You must work alone on your homework, and homework must be written legibly, single-sided on your own lined paper, or typed, with the answers clearly labeled and in the sequential order as assigned. You must write your name and university ID number in the upper right-hand corner of your homework. Staple all pages together and be sure that your name appears on every sheet.

1. (-10 points if wrong) Write your name clearly on each page. Write the time and place of the Final Exam.

2. (12 points) For each of the following $R$ is an equivalence relation on the set $A$. Find the distinct equivalence classes of $R$.
   
   (a) $A = \{a,b,c,d\}$. $R$ is defined on $A$ as follows:
   
   $R = \{(a,a), (b,b), (b,d), (c,c), (d,b), (d,d)\}$.

   (b) $A = \{-4,-3,-2,-1,0,1,2,3,4,5\}$. $R$ is defined on $A$ as follows:
   
   For all $x,y \in A$, $xRy \iff 3 \mid (x-y)$.

   (c) $A = \{(1,3),(2,4),(-4,-8),(3,9),(1,5),(3,6)\}$. $R$ is defined on $A$ as follows:
   
   For all $(a,b),(c,d) \in A$, $(a,b)R(c,d) \iff ad = bc$

3. (10 points) Answer the following:
   
   (a) Let $R$ be the relation of congruence modulo 3. Which of the following equivalence classes are equal?
   
   $[7],[-4],[-6],[17],[4],[27],[19]$

   (b) Let $R$ be the relation of congruence modulo 7. Which of the following equivalence classes are equal?
   
   $[35],[3],[-7],[12],[0],[-2],[17]$

4. (10 points) $F$ is the relation defined on $\mathbb{Z}$ as follows: For all $m,n \in \mathbb{Z}$, $mFn \iff 4 \mid (m-n)$.
   
   (a) Prove that $F$ is an equivalence relation.

   (b) Describe the distinct equivalence classes of $F$.

5. (6 points) Consider the set $A = \{12,24,48,3,9\}$ ordered by the “divides” relation. Is $A$ totally ordered with respect to the relation? Justify your answer.

6. (10 points) Suppose a relation $R$ on a set is reflexive, symmetric, transitive, and antisymmetric. What can you conclude about $R$? Prove your answer.
7. (10 points) Suppose that $A$ is a totally ordered set. Use mathematical induction to prove that for any integer $n \geq 1$, every subset of $A$ with $n$ elements has both a least element and a greatest element.

8. (No points will be awarded for this assignment unless this is done) Sign your name to the following honor code statement: “I pledge on my honor that I have not given or received any unauthorized assistance on this assignment”.