CMSC 250 – Jerry Alan Fails
Due: Tuesday, July 24, 2007

HW #13

Student: ____________________
ID #: ____________________

You must work alone on your homework, and homework must be **written legibly**, single-sided on your own lined paper, **or typed**, with the answers clearly labeled and in the sequential order as assigned. You must write your name and university ID number in the upper right-hand corner of your homework. Staple all pages together and be sure that your name appears on every sheet.

1. (-10 points if wrong) Write your name clearly on each page. Write the time and place of the Final Exam.

2. (12 points) For each of the following R is an equivalence relation on the set A. Find the distinct equivalence classes of R.

   (a) $A = \{a,b,c,d\}$. R is defined on A as follows:
   $R = \{(a,a), (b,b), (b,d), (c,c), (d,b), (d,d)\}$.

   (b) $A = \{-4,-3,-2,-1,0,1,2,3,4,5\}$. R is defined on A as follows:
   For all $x,y \in A$, $xRy \Leftrightarrow 3 \mid (x - y)$.

   (c) $A = \{(1,3),(2,4),(-4,-8),(3,9),(1,5),(3,6)\}$. R is defined on A as follows:
   For all $(a,b),(c,d) \in A$, $(a,b)R(c,d) \Leftrightarrow ad = bc$

3. (10 points) Answer the following:

   (a) Let R be the relation of congruence modulo 3. Which of the following equivalence classes are equal?

   [7],[-4],[-6],[17],[4],[27],[19]

   (b) Let R be the relation of congruence modulo 7. Which of the following equivalence classes are equal?

   [35],[3],[-7],[12],[0],[-2],[17]

4. (10 points) F is the relation defined on $\mathbf{Z}$ as follows: For all $m,n \in \mathbf{Z}$, $mFn \Leftrightarrow 4 \mid (m - n)$.

   (a) Prove that F is an equivalence relation.
   (b) Describe the distinct equivalence classes of F.

5. (6 points) Consider the set $A = \{12,24,48,3,9\}$ ordered by the "divides" relation. Is A totally ordered with respect to the relation? Justify your answer.

6. (10 points) Suppose a relation R on a set is reflexive, symmetric, transitive, and antisymmetric. What can you conclude about R? Prove your answer.

HW #13

Student: ___________________
ID #: ___________________

7. (10 points) Suppose that A is a totally ordered set. Use mathematical induction to prove that for any integer $n \geq 1$, every subset of A with n elements has both a least element and a greatest element.

8. **(No points will be awarded for this assignment unless this is done)** <u>Sign</u> your name to the following honor code statement: "I pledge on my honor that I have not given or received any unauthorized assistance on this assignment".