x,y) \in \mathbf{R} \times \mathbf{R}$, $xRy \Leftrightarrow y = \lfloor x \rfloor$.

   Draw the graphs of $R$ and $R^{-1}$ in the Cartesian plane.

9. (12 points) Determine whether each of the following binary relations are reflexive, symmetric, transitive, or none of these. Justify your answers.

   (a) $F$ is the congruence modulo 5 relation on $\mathbf{Z}$: For all $m,n \in \mathbf{Z}$, $mFn \Leftrightarrow 5 \mid (m-n)$.
   (b) Let $A$ be a nonempty set and $P(A)$ the power set of $A$. Define the "not equal to" relation $R$ on $P(A)$ as follows: For all $X,Y \in P(A)$, $XRY \Leftrightarrow X \neq Y$.

10. (12 points) Let $A$ be a set with eight elements.

    (a) How many binary relations are there on $A$?
    (b) How many binary relations on $A$ are reflexive?
    (c) How many binary relations on $A$ are symmetric?
    (d) How many binary relations on $A$ are both reflexive and symmetric?

11. **(No points will be awarded for this assignment unless this is done)** Sign your name to the following honor code statement: "I pledge on my honor that I have not given or received any unauthorized assistance on this assignment".