CMSC 250
Discrete Structures

Exam #1 Review

Symbols & Definitions for Compound Statements

p	q	$p \wedge q$	$p \vee q$	$p \oplus q$	$p \rightarrow q$	$p \leftrightarrow q$
1	1					
1	0					
0	1					
0	0					

Symbols & Definitions for Compound Statements

p	q	$p \wedge q$	$p \vee q$	$p \oplus q$	$p \rightarrow q$	$p \leftrightarrow q$
1	1	1	1	0	1	1
1	0	0	1	1	0	0
0	1	0	1	1	1	0
0	0	0	0	0	1	1

Prove the following …

P1	$(p \wedge q) \rightarrow (\sim q \vee r)$
P2	$((p \vee \sim s) \wedge q \wedge \sim r \wedge s) \vee q$
P3	$(r \vee \sim s) \rightarrow \sim q$
$\therefore$	$\sim p$

$$\mathbf{P1} \wedge \mathbf{P2} \wedge \mathbf{P3} \rightarrow \sim p$$

(((P ^ Q) -> (~Q v R)) ^ (((P v ~S) ^ Q ^ ~R ^ S) v Q) ^ ((R v ~S) -> ~Q)) -> ~P

Answer to previous proof

P1	$(p \wedge q) \rightarrow (\sim q \vee r)$
P2	$((p \vee \sim s) \wedge q \wedge \sim r \wedge s) \vee q$
P3	$(r \vee \sim s) \rightarrow \sim q$
$\therefore$	$\sim p$

#	STATEMENT	JUSTIFICATION	LINES USED
1	$((p \vee \sim s) \wedge \sim r \wedge s \wedge q) \vee q$	Commutative, associative laws	P2
2	q	Absorption law	1
3	$\sim (r \vee \sim s)$	Double negative, modus tolens	P3
4	$\sim r \wedge s$	DeMorgan's law	3
5	$\sim r$	Conjunctive simplification	4
6	$q \wedge \sim r$	Conjunctive addition	2 & 5
7	$\sim (\sim q \vee r) \rightarrow \sim (p \wedge q)$	Contrapositive	P1
8	$(q \wedge \sim r) \rightarrow (\sim p \vee \sim q)$	DeMorgan's law, double negative	7
9	$\sim p \vee \sim q$	Modus ponens	6 & 8
$\therefore$	$\sim p$	Disjunctive syllogism	2 & 9

Can you prove the following …

P1	$(p \wedge q) \rightarrow (\sim q \vee r)$
P2	q
$\therefore$	$\sim p$

$$\textbf{P1} \wedge \textbf{P2} \rightarrow \sim p$$

Chapter 1

- Statements, arguments (valid/invalid)
- Translation of statements
- Truth tables – special results
- Converse, inverse, contrapositive
- Logical Equivalences
- Inference rules
- Implication – biconditional
- DeMorgan's law
- Proofs (including conditional worlds)
- Circuits

Informal to Formal

- Domain
  - A = set of all food
  - P = set of all people
- Predicates
  - E(x,y) = "x eats y"; D(x) "x is a dessert"
- Examples
  - Someone eats beets
    - $\exists p \in P, \forall a \in A, (a = \text{"beet"}) \rightarrow E(p,x)$
  - At least three people eat beets
    - $\exists p,q,r \in P, \forall a \in A,$
      $(a=\text{"beet"}) \rightarrow E(p,x) \wedge E(q,x) \wedge E(r,x) \wedge (p \neq q) \wedge (p \neq r) \wedge (q \neq r)$
  - Not everyone eats every dessert.
    - $\exists p \in P, \exists a \in A, D(a) \wedge \sim E(p,a)$

Formal to Informal

- Domain
  - D = set of all students at UMD
- Predicates
  - C(s) := "s is a CS student"
  - E(s) := "s is an engineering student"
  - P(s) := "s eats pizza"
  - F(s) := "s drinks caffeine"
- Examples
  - $\forall s \in D, [C(s) \rightarrow F(s)]$
    - Every CS student drinks caffeine.
  - $\exists s \in D, [C(s) \wedge F(s) \wedge \sim P(s)]$
    - Some CS students drink caffeine but do not eat pizza.
  - $\forall s \in D, [(C(s) \vee E(s)) \rightarrow (P(s) \wedge F(s))]$
    - If a student is in CS or Engineering, then they eat pizza and drink caffeine.

Formal to Informal

7. (18 points) Let D be the set of all students at the University of Maryland. Let $C(s)$ be "s is a computer science student", let $E(s)$ be "s is an engineering student", let $P(s)$ be "s eats pizza", and $F(s)$ be "s drinks caffeine". Express each of the following statements using quantifiers, variables, and the predicates $C(s)$, $E(s)$, $P(s)$, and $F(s)$. Then write its negation formally and express it informally.

(a) Every computer science student drinks caffeine.

$\forall s \in D, [C(s) \rightarrow F(s)]$

Negation: $\exists s \in D, [C(s) \wedge \sim F(s)]$
Informal negation: There is a student who is in computer science but does not drink caffeine.

(b) Some computer science students drink caffeine but do not eat pizza.

$\exists s \in D, [C(s) \wedge F(s) \wedge \sim P(s)]$

Negation: $\sim (\exists s \in D, [C(s) \wedge F(s) \wedge \sim P(s)])$
There aren't any students in computer science who drink caffeine, but do not eat pizza.
Or alternatively, $\forall s \in D, [\sim C(s) \vee \sim F(s) \vee P(s)]$
All students eat pizza or are not in computer science or don't drink caffeine.

(c) If a student is in computer science or engineering then they eat pizza and drink caffeine.

$\forall s \in D, [(C(s) \vee E(s)) \rightarrow (P(s) \wedge F(s))]$

Negation: $\exists s \in D, [C(s) \vee E(s) \vee \sim P(s) \vee \sim F(s)]$
Informal negation: There is a student who is not in computer science or does not drink caffeine.

Quiz #2 Solution

P1	$\forall x \in D, [A(x) \wedge B(x)] \rightarrow [M(x) \vee N(x)]$
P2	$\exists y \in D, A(y) \wedge \sim N(y)$
$\therefore$	$\exists z \in D, M(z) \vee \sim B(z)$

#	Statement	Justification	Lines Used
0	Assume p and q are in D		
1	$A(p) \wedge \sim N(p)$	Existential instantiation	P2
2	$[A(p) \wedge B(p)] \rightarrow [M(p) \vee N(p)]$	Universal instantiation	P1
3	$\sim [A(p) \wedge B(p)] \vee [M(p) \vee N(p)]$	Definition of implication	2
4	$[\sim A(p) \vee \sim B(p)] \vee [M(p) \vee N(p)]$	DeMorgan's law	3
5	$[\sim A(p) \vee N(p)] \vee [M(p) \vee \sim B(p)]$	Commutative and associative laws	4
6	$\sim [A(p) \wedge \sim N(p)] \vee [M(p) \vee \sim B(p)]$	DeMorgan's law	5
7	$[M(p) \vee \sim B(p)]$	Disjunctive syllogism	1 & 6
$\therefore$	$\exists z \in D, M(z) \vee \sim B(z)$	Existential generalization	7

Chapter 2

- Predicates – free variable
- Translation of statements
- Multiple quantifiers
- Arguments with quantified statements
  - Universal instantiation
  - Existential instantiation
  - Existential generalization

$\forall a, n \in \mathbf{Z}$, $6 | 2n(3a + 9)$

- Suppose b is an arbitrary, but particular integer that represents a above.
- Suppose m is an arbitrary, but particular integer that represents n above.
- $6 | 2m(3b + 9)$, original
- $6 | 2m \cdot 3(b + 3)$, by algebra (distributive law)
- $6 | 6m(b + 3)$, by algebra (associative, commutative, multiplication)
- Let $k = m(b + 3)$; $k \in \mathbf{Z}$ by closure of **Z** under addition and multiplication
- $6 | 6k$ by definition of divisible
  - ($d | n \leftrightarrow \exists q \in \mathbf{Z}$, such that $n = dq$), where k is the integer quotient

$\forall n \in \mathbf{Z}, (n + n^2 + n^3) \in \mathbf{Z}^{even} \rightarrow n \in \mathbf{Z}^{even}$

- Contrapositive (which is equivalent to proposition)
  - $\forall n \in \mathbf{Z}, n \in \mathbf{Z}^{odd} \rightarrow (n + n^2 + n^3) \in \mathbf{Z}^{odd}$
- Suppose m is an arbitrary, but particular integer that represents n above.
- Let $m = 2k + 1$, by definition of odd, where $m,k \in \mathbf{Z}$
- $(m + m^2 + m^3) = [(2k+1) + (2k+1)^2 + (2k+1)^3]$, by substitution
- $[(2k+1) + (4k^2+4k+1) + (8k^3+8k^2+2k+4k^2+4k+1)]$, by algebra (multiplication)
- $[(2k+1) + (4k^2+4k+1) + (8k^3+12k^2+6k+1)]$, by algebra (addition)
- $[8k^3+16k^2+12k+3]$, by algebra (associative, commutative, addition)
- $[8k^3+16k^2+12k+2+1]$, by algebra (addition)
- $[2(4k^3+8k^2+6k+1)+1]$, by algebra (distributive)
- Let $b = 4k^3+8k^2+6k+1$; $b \in \mathbf{Z}$ by closure of $\mathbf{Z}$ under addition and multiplication
- $(n + n^2 + n^3) = 2b + 1$, which is odd by definition of odd
- Therefore we have shown, $\forall n \in \mathbf{Z}, n \in \mathbf{Z}^{odd} \rightarrow (n + n^2 + n^3) \in \mathbf{Z}^{odd}$, which is equivalent to the original proposition because it is its contrapositive, therefore, the original proposition is true
  - $\forall n \in \mathbf{Z}, (n + n^2 + n^3) \in \mathbf{Z}^{even} \rightarrow n \in \mathbf{Z}^{even}$

Chapter 3

- Proof types
  - Direct
  - Counterexample
  - Division into cases
  - Contrapositive
  - Contradiction
- Number Theory
  - Domains
  - Rational numbers
  - Divisibility – mod/div
  - Quotient-Remainder Theorem
  - Floor and ceiling
  - Sqrt(2) and infinitude of set of prime numbers

$\forall x \in \mathbf{Z}, \forall y \in \mathbf{Q}, (y/x) \in \mathbf{Q}$

$\forall x, y, z \in \mathbf{Z}^{\text{even}}, (x + y + z)/3 \in \mathbf{Z}^{\text{even}}$

$$\forall a,b,c \in \mathbf{Z},\ (a|b \wedge a|c) \rightarrow (b|c \wedge c|b)$$

$$\exists n \in \mathbf{Z},\ (2n^2 - 5n + 2) \in \mathbf{Z}^{\text{prime}}$$

$$\forall n, x \in \mathbf{Z},\ \forall p \in \mathbf{Z}^{\text{prime}},\ p \mid n^x \rightarrow p \mid n$$